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REFERENCE COORDINATE SYSTEMS AND FRAMES : CONCEPTS AND REALIZATION

Abstract

Geodynamics has become the subject of intensive international research during the last decade, involving plate tectonics, both on the intra–plate and inter–plate scale, i.e., the study of crustal movements, and the study of earth rotation and of other dynamic phenomena such as the tides. Interrelated are efforts improving our knowledge of the gravity and magnetic fields of the earth. A common requirement for all these investigations is the necessity for a well–defined reference coordinate system (or systems) to which all relevant observations can be referred and in which theories or models for the dynamic behavior of the earth can be formulated. In view of the unprecedented progress in the ability of geodetic observational systems to measure crustal movements and the rotation of the earth, as well as in theory and model development, there is a great need for the theoretical definition, practical realization, and international acceptance of suitable coordinate system(s) to facilitate such work. This article deals with certain aspects of the establishment and maintenance of such a coordinate system.

Ideal and Conventional Reference Systems and Frames

In order to clarify some of the conceptual aspects of various reference systems and frames, we propose to use specific terms proposed in [Kovalevsky and Mueller, 1981] that have been used somewhat inconsistently in the past.

The purpose of a reference frame is to provide the means to materialize a reference system so that it can be used for the quantitative description of positions and motions on the earth (terrestrial frames), or of celestial bodies, including the earth, in space (celestial frames). In both cases the definition is based on a general statement giving the rationale for an ideal case, i.e. for an ***ideal reference system.*** For example, one would have the concept of an ideal terrestrial system, through the statement that with respect to such a system the crust should have only deformations (i.e., no rotations or translations). The ideal concept for a celestial system is that of an inertial system so defined that in it the differential equations of motion may be written without including any rotational term. In both cases the term "ideal" indicates the conceptual definition only and that no means are proposed to actually construct the system.

The actual construction implies the choice of a physical structure whose motions in the ideal reference system can be described by physical theories. This implies that the environment that acts upon the structure is modeled by a chosen set of

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parameters. Such a choice is not unique : there are many ways to model the motions or the deformations of the earth ; there are also many celestial bodies that may be the basis of a dynamical definition of an inertial system (moon, planets, or artificial satellites). Even if the choice is based on sound scientific principles, there remains a part of imperfection or arbitrariness. This is one of the reasons why it is suggested to use the term "conventional" to characterize this choice. The other reason is related to the means, usually conventional, by which the reference frames are defined in practice.

At this stage, there are still two steps that are necessary to achieve the final materialization of the reference system so that one can refer coordinates of objects to them. First, one has to define in detail the model that is used in the relationship between the configuration of the basic structure and its coordinates. At this point, the coordinates are fully defined, but not necessarily accessible. Such a model is called a ***conventional reference system.*** The term "system" thus includes the description of the physical environment as well as the theories used in the definition of the coordinates. For example, the FK4 (conventional) reference system is defined by the ecliptic as given by Newcomb's theory of the sun, the values of precession and obliquity, also given by Newcomb, and the Woolard theory of nutation. Once a reference system is chosen, it is still necessary to make it available to the users. The system usually is materialized for this purpose by a number of points, objects or coordinates to be used for referencing any other point, object or coordinate. Thus, in addition to the conventional choice of a system, it is necessary to construct a set of conventionally chosen (or arrived at) parameters (e.g., star positions or pole coordinates). The set of such parameters, materializing the system, define a ***conventional reference frame.*** For example, the FK4 catalogue of over 1500 star coordinates define the FK4 frame, materializing the FK4 system. Another example is the BIH Conventional Terrestrial Frame, whose pole is the origin of the polar motion derived (and published) by the BIH, and whose longitude origin is the Greenwich Mean Astronomical Meridian, in reality the point on the equator of the above pole, used by the BIH for deriving UT1. This frame materializes the BIH Conventional Terrestrial System (CTS), which itself until recently was defined by the FK4 frame, Newcomb's constants of precession and obliquity, Woolard's series of nutation, and by all the assumptions made regarding the reference coordinates of the participating observatories and their relative weights, etc. The current BIH system is based on the IAU 1976 precession constant and the IAU 1980 (Wahr) series of nutation.

Another way of defining the CTS for the deformable earth is through the time varying positions of a number of terrestrial observatories whose coordinates are periodically reobserved by some international service. The frame of this CTS could then be derived from the changing coordinates through transformations containing rotational (and possible translational) parameters. These transformation parameters computed and published by the service would then define the frame of the system. The service, as part of the system definition, thus would have to make the assumption that the progressive changes of the reference coordinates of the observatories do not represent rotations (and translations) in the statistically significant sense. This mode seems to be the consensus for the establishment of the future CTS frame.

It is also necessary to point out that celestial reference systems may be defined ***kinematically*** (through the positions of extragalactic radio sources), or ***dynamically*** (through the geocentric or heliocentric motions of artificial satellites, moon, planets). Stellar systems, such as the FK5 , are hybrid. Furthermore, approximations must be introduced in the model so that it is not true to say that these systems are realizations of an ideal inertial system. This is why it is appropriate to use the term conventional "quasi" inertial system (CIS) as a common term for all such celestial systems. The corresponding frames would be defined by either the adopted positions of a set of radio sources (kinematic frame) or the adopted geocentric or heliocentric ephemerides (dynamic frames), all serving the materialization of the CIS with greater or lesser success (accuracy).

There seems to be general agreement that only two basic coordinate systems are needed : a Conventional Inertial System (CIS), which in some "prescribed way" is attached to extragalactic celestial radio sources, to serve as a reference for the motion of a Conventional Terrestrial System (CTS), which moves and rotates in some average sense with the earth and is also attached in some "prescribed way" to a number of dedicated observatories operating on the earth's surface [Mueller, 1981]. In the latter, the geometry and dynamic behavior of the earth would be described in the relative sense, while in the former the movements of our planetary system (including the earth) and our galaxy could be monitored in the absolute sense. There also seems to be a need for certain interim systems to facilitate theoretical calculations in geodesy, astronomy, and geophysics as well as to aid the possible traditional decomposition of the transformations between the frames of the two basic systems.

As we will see later, there already seems to be understanding in principle on how the two basic reference systems should be established ; certain operational details need to be worked out and an international agreement is necessary. There are, however, a number of more or less open questions which will have to be discussed further. These include the type of interim systems needed and their connections to both CIS and CTS, the type(s) of observatories, their number and distribution, whether all instruments need to be permanently located there or only installed at suitable regular intervals to repeat the measurements ; how far the model development should go so as not to become impractical and unmanageable ; and how independent observations should be referenced to the CTS, i.e., what kind of services need to be established and by whom. This discussion deals only with questions related to the CTS.

Conventional Terrestrial Systems (CTS) of Reference

As mentioned, the frame of the CTS is in some "prescribed way" attached to observatories located on the surface of the earth. The connection between the CTS and CIS frames by tradition (to be preserved) is through the conventional rotations expressed as [Mueller, 1969]

$$[\underline{CTS}] = SNP\ [\underline{CIS}] \qquad (1)$$

where P is the matrix of rotation for precession, N for nutation, and S for earth rotation (polar motion and sidereal time). Polar motion thus is defined as the angular separation of the third axis of the CTS, the Conventional Terrestrial Pole (CTP), and the axis of the earth for which the nutation (N) is computed (e.g., instantaneous rotation axis, Celestial Ephemeris Pole, Tisserand mean axis of the mantle (see [Mueller, 1981]).

Geodynamic requirements for a CTS may be discussed in terms of global or regional problems. The former are required for monitoring the earth's rotation, while the latter are mainly associated with crustal motion studies in which one is predominantly interested in strain or strain rate, quantities which are directly related to stress and rheology. Thus for these studies, global reference systems are not particularly important although it is desirable to relate regional studies to a global frame.

For the rotation studies one is interested in the variations of the earth's rotational rate and in the motions of the rotation axis both with respect to space (CIS) and the crust or the CTS. The problem therefore is threefold : (1) to establish a geometric description of the crust, either through the coordinates of a number of points fixed to the crust, or through polyhedron(s) connecting these points whose side lengths and angles are directly estimable from observations using the new space techniques (laser ranging or VLBI). The latter is preferred because of its geometric clarity. (2) To establish the time–dependent behavior of the polyhedron due to, for example, crustal motion, surface loading or tides. (3) To relate the polyhedron to both the CIS and the CTS. For the global tectonic problems only the first two points are relevant although these may also be resolved through point (3).

In the absence of deformation, the definition of the CTS is arbitrary. Its only requirement is that it rotates with the rigid earth, but common sense suggests that the third axis should be close to the mean position of the rotation axis and the first axis be near the origin of longitudes. An arbitrary choice, such as the one presently defined by the BIH–published polar coordinates and UT1 is appropriate.

In the presence of deformations, particularly long periodic or secular ones, the definition is more problematical, because of the inability to separate rotational (and translational) crustal motions of the crust from those of the CTS. This is why the consensus seems to be the CTS described earlier. If such a system is adapted, the secular type motions mentioned above will be absorbed in the future CTS, by definition. Residuals with respect to such a CTS will provide estimates of relative motions between stations, i.e., of the deformations.

One geophysical requirement of the reference system is that other geophysical measurements can be related to it. One example is the gravity field. The reference frame generally used when giving values of the spherical harmonic coefficients is tied to the axes of figure of the earth. This frame should be simply related with sufficient accuracy to the CTS as well as to the CIS in which, for example, satellite orbits are calculated. Another example is height measurements with respect to the geoid.

The vertical motions may require some special attention, because absolute motions with respect to the center of mass have an immediate geophysical interest and are realizable. Again, if the center of mass has significant motions with respect to the crust, such a motion will be absorbed in the future CTS, if defined as suggested above. At present there is no compelling evidence that the center of mass is displaced significantly at least at the decade time scale.

Apart from the geometrical considerations the configuration of observatories should be such that (1) there are stations on most of the major tectonic plates in sufficient number to provide the necessary statistical strength, (2) the stations lie on relatively stable parts of the plate so as to reduce the possibility that tectonic shifts in some stations will not overly influence, at least initially, the parameters defining the CTS frame.

Finally one should realize that the problem of the geometric origin of the CTS frame is linked to that of a geocentric ephemeris frame. The center of mass of the earth is directly accessible to dynamical methods and is the natural origin of a geocentric satellite–based dynamical system. But, as such, it is model dependent. And, unless the terrestrial reference frame is also constructed from the same satellites (as is the case in various earth models such as GEM, SAO, GRIM), there may be inconsistencies between the assumed origin of a kinematically obtained terrestrial system and the center of mass. A time–dependent error in the position of the center of mass, considered as the origin of a terrestrial frame, may introduce spurious apparent shifts in the position of stations that may then be interpreted as erroneous plate motions. To avoid this problem the parameters defining the CTS frame should include translational terms as suggested earlier.

Current Situation

Until 1984 the internationally accepted Woolard series of nutation was used to compute the position of the instantaneous rotation axis of the rigid earth, and the CTP was the Conventional International Origin (CIO), defined by the adopted astronomic latitudes of the five International Latitude Service (ILS) stations [Mueller, 1969].

From 1984 onward the IAU 1980 [Wahr, 1981] series of nutation for the nonrigid earth gives the space position of the Celestial Ephemeris Pole (CEP). The CTP officially remains the same as before. Thus, conceptually, polar motion should be determined from latitude observations only at the ILS stations. This has been done for 80 years, and the results are the best available *long–term* polar motions, properly, but not very accurately, determined. The first axis of the CTS, the Greenwich Mean Astronomical Meridian, is defined by the assigned astronomic longitudes of time observatories participating in the work of the Bureau International de l'Heure (BIH).

For reasons explained elsewhere (e.g., [Mueller, 1981]), the use of the CIO is no longer a reality. The common denominator being the series of nutation, observationally the CTP is defined by the coordinates of the pole as published by the IPMS or by the BIH. Thus it is legitimate to speak of IPMS and BIH CTP's. The situation recently has become even more complicated because Doppler and laser satellite tracking, VLBI observations, and lunar laser ranging also can determine earth rotation parameters, some of which are incorporated in the BIH computations. Further confusion arises due to the fact that the BIH has two systems : the BIH 1968 and the BIH 1979, the latter due to the incorporation of certain annual and semiannual variations of polar motion determined from the comparisons of astronomical (optical) results with those from Doppler and lunar laser observations [Feissel, 1980].

Though naturally every effort has been made to keep the IPMS and BIH pole of the CTS as close as possible to the CIO, the situation cannot be considered satisfactory from the point of view of the geodynamic accuracy requirement of a few parts in 10^8 .

The Future Conventional Terrestrial Reference Frame

There seems to be general agreement that the new CTS frame conceptually be defined similarly to the CIO–BIH system [Bender and Goad, 1979 ; Guinot, 1979 ; Kovalevsky, 1979 ; Mueller, 1975, 1981 ; Kovalevsky and Mueller, 1981], i.e., it should be attached to observatories located on the surface of the earth. The main difference in concept is that these can no longer be assumed motionless with respect to each other. Also they must be equipped with advanced geodetic instrumentation like VLBI or lasers, which are no longer referenced to the local plumblines. Thus the new transformation formula may have the form

$$[\underline{OBS}]_j = \underline{L}_j + [\underline{CTS}]_j + \underline{v}_j \quad (2)$$

where $\underline{L}_j$ is the vector of the "j" observatory's movement on the deformable earth with respect to the CTS.

The $[\underline{OBS}]_j$ is related to the observatory coordinates $(\overline{X}^\circ j)$ determined in the terrestrial frame inherent in the observational technique (e.g., SLR) "0", through the well–known transformations involving three translation components $(\overline{\delta}^\circ)$, three (usually very small) rotations $(\overline{\beta}^\circ)$ and a differential scale factor (c) :

$$[\underline{OBS}]_j = \overline{X}_j^\circ + \overline{\delta}^\circ + R_1(\beta_1^\circ) R_2(\beta_2^\circ) R_3(\beta_3^\circ) \overline{X}_j^\circ + c\overline{X}_j^\circ \qquad (3)$$

Naturally in case of techniques which observe directions only (e.g., astrometry), the terms containing translation and scale will be omitted. Eqs. (2) and (3) together with (4) below (and possibly others) may form the observation equations to be used when realizing the future CTS. The latter equation [Zhu and Mueller, 1983] relates an earth rotation parameter (ERP) series (x_p, y_p, and UT1) determined by the technique "0", within its own frames of reference, with the parameters of rotation above :

$$\begin{aligned} x_p - \beta_2^\circ + a_1^\circ \sin\theta + a_2^\circ \cos\theta &= x_p^\circ + v_{x_p} \\ y_p - \beta_1^\circ - a_1^\circ \cos\theta + a_2^\circ \sin\theta &= y_p^\circ + v_{y_p} \\ \omega_c \mathrm{UT1} + \beta_3^\circ - a_3^\circ &= \omega_c \mathrm{UT1}^\circ + v_{\mathrm{UT1}} \end{aligned} \qquad (4)$$

where a_1°, a_2°, a_3° are the small rotations between the frames of the CIS of the technique "0" and that of the service, θ is the sidereal time, ω_c the conversion factor between sidereal and solar times, and v the residuals.

The unknowns in the above system of equations to be solved for, in a least squares solution minimizing the square sum of the residuals v, are $[\underline{CTS}]_j$ and $\underline{L}_j$ for the observatories; $\overline{\delta}^\circ$, $\overline{\beta}^\circ$ and c° for the terrestrial frames of the techniques; $\overline{a}^\circ$ for their inertial frames; and finally, the ERP parameters (x_p, y_p and UT1) for the service. If, however, in eq. (3) the ERP's (x_p°, y_p°, UT1°) are mean values averaged over intervals longer than a day, a_1° and a_2° cannot be determined, because the *sin* θ and *cos* θ terms average to zero in one sidereal day.

As mentioned, the parameters pertaining to the observatories ($[\underline{CTS}]_j$ and L_j) define the CTS. The others give the relationships of the CTS to the technique "0" terrestrial frame ($\overline{\delta}^\circ$ $\overline{\beta}^\circ$, c); to the CIS (x_p, y_p, UT1); and the latter's relationship to the technique "0" inertial frame $(\overline{a}^\circ)$.

The rotations in eq. (3) can either be determined from the Cartesian coordinates (e.g., [Moritz, 1979]) or, for possible better sensitivity, since the rotation is least sensitive to variations in height, only from those of the horizontal coordinates (geodetic latitude and longitude) (e.g., [Bender and Goad, 1979]). It is, however, unlikely that the rotations will continue to be determined (as presently) from astronomical coordinates, i.e., from the direction of the vertical, for the reasons of inadequate observational accuracy. Note that when using this method, the deformations (and the residuals) by definition cannot have common rotational (or translational) components.

As far as the origin of the CTS is concerned, it could be centered at the center of mass of the earth, and its motion with respect to the stations can be monitored either through observations to satellites or the moon, or, probably more sensitively, from continuous global gravity observations at properly selected observatories [Mather et al., 1977]. For the former method, the condition

$$\sum_{D} w_D \bar{\delta}^{\bullet}_D = 0$$

could be imposed on the above adjustment. The summation would be extended to all the above dynamic techniques D with given relative weights w_D . A similar condition could also be imposed on the scale extended to techniques defining the best scales (probably VLBI).

The above method of determining ERP or some variation thereof needs to be initialized in a way to provide continuity. This could be done through the IPMS or BIH poles, and the BIH zero meridian, at the selected initial epoch (or averaged over a well-defined time interval, say 1–1.2 years), uncertainties in their definition mentioned earlier mercifully ignored.

It is probably not useless to point out that if such a system is established, the most important information for the users will be the ERP and the transformation parameters, but for the scientist new knowledge about the behavior of the earth will come from the analysis of the residuals after the adjustment.

It is hoped that the IAU and IUGG will make practical recommendations on the establishment of such or a very similar Conventional Terrestrial System, including the necessary plans for supporting observatories and services. One of the recommendations ought to be that due to the fact that the ultimate goal is the determination of the total transformation between the CTS and CIS, the future service must publish not only the ERP's determined from the repeated comparisons (the situation at present), but also the models and parameters discussed above, i.e., the parameters defining the whole *system.*

In conclusion, there is little doubt that the terrestrial reference frame presently adopted is of very little practical use because of its insufficient accessibility. Further, the astronomical observations should be replaced by methods which are not tied to the direction of the vertical but rather to directions tied to the crust. Such methods are the laser observations to satellites and to the moon, and VLBI. Portable systems can establish the polyhedron(s) discussed earlier, while permanent stations at suitably chosen locations would become the observatories for the maintenance of the CTS using the method described above.

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REFERENCES

P. BENDER and C. GOAD (1979) : "Probable LAGEOS Contributions to World Wide Geodynamics Control Network." *The Use of Artificial Satellites for Geodesy and Geodynamics, Vol. II, Proc. of International Symposium,* Athens, May 29 – June 2, 1978, G. Veis and E. Livieratos, eds., National Technical Univ. Athens, pp. 145–161.

M. FEISSEL (1980) : "Determination of the Earth Rotation Parameters by the Bureau International de l'Heure, 1962–1979." Bull. Geod., *54,* 1, pp. 81–102.

B. GUINOT (1979) : "Basic Problems in the Kinematics of the Rotation of the Earth." McCarthy and Pilkington, eds., *Time and the Earth's Rotation,* D. Reidel, Dordrecht, pp. 7–18.

J. KOVALEVSKY (1979) : "The Reference Systems." McCarthy and Pilkington, eds., *Time and the Earth's Rotation,* D. Reidel, Dordrecht, pp. 151–163.

J. KOVALEVSKY and I.I. MUELLER (1981) : "Comments on Conventional Terrestrial and Quasi–Inertial Reference Systems." Gaposchkin and Kolaczek eds., *Reference Coordinate Systems for Earth Dynamics,* D. Reidel, Dordrecht.

R.S. MATHER, E.G. MASTERS and R. COLEMAN (1977) : "The Role of Non–Tidal Gravity Variations in the Maintenance of Reference Systems for Secular Geodynamics." *Uniserv G* 26, Univ. of New South Wales, Sidney.

H. MORITZ (1979) : "Concepts in Geodetic Reference Frames." Dept. of Geodetic Science and Surveying Rep. 294, Ohio State Univ., Columbus.

I.I. MUELLER (1969) : *Spherical and Practical Astronomy as Applied to Geodesy.* Ungar Publ., New York.

I.I. MUELLER (1975) : "Tracking Station Positioning from Artificial Satellite Observations." *Geophys. Surveys, 2,* 3, pp. 243–276.

I.I. MUELLER (1981) : "Reference Coordinate Systems for Earth Dynamics : A Preview." Gaposchkin and Kolaczek, eds., *Reference Coordinate Systems for Earth Dynamics,* D. Reidel, Dordrecht.

J.M. WAHR (1981) : "The Forced Nutations of an Elliptical, Rotating, Elastic and Oceanless Earth." *J.R. Astr. Soc., 64,* pp. 705–727.

S.–Y. ZHU and I.I. MUELLER (1983) : "Effects of Adopting New Precession, Nutation and Equinox Corrections on the Terrestrial Reference Frame." *Bull. Geod., 57,* pp. 29–42.

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