II. NOTICES SCIENTIFIQUES

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THE ISOSTATIC REDUCTION AND THE INDIRECT OR BOWIE EFFECT

The discussions on the subject of the indirect isostatic reduction during the General Assembly of the U.G.G.I. in Oslo have made it clear how widely divergent the views on this matter are and an attempt seems, therefore, useful to set forth the principles and problems involved in this matter in a short paper that avoids complicated mathematical formulas. During the war the writer was brought to a study of these problems because of the need he was in to reduce the results of the gravity observations at sea in such a way that they would be comparable to the results obtained on the continents. In discussing these problems we shall also have to touch on some related problems of the isostatic reduction itself.

As it is well known the need for the indirect isostatic reduction is brought about by the fact that the taking away of the topographic masses i. e. of all the masses between the physical surface of the Earth and the geoid and of their isostatic compensation involves a change of the potential field of the Earth and, therefore, a shift of the geoid. The result is that thus there are still topographic masses left with regard to this new position of the geoid, viz. the masses between the original geoid and the new one which according to DE GRAAFF HUNTER's proposal, we shall call the co-geoid.

This confronts us with two problems. In the first place we shall have to apply a new free-air reduction to the gravity values over the distance between both geoids for reducing them to the co-geoid. In the second place we shall have to take away the effect on the gravity values and on the deflections of the vertical of the mass-layer between both geoids. If this last mass-layer should be isostatically compensated we shall also have to take away the effect of this compensation. This last reduction would not, however, constitute an increase of the work but, on the contrary, a simplification as we may assume that the mass-layer under consideration is so gradually changing its thickness that the effects of the layer and of its compensation can safely be supposed to cancel each other. We shall presently come back to this question.

After this short summary of the main problems which, as we shall see, is not yet complete we shall look for a moment at the aims we have in applying the topographic and isostatic reductions giving rise to these problems. These aims are twofold. In the first place the determination of the geoid by means of the theorem of Stokes which makes it not only necessary to remove from the Earth all masses outside the geoid but also desirable to remove as far as possible all irregularly distributed masses because they make the anomaly-field likewise irregular and so the limited number of available gravity-data would too poorly represent this field; this consideration leads to the wish to reduce for the whole topography and for its isostatic compensation.

In the second place we may wish to use the gravity data and the deflections of the vertical for the geophysical study of the Earth, i. e. for the investigation of the way the masses are distributed in the Earth and of the possible deviations from equilibrium. For this purpose we shall likewise wish not only to remove the effects of the topography but also of its compensation, because the latter represents the masses accompanying the topography if the Earth's crust is in hydrostatic equilibrium on the underlying plastic layer. If the Earth would be in equilibrium the gravity values after these reductions would correspond to an equilibrium distribution in the whole Earth which in itself would be an important object for our investigations.

It would obviously be recommandable that no separate reductions were made for these two purposes and that, therefore, the same method would be adopted in both cases for the isostatic reduction as well as for the corresponding indirect reduction. Besides the advantage thus obtained that only one reduction would be required we should avoid the unhappy consequences of two different sets of gravity anomalies which would certainly lead to confusion. We shall afterwards come back to this question.

If this line would be adopted for the isostatic reduction itself, we should have to base it on the hypothesis of hydrostatic equilibrium and not on that of the equality of the masses of the topography and the compensation. For the local anomaly-field this does not make much difference but for the figure of the Earth in its great lines it gives wide-spread deviations that can not be neglected. For the Airy reduction with a thickness T of the rigid crust of 30 km it gives an increase of the anomalies over the oceans with regard to the continents of about 3 mgals and for the Hayford reduction of about 7 mgals. It is true that these figures are of the order of the mean errors in the gravity determinations at sea but they have a systematic character and that makes it necessary to take them into accaunt. The resulting deviations of the geoid may lie between 5 and 10 meters and that seems too much to neglect.

Resuming we may say that the methods for isostatic reduction based on the hydrostatic equilibrium of the Earth's crust must be preferred to those founded on the equality of the masses of topography and compensation, especially for the reduction of the gravity results at sea where these masses are larger and more extensive than on the continents and where, therefore, the effect of the deviation is more serious.

We must, however, realize that this decision has one inconvenience which is often raised as an objection against it. This is the fact that the inequality of the masses of the topography and the compensation results in a change of the total mass of the Earth when applying this reduction. The writer does not think, however, that this objection is serious enough for preventing us from following this line. We can easily determine the total change of mass of the Earth for a special system of isostatic reduction and derive the correction we have, therefore, to apply to the formula for normal gravity. For the Hayford reduction it amounts to a correction of —5.7 mgals and for the Airy reduction to $-0.10\ T + 1.0$ mgals (1), i. e. for $T = 30$ km to —2.0 mgals. For the determination of the geoid there is no need to make any change; as Lambert has recommanded in his valuable paper on *The Reduction of observed values of Gravity to Sea-level* (2) we may well decide that the zero order harmonic term of the shift of the geoid is zero under all conditions or, in other words, that the volume of the geoid is not affected by a change of the mass of the Earth.

Before leaving the subject of the isostatic reduction itself we shall for a moment examine the first order term of the development in spherical harmonics of the topography : the isostatic reduction for this term reveals a peculiar feature as it seems to bring about a shift of the centre of gravity of the Earth. In 1930 LAMBERT (2) has already pointed this out for the isostatic reduction according to the supposition of the equality of the masses of topography and compensation. As this reduction comes to the same as the transfer of the topographic masses towards the inside of the Earth and as for a first order spherical harmonic topography this transfer multiplied by the mass has everywhere a component in the same sense, we at once recognize that a shift of the centre of gravity must occur. Lambert suggests that if this shift is more than a mathematical fiction, the originating of the first order term of the topography must have been due to external forces. We may add that if we apply an isostatic reduction according to the hypothesis of hydrostatic equilibrium we find an even greater shift of the centre of gravity viz. three times more.

(1) In the writer's paper on *The Indirect isostatic or Bowie reduction* in the *Bulletin Géodésique*, N. S., n° 1, a printing error got in in this formula (see p. 76, form. 80).

(2) Walter D. LAMBERT. The reduction of observed values of gravity to sea-level, *Bulletin Géodésique*, n° 26, 1930 (p. 159 sqq).

By means of the following reasoning, for the details of which the writer may refer to § 4 of his paper in the *Bulletin Géodésique*, N. S., n° 1 (1), we can conclude that such a shift does not occur. Basing us on the Airy system of compensation we can determine the position of the geoid in a theoretical Earth composed of a core of plastic matter with a density equal to that of the substratum and a crust of normal density having a thickness varying over the surface as a first order spherical harmonie or, in other words, bounded on the in- and outside by two not-concentric spheres (see fig. 1).

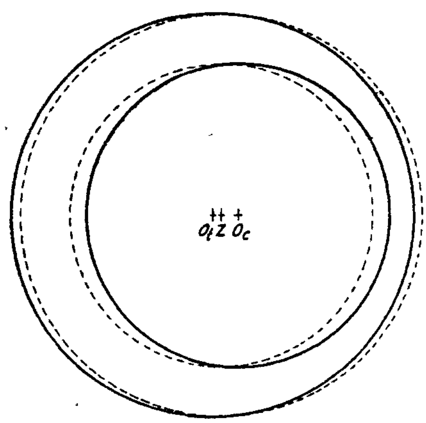

Fig. 1

It is simple to prove that the geoid of such an Earth is again a sphere with its centre coinciding with the centre of gravity. The same result is found for the somewhat more complicated case that the inside of the Earth is constituted in the same way as for the actual Earth with an increasing density towards the centre.

If we should now determine the height of the compensation by assuming it to be bounded by a concentric sphere with a radius equal to that of the geoid minus the mean thickness T of the crust we would arrive at the remarkable result that the compensation would neither correspond to the hypothesis of hydrostatic equilibrium of the crust nor to that of the equality of the topography and ths compensation.

Since 1942 when he pointed this out in his above-mentioned paper, the writer found the explanation of this queer result. Besides another effect which we shall not deal with here is caused by the fact that

(1) In formulas 82 and 83 of this paper a printer's error occurs, the + sign has to be changed into a — sign.

the potential surface at a depth T which may be considered to be the boundary of the compensation is not concentric with the geoid. Adopting this boundary the compensation corresponds exactly to the assumption of hydrostatic equilibrium. We thus find three different masses all distributed according to a first order spherical harmonic, viz. the topography, the compensation and a crustal layer comprised between a sphere at a depth T below the geoid and the potential surface at the same mean depth. The presence of this last layer brings back the centre of gravity which, as we have seen above, would undergo a shift if only the topography and the compensation were present.

We may conclude that we can only find a satisfactory view-point corresponding to the gravity-centre remaining at its place or, in other words, to the geoid having no first order spherical harmonic with regard to that centre, by basing ourselves on the physical background of the principle of isostasy, viz. the principle of hydrostatic equilibrium combined with the presence of an uncompensated layer caused by the potential surfaces. This layer is obviously held in its place by the presence of the masses of topography and compensation. We see that we thus have already been brought to deal with problems of the indirect reduction.

We shall now take up these problems for all the other spherical harmonic terms of the topography. We have mentioned that besides the topography and its compensation according to the hypothesis of hydrostatic equilibrium we have a mass-layer between the geoid and the co-geoid and the writer is convinced that this layer likewise is not compensated. We must again assume this layer to be held in its place by these masses themselves and not by some hydrostatic balance with regard to the co-geoid which in the actual Earth has no physical meaning; this co-geoid can, therefore, play no part in the equilibrium conditions of the existing Earth.

The non-compensated mass-layer between the geoid and the co-geoid is not the only effect of the presence of the topography and the compensation; these masses must also cause a shift of the internal equipotential surfaces of the Earth and as these surfaces determine the distribution of the density inside the Earth, this must likewise bring about extra masses, positive and negative, in the Earth. One instance of such a layer is already found at the lower boundary of the crust in the same way as it has already been mentioned above for the first order spherical harmonic (1). It is clear that for these extra

(1) The uncompensated crustal layer present for the first order spherical harmonic term may in fact be considered as a combination of two layers which in this case can be treated as a single one; for one part it is a layer between the geoid and a co-geoid and for the second part a layer between the undisturbed potential surface at a depth T and this same potential surface as shifted by the topography and the compensation.

masses there can not be any question of isostatic compensation and this may perhaps give a further indication in favour of the view-point of non-compensation of the layer between the geoid and the co-geoid which, in fact has the same character.

Resuming the writer thinks that there are four groups of masses which disturb the regular mass-distribution of the Earth, the topographic masses, the isostatic compensation, the masses between the geoid and the co-geoid and the inside masses brought about by the shift of the internal equipotential surfaces. All these masses affect the potential field of the Earth and so the shifts of the geoid (1) and of the internal equipotential surfaces are a function of them all. It is possible to determine them separately for each spherical harmonic of the topography and then to derive an approximate formula for the sum of all these terms. We thus may obtain the total result for the shift of the geoid and for the indirect reduction to gravity. For these resulting formulas the writer may refer to his paper in the *Bulletin Géodésique,* N. S., n° 1, 1946. They allow a quick numerical computation for special cases.

It may be of interest to point out that these complications in deriving the equilibrium of the crust with its topography floating on the plastic Earth are in theory also present in the simple case of a body floating on water. It is usually assumed that the conditions of equilibrium in this case are represented by the law of Archimedes but it is seldom realized that this law in the simple shape usually employed is only an approximation. It may be true that for small dimensions it is a close approximation but still we may recognize that also in this case the Newtonian attraction brings about a shift of the equipotential surfaces and that, therefore, the formulas for the hydrostatic equilibrium are more complicated than those usually applied. We need not be surprised that for large topographic features on the Earth's surface these deviations become appreciable and the maximum deviation for the first order spherical harmonic of the Earth's topography amounts to several meters.

For giving an idea about the dimensions of this last deviation we shall give the figures for the shift of the gravity-centre that would occur if in applying the isostatic reduction we should only take into account the topography and the compensation. For the two hypothesis viz. for the assumption of equality of the masses of the topography and the compensation and for the supposition of hydrostatic equilibrium we find the figures given below. For their deduction we have availed ourselves of the development in spherical harmonics of the topography published by Prey in 1922 (2); the first order term of this

(1) We shall continue to indicate the shifted geoid as the co-geoid.
(2) A. Prey, Darstellung der Höben und Tiefenverhältnisse der Erde, *Abh. Ges. d. Wiss. Göttingen, Math. Phys. Kl.,* N. F. Bd. XI, 1, 1922.

development shows a maximum topography of 1204 m. at a point in the Black Sea at $\varphi = 43°57'$ N and $\lambda = 31°5'$ E and the same negative value at its antipode. Using this value we find for the shift of the gravity-centre which would occur in the same direction viz. towards the Black Sea.

	hydrostatic equilibrium	equality of mass
Airy reduction, T = 30 km.....	8.2 m	2.8 m
Hayford reduction.............	15.6 m	5.2 m

We see that none of these values is negligible.

Another group of figures worth while mentioning are those relative to the total layer between the geoid and the co-geoid, i. e. the sum of the figures for all the spherical harmonic terms. We find a maximum thickness of this layer of 32 m. in Central Asia at about $\varphi = 43°$ N and $\lambda = 80°$ E and another high value of 30 m. in the Andes at $\varphi = 15°$ S and $\lambda = 70°$ W. A minimum value of —11 m. occurs in the Indian Ocean east of Madagascar at $\varphi = 20°$ S and $\lambda = 52°$ E and three low values of —10 m. in the North Pacific at $\varphi = 40°$ N and $\lambda = 180°$ and in the Atlantic at $\varphi = 0°$ and $\lambda = 5°$ W and at $\varphi = 20°$ N and $\lambda = 40°$ W. These figures may give us an idea about the size of the non-compensated layer between the geoid and the cogeoid which accompanies the topography and its compensation.

A number of geodesists among whom I may mention Dr. DE GRAAFF HUNTER, the Brigadier BOMFORD and the R. P. Pierre LEJAY object to the complicated reductions mentioned above. They have pointed out that if we restrict ourselves to the problem of the determination of the geoid by means of Stokes' theorem, we can solve it more simply and obtain a higher accuracy.

There is no doubt that this is true. Instead of applying the isostatic reduction according to the principle of hydrostatic equilibrium we can assume the exact equality of the masses of topography and compensation which leads to the simplification that the reduction for both groups of masses means only a transfer of the topographic masses inside the geoid as it is wanted for Stokes' theroem. This has the advantage of leaving the total mass of the Earth unchanged. It is true that the masses between the geoid and the co-geoid are partly left outside the geoid and so they have also to be removed but we may likewise push them inside the geoid. As we have already remarked in the beginning this does not bring about much change for the gravity anomalies as this layer has a very regular thickness without any abrupt changes and so LEJAY (1) is no doubt right in neglecting the corresponding reduction of gravity. Of course the free air reduction over the distance between geoid and co-geoid remains necessary.

DE GRAAFF HUNTER and BOMFORD (2) combine the reduction for the masses between geoid and co-geoid with that for the topography and the compensation and for the same reason as mentioned above we can overlook the fact that the thickness of the layer they take into account deviates somewhat from the real value as derived before in this paper because they compute it by assuming this layer to be isostatically compensated.

As, however, this layer is not compensated the transfer of it inside the geoid brings extra masses there, which generally speaking, tend to be positive for the continents and negative for the oceans. The substitution for the isostatic reduction of the principle of equal masses of topography and compensation to that of hydrostatic equilibrium means also an excess of mass of positive sign in the continents and so both effects deviate in the same sense from the hypothesis of equilibrium which we have assumed for our deductions and considerations of this paper.

The geoid derived from the isostatic anomalies thus obtained differs from that found along the lines developed in this paper on the assumption of equilibrium. It is clear, however, that if we apply to this geoid the correction corresponding to the removal of the topography and compensation we must find the same result for the actual geoid as it exists in the Earth as we would have found if we should correct the geoid obtained along the lines of this paper for the effect brought about by the removal of the masses mentioned in that connection, i. e. the topography, the compensation, the layer between the geoid and the co-geoid and the inside layers caused by the shift of the internal equipotential surfaces.

So, for the determination of the actual geoid the way recommended by DE GRAAFF HUNTER, BOMFORD or LEJAY must give the same final result and as their way of reduction of the available gravity values will no doubt make them just as widely representative as the method of reduction given by the writer and as their tables can be more accurate because the formulas are simpler and have indeed already been derived with great accuracy by LAMBERT and DARLING (3), it is clear that many are inclined to prefer this method. Still the writer thinks that the drawbacks are so serious that we must

(1) R. P. Pierre LEJAY, Développements modernes de la Gravimétrie, *Paris*, Gauthier-Villars, 1947 (p. 130).
(2) *Bulletin Géodésique*, n° 29.
(3) Walter D. LAMBERT and Fred. W. DARLING, Tables for determining the form of the geoid and its indirect effect on gravity, *Special Publication of the Coast and Geodetic Survey*, n° 199, 1936.

prefer the more complicated method indicated in this paper which, moreover, in practice does not make much difference for the computations while it gives sufficient accuracy. Because of the importance of this question he wants to go somewhat deeper here into it than he did in the beginning of this paper.

There can not be any doubt about the fact that because of the wide-spread masses added inside the geoid to the system of masses which would be present according to the equilibrium conditions the system of anomalies must show wide-spread disturbances which may certainly attain to values of more than 5 mgals for the Airy reduction and to more for the Hayford reduction. As the masses are connected with the topographic features of great extent on the Earth's surface the same must be true for these disturbances and so we can not e. g. use the anomalies for comparative studies between the gravity fields in continents and oceans. Nor can we employ them for accurate studies about the general problem of the figure of the Earth in its principal lines, i. e. about the question of a longitude term and for the deriving of the flattening as caused by the distribution of the densities inside the Earth. Neither can we thus satisfactorily investigate the question whether there still are wide-spread temperature deviations in the Earth as would be revealed by extensive fields of gravity anomalies at the surface; the writer thinks this to be the case e. g. along the eastern and south-eastern border of the Asiatic continent. We may add here the problem whether there are convection-currents of great extent going on in the Earth which likewise could hardly be attacked by means of this gravity material. In short we may say that no phenomena that may be expected to cause gravity anomalies of small value but of great horizontal extent could thus well be investigated.

It seems hardly acceptable that if the gravity results would be present they would not be employed for such studies and so we may be sure that at one time or another these results would indeed be subjected to the reductions needed for attacking these problems. But in that case the application of the other method for the purpose of determining the geoid by means of Stokes' theorem would simply mean double work. Besides it would create two sets of gravity anomalies over the Earth for the same method of isostatic reduction and this would no doubt cause confusion.

Resuming the writer should wish to lay great emphasis on the desirability to keep to one system of indirect reduction and one assumption about the isostatic compensation and to choose for both the base of hydrostatic equilibrium (1).

(1) In his valuable book on *Développements modernes de la Gravimétrie*, the R. P. LEJAY in dealing with the isostatic reduction recognizes the disadvantage from a physical viewpoint of the assumption of equality of mass of topography and compensation with regard to the assumption of hydrostatic equilibrium but he draws attention to the fact that the converging of the erosion or sedimentation, brings about vertical movements for the readjustment

Before closing these considerations we must still touch on one more subject which has hitherto been neglected; the problem has already been raised by LAMBERT in his paper of 1930 in the *Bulletin Géodésique* n° 26. It concerns the changes of the second order spherical harmonic terms brought about by the removal of the topography and the compensation. This involves a change of the moments of inertia of the Earth which must in two ways affect its kinematic properties; it must bring about a change in the speed of rotation and, therefore, of the flattening and it must affect the position of the polar axis. By using PREY's figures for the second order spherical harmonic terms and applying HAYFORD's isostatic reduction for a depth of compensation of 100 km, LAMBERT finds for the angular displacement of the north pole a value of 60″.6 along the meridian of 132° W. The question is whether we have to take this into account. The writer thinks we can answer this question in the negative as long as we are occupied by our actual problems regarding the Earth. We know that for the kinematic properties of the present Earth the masses of topography and compensation have to be included and so the equilibrium figure of the Earth must apply to these circumstances. The removal of topography and compensation is a theoretical abstraction which does not change those kinematic conditions.

As far as the writer can see this problem would only arise when we should require a study of the changes of the polar axis and of the flattening brought about by the development of the complete topography on the Earth's surface. As this development has no doubt taken place during the whole age of the crust and as we may well suppose that during that time other mass-changes yet unknown have also occurred it does not seem likely that the solution of this problem has much practical value.

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verticals introduces lateral stresses if a change in the topography, e. g. by of the hydrostatic equilibrium. The complications and uncertainties thus involved for the physical picture reconcile him with the assumption of equality of mass which he otherwise prefers for its greater simplicity.

Although the writer quite agrees to Lejay's conclusion of lateral effects in the crust he does not think that this ought to lead us towards giving up the principle of hydrostatic equilibrium. It is clear that one of the causes of gravity anomalies can be found in lateral compression of the crust which by its vaulting effect must disturb the floating balance fo the crust it is important first to take away all effect of normal hydrostatic equilibrium in order to be able to study this and other disturbing effects.