PARAMETERS OF COMMON RELEVANCE OF ASTRONOMY, GEODESY, AND GEODYNAMICS

by B.H. Chovitz, former president of SSG 5.100

The 1983 IAG General Assembly charged SSG 5.100 to :

1. Review the most current information available on the values of physical quantities which are fundamentally important in the fields of astronomy, geodesy, and geodynamics.

2. Reconcile this information into a uniform, compatible set.

3. Recommend to the 1987 General Assembly the publication of a set of most up-to-date representative values.

4. Consider the status of the Geodetic Reference System 1980, and if any changes in this reference system should be recommended to the General Assembly.

Since only "parameters of common relevance" are to be considered, these are restricted to those which refer to the Earth as a ***body.*** Thus, those which relate only to the Earth's center of mass, or, equivalently, do not distinguish between topocentric and geocentric orientation are not discussed. Thus eliminated are astronomical parameters like the astronomical unit, star catalog information, and ephemerides of the Sun and Moon.

Furthermore, only parameters relating to the Earth, as opposed to corresponding ones for the Moon and planets, are reviewed, because the latter are presently only marginally relevant to geodesy. The parameters expressing the gravitational field of the Moon affect the determination of geodetic data from lunar ranging, but an up-to-date list can be found in Project MERIT Standards (1983), pp. 11–12.

The International Astronomical Union (IAU) at its 19th General Assembly in November 1985 authorized (Resolution C1) the formation of a working group to review the system of astronomical constants "in collaboration with the appropriate special study group of the International Association of Geodesy", which is, of course, SSG 5.100. It is essential that these two groups complement, rather than compete with, each other. Therefore, this report will concentrate on those parameters which are primarily geodetic in nature, in particular, the parameters which constitute the traditionally accepted definition of a geodetic reference system (GRS), and, furthermore, include geodynamical considerations by examining their time-varying aspects. It is hoped that a more comprehensive set of parameters, encompassing astronomical, planetary, relativistic, etc., domains, can be issued later as a joint venture of both study groups.

I. Time-invariant values

1. Angular velocity of the Earth's rotation, ω.

Extending the annual sequence of values for ω displayed in the previous SSG survey (Rapp, 1983) obtained from the annual reports of the Bureau International de l'Heure :

year	ω
1978	$7\,292\,114.903\,(10^{-11})$ rad/s
1979	4.925
1980	4.952
1981	4.964
1982	4.964
1983	4.954
1984	5.019
1985	5.025

The change in ω over this 8 year span is (10^{-12}) rad/s. Since seasonal variations are of the same order of magnitude, this reinforces the longstanding IAG decision to present a fixed value of ω to no more than 7 significant digits. Thus :

$$7.292\,115\,(10^{-5})$$

is recommended for retention as the representative value.

2. Geocentric gravitational constant, GM .

The most productive source of information for determination of GM has been Lageos. Reduction of observations by NASA Goddard Space Flight Center (D. Smith, personal communication, November 1986) and by the University of Texas (Tapley et al., 1985) has yielded

$$3\,986\,004.40 \pm .02\,(10^8)\,\mathrm{m}^3/\mathrm{s}^2$$

Lunar laser ranging (LLR) provides another source of data, but these observations are referred to a coordinate system with origin at the Earth-Moon barycenter, and subject to a relativistic correction to transfer to a geocentric origin of $(+.06)\,(10^8)\,\mathrm{m}^3/\mathrm{s}^2$ (Martin et al., 1985). The most recent determination of GM by the Jet Propulsion Laboratory (JPL) based on LLR data, and shifted from the barycentric to the geocentric origin is (J. Dickey, personal communication, January 1987)

$$3\,986\,004.43 \pm .06\,(10^8)\,\mathrm{m}^3/\mathrm{s}^2$$

The above two values are compatible since the error bar range of the latter includes that of the former. Therefore, as the current representative value the Lageos determination has been chosen, but with a slightly more conservative standard error which overlaps the LLR estimate :

$$3\,986\,004.40 \pm .03\,(10^8)\,\mathrm{m}^3/\mathrm{s}^2$$

Of course, in accordance with the GRS 80 convention, this includes the atmospheric contribution.

3. Equatorial radius, a .

As implied in Rapp (1983), improvement in the determination of a depends more on refining its definition than in better measurement – a consequence of this parameter lacking the natural relationship to physical reality enjoyed by the other three of the Geodetic Reference System. (In this respect, the potential of a particular level surface would serve as a more complementary substitute.) Some additional information on the size of a has accumulated over the past 4 years, but the uncertainties associated with these determinations have not decreased, due mainly to the difficulty in precisely relating the ellipsoidal model to the geoid. The recent **NASA** Goddard **GEM-T1** satellite solution for the Earth's gravitational field (**J. Marsh**, personal communication, November, 1986) yielded

$$6\,378\,137.8 \ \pm 2.6 \text{ m}$$

based on comparison of geoidal and ellipsoidal heights at 30 tracking stations. A combination of Doppler data and gravimetry at the Ohio State University (**R. Rapp**, personal communication, November, 1986) resulted in

$$6\,378\,136.2 \text{ m}$$

estimated to be good to ± 0.5 to ± 1 m .

Another possible source of evidence for a is the global mean equatorial value of terrestrial gravity, γ . To the first-order, a is a function of both GM and γ . Since GM is known to two orders of magnitude better than a , its value can be fixed at the representative value listed under para. 1., leaving a to vary just with γ . This yields the following tabular relationship :

γ (mgal)	a (m)
978 033.1	6 378 135
2.8	6
2.5	7
2.2	8

Rapp (ibid.) has supplied two recent values of γ , one from terrestrial data, and one utilizing comparisons with altimetric data found in **Rapp** (1986). The first reduces to

$$(978\,032.4 \ + \ c) \text{ mgal}$$

where $0 < c < 1.5$ represents an imperfectly determinable terrain correction. The second furnishes

$$978\,032.6 \text{ mgal}$$

with an unstated standard error. These values of γ indicate that they are hardly reliable to 7 significant figures and thus cannot yield definitive information to what is already known about a . However, they are sufficiently consistent with each other to lend credence to a value for a of 6 378 136 m (**Rapp**, 1983) or the **GRS 80** value of 6 378 137 m . Since there is no real evidence to support a change in the 1983 recommendation, we retain as the current representative value

$$6\,378\,136 \ \pm 1 \text{ m}$$

However, it has been rewritten to display only seven significant figures, in order to emphasize the limits of our present information. The most promising approach over the next quadrennium for improving knowledge of a should be utilization of new altimetry data along with the means of filtering out the sea surface topography at the 10 cm level.

4. Second-degree zonal harmonic coefficient, J_2 .

The most recent comprehensive satellite-only solution for the Earth's gravitational field is GEM-T1 (Marsh, ibid.), which produced for J_2, corresponding to a scale of $a = 6\,378\,136\,\mathrm{m}$:

$$(1\,082\,625.6 \pm 0.9)(10^{-9})$$

The relation between J_2 and a is given by

$$dJ_2/da = -0.3\,(10^{-9})\,\mathrm{m}^{-1}$$

The uncertainty in a will slightly increase the formal uncertainty in the above value of J_2 to ± 1.0.

The change in J_2 to GEM-T1 from the previously best accepted Goddard model, GEM-L2, quoted in Rapp (1983), is $-3.2\,(10^{-9})$. Although GEM-T1 is deemed the superior model, principally because of its more comprehensive modelling of ocean tides, it is definitely capable of improvement by addition of altimetric and terrestrial gravity data. These changes could very well affect the seventh significant digit in J_2. Therefore, a current representative value of J_2 is conservatively taken to be

$$(1\,082\,626 \pm 2)(10^{-9})$$

In accordance with the accepted GRS definition, the zero-frequency tidal effect on J_2 of $+9.3\,(10^{-9})$ is excluded from the above value.

II. Time-varying values

This section will be devoted to consideration of the GRS parameters in the form

$$p = p_0 + \dot{p}\,t$$

where p represents ω, GM, a, or J_2; t is time elapsed from some epoch, and $\dot{p}$ is a constant rate of change for the parameter p. The only other sort of time variation for which there is firm evidence is periodicity in ω which will be discussed under that heading.

1. $\dot{\omega}$.

As mentioned before, periodic changes affect the eighth significant digit of ω. Previous SSG reports have stated that these do not affect most geodetic applications. However, they do entail important consequences in geodynamical processes. There are an extremely complex and very wide range of frequencies : daily, seasonal, annual, multi-year, decadal, etc. A great deal of fresh evidence has accumulated over the past several years on the shorter periods especially from VLBI observations. The annual reports of the Bureau International de l'Heure remain the best and most comprehensive source of data on this subject. A recent general review of the causes and consequences of periodic variations in ω can be found in Dickey and Eubanks (1985). It is felt that it would be premature for this SSG to attempt to extract and generalize from these and other reports, so this discussion will be confined to the secular variation only.

The secular change in ω consists of two parts : a tidal deceleration, $\dot{\omega}_T$, and a non-tidal acceleration, $\dot{\omega}_{NT}$, the possible causes of which are discussed in Lambeck (1980). Determination of $\dot{\omega}_T$ are derived from observations of $\dot{n}$, the change in the Moon's mean motion, or from Earth-satellite solutions for tidal components. The prime source of recent information on $\dot{\omega}_{NT}$ has been $\dot{J}_2$, which reflects what currently may be the main cause of $\dot{\omega}_{NT}$: post-glacial backsurge in the Earth's mantle, affected by the gravitational attraction of the core. Total $\dot{\omega}$ is measured directly, and can be inferred from geological evidence and astronomic records. Thus there are three separate sources of data connecting the equation

$$\dot{\omega} = \dot{\omega}_T + \dot{\omega}_{NT}$$

providing a check on the reliability of the data and the underlying hypotheses.

Recent studies of $\dot{\omega}$ have been carried out at JPL and Goddard. The former (Yoder et al., 1983, revised by Dickey, personal communication, 1987) obtained

$$\dot{\omega}_T = -6.0 \pm 0.3\,(10^{-22})\,\mathrm{rad/s^2}$$

$$\dot{\omega}_{NT} = +1.5 \pm 0.2\,(10^{-22})\,\mathrm{rad/s^2}$$

The first is based on $\dot{n} = (-24.9 \pm 1.0)''/\mathrm{cy}^2$ obtained from LLR data (Newhall et al., 1986). The second is due to Morrison and Stephenson (1982) derived from long-term astronomic data.

The Goddard (Christodoulidis et al., 1987) values are

$$\dot{\omega}_T = -5.98 \pm 0.22\,(10^{-22})\,\mathrm{rad/s^2}$$

$$\dot{\omega}_{NT} = +1.29 \pm 0.28\,(10^{-22})\,\mathrm{rad/s^2}$$

The first is based on a detailed tidal analysis derived from a multi-Earth-satellite solution, and the second utilizes $\dot{J}_2 = -2.8\,(10^{-11})/\mathrm{yr}$ (Rubincam, cf. para. 4 below).

Another recent solution for $\dot{\omega}_{NT}$ is by Burša (1986a) who obtains $+1.2\,(10^{-22})\,\mathrm{rad/s^2}$ based on $\dot{J}_2$ of $-2.6\,(10^{-11})/\mathrm{yr}$.

These results yield confidence that the components of $\dot{\omega}$ are now known to better than 5 in the second significant figure. Thus current representative values are assigned as

$$\dot{\omega}_T = -6.0 \pm 0.3\,(10^{-22})\,\mathrm{rad/s^2}$$

$$\dot{\omega}_{NT} = +1.4 \pm 0.3\,(10^{-22})\,\mathrm{rad/s^2}$$

yielding the composite result

$$\dot{\omega} = -4.6 \pm 0.4\,(10^{-22})\,\mathrm{rad/s^2}$$

It should be noted that this magnitude is 15 % smaller than the value given in Lambeck (1980).

The validity of the value and standard error assigned to $\dot{\omega}_{NT}$ should be understood as being limited by the following considerations. On the one hand, inter-decadal (10 to 100 years) fluctuations which can be an order of magnitude larger are supposed to be removed; on the other hand, the best source of evidence, $\dot{J}_2$, refers to a phenomenon (post-glacial rebound) which holds for a period of less than 10^5 years.

Quoting from **Morrison** and **Stephenson** (1982) who refer to their estimate of $\dot{\omega}_{NT}$ given above, "This value ... is clear evidence of a long-term, non-tidal acceleration component in the Earth's rotation which probably arises from a fractional change of the opposite amount in the moment of inertia. This is the average value over 2500 years and it is not necessarily constant over the intervening period."

2. $\dot{G}$

The scientific interest in change in GM centers on $\dot{G}$, because of its wide ramifications in physics, and because no evidence has turned up for any discernible change in $\dot{M}$. Based principally on Mars Viking Lander data, both **Reasenberg** (1983) and **Hellings** et al., (1983) have separately determined values that correspond at the level of

$$\dot{G}/G < \pm 1\,(10^{-11})/\mathrm{yr}$$

These values are compatible with $\dot{G} = 0$, representing upper bounds on $\dot{G}$. The uncertainty is due mainly to uncertainties in knowledge of the masses of the asteroids.

A non-zero value of $\dot{G}$ would affect $\dot{\omega}_{NT}$. **Lambeck** (1979) gives the relation, based on eclipse data

$$\dot{\omega}_{NT}/\omega = -1.8\,\dot{G}/G$$

yielding, for given $\dot{\omega}_{NT} = 1\,(10^{-22})\,\mathrm{rad/s^2}$,

$$\dot{G}/G = -2.4\,(10^{-11})/\mathrm{yr}$$

This is near the bound stated above for $\dot{G}/G$ from Viking data, which is not affected by $\dot{\omega}_{NT}$. **Lambeck** (1980) obtains from differencing $\dot{\omega}$ and $\dot{\omega}_T$

$$\dot{\omega}_{NT} = +1.6\,(10^{-22})\,\mathrm{rad/s^2} \quad \text{if } \dot{G} = 0$$
$$= +0.5\,(10^{-22})\,\mathrm{rad/s^2} \quad \text{if } \dot{G} \neq 0\,.$$

This difference appears to correspond to $\dot{G}/G = 2.4\,(10^{-11})/\mathrm{yr}$ given above. Since the discussion under $\dot{\omega}$ indicated that $\dot{\omega}_{NT}$ due to $\dot{J}_2$ alone is $+1.4\,(10^{-22})\,\mathrm{rad/s^2}$, this puts a further constraint on the size of $\dot{G}$.

In summary, there is no real evidence at present that $\dot{G} \neq 0$.

3. $\dot{a}$

Lambeck (1979) lists $\dot{a}$ as one of the possible generators of $\dot{\omega}_{NT}$, and deduces that $\dot{a} = -1$ mm/yr corresponds to $\dot{\omega}_{NT} = (10^{-22})\,\mathrm{rad/s^2}$. Since this is the approximate size of $\dot{\omega}_{NT}$ which is caused by other factors, primarily $\dot{J}_2$, this imposes an upper limit on the possible change in size to less than 1 mm/yr.

Burša (1984) shows that the Earth's principal moment of inertia cannot be increasing, based on derivations from the values of $\dot{n}$ and $\dot{\omega}$ determined from LLR, and as implied in the discussion of $\dot{\omega}_{NT}$.

Thermal evolution models of the Earth (**Christensen**, 1985) indicate a rate of cooling of about $0.1\,^\circ\mathrm{C/my}$. From this can be inferred a maximum $\dot{a}$ of less than $-.01$ mm/yr.

Thus no evidence appears to exist for a non-zero value of $\dot{a}$.

It should be acknowledged, however, that some geophysicists have held a different view; cf., e.g., **Wesson** (1975) and **Carey** (1976).

4. $\dot{J}_2$

The principal results have again been obtained at JPL and Goddard. From **Yoder** et al., (1983) :

$$\dot{J}_2 = -3.1\ (10^{-11})/\mathrm{yr} \quad \text{based on glaciation rebound of } 2\,800\ \mathrm{yr}$$

Dickey (personal communication, January 1987) states that a revised solution is underway which is anticipated to fall between -2.6 and $-3.1\ (10^{-11})/\mathrm{yr}$.

From **Rubincam** (1984) :

$$\dot{J}_2 = -2.6 \pm .6\ (10^{-11})/\mathrm{yr}$$

Rubincam's latest result (personal communication, November 1986) is

$$\dot{J}_2 = -2.8 \pm 0.3\ (10^{-11})/\mathrm{yr}$$

Since this corresponds very well with the latest JPL results, it is recommended as the current representative value.

III. Related results

In this section is covered related results pertaining to the GRS coordinate system and derived parameters, mostly due to studies by **Milan Burša** over the past 4 years.

Burša and **Sidlichovsky** (1985) show that $\dot{J}_2 = -3\ (10^{-9})/\mathrm{cy}$ affects the Eulerian period of polar motion by $+70$ s/cy and the amplitude by about -2 cm/cy.

Burša (1986b) quotes **Vondrak** (1985) that the secular motion of the pole based on astrometric observations during 1900 to 1984 has been .0033″/yr along the meridian 281.8° E with reference to the CIO pole.

Burša (1985) presents an interesting table on how the Earth's moments of inertia and the flattening, f, vary over the period covering -10^6 to $+4\,(10^5)$ yr with respect to the present. For example, $1/f$ ranges from 294.7 to 299.7. (In doing this, however, he extrapolates the currently known value for $\dot{J}_2$ to apply over that entire time interval.)

Burša (1986c) computes $\dot{C}$, the secular variation in the Earth's principal moment of inertia, to be :

$-3.8 \pm 0.6\,(10^{29})\ \mathrm{kg\,m^2/cy}$	from the constraint of angular momentum balance of the Earth-Moon-Sun system

$$-4.2 \pm 0.9\ (10^{29})\ \mathrm{kg\ m^2/cy} \qquad \text{from observed } \dot{J}_2$$

Also, from $\dot{J}_2$ he derives

$$(\dot{1/f}) = -4.1\ (10^{-9})/\mathrm{cy}$$

This last value is also affected to a lesser extent by the secular motion of the pole, and to an even lesser extent by $\dot{\omega}$.

Nagy (1985) has compared the GRS 1980 gravity formula with a revision obtained by substituting the representative values given by Rapp (1983), and shows that there is no practical difference.

Finally, detailed discussions on the distinction between the barycentric and geocentric coordinate systems as a result of the relativistic effect can be found in Hellings (1986), Martin et al., (1985), and Ashby and Bertotti (1986).

IV. Summary

It is recommended that a current set of representative values for the GRS parameters now include their secular changes. The following list is submitted :

$\omega = 7.292\,115 \pm 0.1\ (10^{-5})\ \mathrm{rad/s}$

$\dot{\omega}_T = -6.0 \pm 0.3\ (10^{-22})\ \mathrm{rad/s^2}$

$\dot{\omega}_{NT} = +1.4 \pm 0.3\ (10^{-22})\ \mathrm{rad/s^2}$

$GM = 3\,986\,004.40 \pm 0.03\ (10^8)\ \mathrm{m^3/s^2}$

$\dot{G}/G = 0 \pm 1\ (10^{-11})/\mathrm{yr}$

$a = 6\,378\,136 \pm 1\ \mathrm{m}$

$\dot{a} = 0$

$J_2 = 1\,082\,626 \pm 2\ (10^{-9})$

$\dot{J}_2 = -2.8 \pm 0.3\ (10^{-11})/\mathrm{yr}$

It is not recommended that any change be made at present in GRS 80. However, serious consideration should be given over the next quadrennium to revising GRS 80 in two ways :

1. Establish time-varying rates (with an epoch) where warranted (J_2 being most likely).

2. Define and fix a reference coordinate system most compatible with modern measurements. (This might be most appropriately considered by the new Earth Rotation Service.)

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