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IMPACTS OF GEODYNAMIC PHENOMENA ON SYSTEMS FOR HEIGHT AND GRAVITY

Abstract

Geodynamic phenomena of permanent or secular characters play a significant role when defining height systems and gravity systems. A treatment is here given of the permanent earth tide, postglacial land uplift, sea level changes and polar drift from this point of view.

1. *The various gravity and geoid concepts originating from differing ways of handling the permanent tide are treated, and transformations between them are given. Numerical applications are made to height and gravity systems in the Nordic countries, including determination of land uplift. The oceanographic deviation of mean sea level in the Baltic Sea is discussed with respect to the permanent tide. Problems caused by the permanent tide in height determination based on* GPS *are illustrated.*

2. *The effects of postglacial land uplift and sea level changes are dealt with together. Again, numerical applications are made to height and gravity systems in the Nordic countries. It is discussed how sea level data should be included in the determination of land uplift. An attempt to estimate the remaining land uplift is made.*

3. *A few words are said on the role of polar drift when defining a gravity system.*

1. The Permanent Earth Tide

1.1. Introduction : Mean, non-tidal and zero geoids

A considerable part of the tides does not vary periodically with time but is permanent. The permanent tide, low in the polar areas and high in the equatorial area, is a consequence of the Moon and the Sun moving fairly close to the equator.

The permanent tide, forming a part of the long period tide, was discovered already by Darwin (1899). Its role in geodetic systems for height and gravity was first discussed by Jensen (1949) and Honkasalo (1964). Both of them, however, seem to have been unaware of the work of their predecessor half a century earlier.

The permanent tidal deformation of the geoid, $\bar{N} = \bar{W}/g$, as well as the permanent tidal gravitation, $\bar{g} = -\partial\bar{W}/\partial r$, are illustrated in ***Figure 1*** assuming, for the moment, a rigid Earth. Within 0.1 cm and 0.1 μgal we have (cf. Heikkinen, 1978)

$$\bar{N} = \frac{\bar{W}}{g} = 9.9 - 29.6\, sin^2\varphi \text{ cm} \tag{1}$$

Bull. Géod. 63 (1989) pp. 281–296.

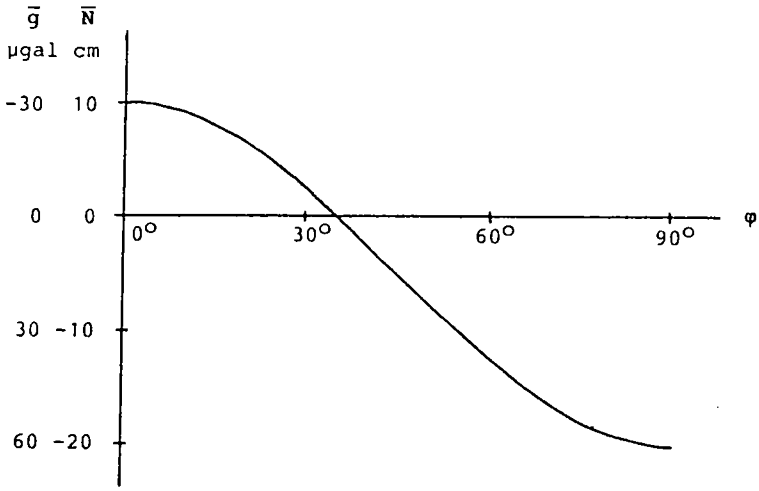

Fig. 1 — The permanent tide as a function of latitude (rigid Earth).

and

$$\bar{g} = -\frac{\partial \bar{W}}{\partial r} = -30.4 + 91.2\, sin^2 \varphi \ \mu\text{gals} \tag{2}$$

$\bar{W}$ being the permanent tidal potential, g gravity, and $\partial/\partial r$ the radial derivative.

Traditionally, a gravity measurement was tidally corrected by simply subtracting

$$g' = -\delta \frac{\partial W}{\partial r} \tag{3}$$

where W is the tidal potential and $\delta \approx 1.16$. In doing so one eliminated the whole tidal effect, both the periodical and permanent parts. Honkasalo (1964) suggested that the permanent part should be restored so that a gravity measurement should be tidally corrected by subtracting

$$g'' = -\delta \frac{\partial W}{\partial r} + \delta \frac{\partial \bar{W}}{\partial r} \tag{4}$$

Honkasalo's suggestion was adopted in the International Gravity Standardization Net 1971 (IGSN 71) . We name this kind of gravity ***mean gravity,*** and the corresponding geoid the ***mean geoid.***

The height of the mean geoid above the geoid corresponding to (3) is

$$\bar{N}' = (1 + k)\frac{\bar{W}}{g}$$

where the Love number $k \approx 0.30$ (cf. Melchior, 1978). As a maximum we have, for the poles, $\bar{N}' = -26$ cm . An attempt to calculate the same quantity through Stokes' formula will produce an erroneous result, the error being 72 cm . This is due to the fact that mean gravity includes the permanent tidal attraction caused by masses which are outside the geoid. To avoid this, Heikkinen (1979) proposed to revert to the traditional correction (3), which was also resolved by the IAG in 1979. We name this kind of gravity ***non-tidal gravity,*** and the corresponding geoid the ***non-tidal geoid.***

Whereas the Earth responds to the periodical tidal force as an (almost) elastic body it should respond to the permanent tidal force as a body close to hydrostatic equilibrium; see Groten (1970) and Zschau & Wang (1985). For the fluid Love numbers Lambeck (1980) has found $h_f = 1.93$, $k_f = 0.93$, yielding $\gamma_f = 0$ and $\delta_f = 1.53$. This would imply that the ***true non-tidal gravity and geoid*** should be determined using $\delta \approx 1.53$ instead of 1.16 for the permanent part contained in (3). A problem, however, is that the δ factor for the permanent tide cannot be determined from any observations. Furthermore, if we try to eliminate the permanent tidal deformation we also have to change the Earth's moments of inertia, its rotational velocity and its centrifugal force. The change of the rotational velocity would correspond to shortening the day by 5 ms . To avoid all the above problems, Ekman (1979, 1981) and Groten (1980) proposed a third gravity concept, resolved by the IAG in 1983 (cf. also Rapp, 1983) : The permanent tidal attraction is eliminated whereas the permanent tidal deformation is retained. To achieve this a gravity measurement is tidally corrected by subtracting

$$g''' = -\delta \frac{\partial W}{\partial r} + (\delta - 1) \frac{\partial \overline{W}}{\partial r} \tag{5}$$

We name this kind of gravity ***zero gravity,*** and the corresponding geoid the ***zero geoid.***

We note that there have also been published two proposals to revert to mean gravity, in spite of the problems with the validity of Stokes' formula in this case. Šimon (1980) and Yurkina et al. (1986) suggest that a special correction be made to all gravity anomalies before they go into Stokes' formula. Grafarend & Sansò (1984) suggest that Stokes' formula be replaced by a more complicated formula allowing masses outside as well as inside the geoid. For an attempt to solve the problem by redefining the normal gravity field see Zeman (1987).

Finally we set up the tidal corrections analogous to (3), (4) and (5) for a levelled height, i.e. a height above the geoid. To obtain the ***height above the non-tidal geoid*** we subtract

$$H' = -\gamma \frac{W}{g} \tag{6}$$

where $\gamma \approx 0.68$. To obtain the ***height above the mean geoid*** we subtract (cf. also Jensen, 1949, and Simonsen, 1965)

$$H'' = -\gamma \frac{W}{g} + \gamma \frac{\overline{W}}{g} \tag{7}$$

We may note that the height above the mean geoid is identical with the height above the ***true*** non-tidal geoid, provided that $\gamma = 0$ should be applied instead of $\gamma \approx 0.68$ to the permanent part contained in (6). To obtain the ***height above the zero geoid*** we subtract

$$H''' = -\gamma \frac{W}{g} - (1 - \gamma) \frac{\overline{W}}{g} \tag{8}$$

1.2. Tidal transformations of heights and gravity

The various treatments of the permanent tide presented in the preceding section have to be taken into account in the following cases : 1. Comparison of different height systems or gravity systems, e.g. of two neighbour countries. 2. Computation of land uplift from two levellings within a country. 3. Study of mean sea level, i.e. sea surface topography. 4. Comparison of satellite (GPS) determined heights with levelled heights. 5. Computation of geoid heights with Stokes' formula. To do this we need to be able to transform one kind of gravity to another, or one kind of height to another.

The relations between the different ***gravity*** concepts are easily found from (3), (4), (5) and (2). Thus, the transformations between zero gravity g_z, mean gravity g_m and non-tidal gravity g_n are given by

$$g_m - g_z = -30.4 + 91.2 \sin^2 \varphi \ \mu\text{gals} \tag{9}$$

$$g_z - g_n = (\delta - 1)(-30.4 + 91.2 \sin^2 \varphi) \ \mu\text{gals} \tag{10}$$

$$g_m - g_n = \delta\,(-30.4 + 91.2 \sin^2 \varphi) \ \mu\text{gals} \tag{11}$$

(For those using Wahr's (1981) theory when making tidal gravity corrections a slightly more accurate version of (10) may be found in Rapp (1983); it should, however, be amended according to Dehant & Ducarme (1987)).

The relations between the different ***height*** concepts are found from (6), (7), (8) and (1). Thus, the transformations between a height difference ΔH_z above the zero geoid, a height difference ΔH_m above the mean geoid, and a height difference ΔH_n above the non-tidal geoid, between a northern and a southern station, are given by

$$\Delta H_m - \Delta H_z = 29.6\,(\sin^2 \varphi_N - \sin^2 \varphi_S) \ \text{cm} \tag{12}$$

$$\Delta H_z - \Delta H_n = 29.6\,(\gamma - 1)(\sin^2 \varphi_N - \sin^2 \varphi_S) \ \text{cm} \tag{13}$$

$$\Delta H_m - \Delta H_n = 29.6\,\gamma\,(\sin^2 \varphi_N - \sin^2 \varphi_S) \ \text{cm} \tag{14}$$

In a similar manner relations between the various ***geoid heights*** above the ellipsoid are established. The transformations between a zero geoid height N_z, a mean geoid height N_m and a non-tidal geoid height N_n are

$$N_m - N_z = 9.9 - 29.6 \sin^2 \varphi \ \text{cm} \tag{15}$$

$$N_z - N_n = k\,(9.9 - 29.6 \sin^2 \varphi) \ \text{cm} \tag{16}$$

$$N_m - N_n = (1 + k)(9.9 - 29.6 \sin^2 \varphi) \ \text{cm} \tag{17}$$

In the following sections we will illustrate items 1 – 5 above by applying some of the formulae (9) – (17) to height and gravity in the Nordic countries.

1.3. On height and gravity systems in the Nordic countries

The height systems of the Nordic countries do not have the same kind of geoid as reference surface. Neither do the gravity systems refer to the same kind of gravity. We have examined how the permanent tide has been treated in the fundamental height and gravity networks in the Nordic countries (except Iceland); for details see Ekman (1988a). A summary of the reference geoids used in the present height systems is given in ***Table 1.*** A summary of the gravity concepts used in the present gravity systems is given in ***Table 2.*** Since no tidal corrections at all were applied earlier, all older height systems refer to the mean geoid (at least approximately, cf. Jensen, 1949) and older gravity systems contain mean gravity.

Table 1

Reference geoids in the Nordic height systems

Country	Height system	Geoid
Denmark	DNN GI 1950.5	Non-tidal ($\gamma = 0.7$)
Norway	NN 1954	Mean
Sweden	RH 70	Non-tidal ($\gamma = 0.8$)
Finland	N 60	Mean

Table 2

Gravity concepts in the Nordic gravity systems

Country	Gravity system	Gravity
Denmark	(No name yet)	Zero
Norway	IGSN 71	Mean
Sweden	RG 82	Zero
Finland	FOGN	Non-tidal ($\delta = 1.14$)

As a numerical example for ***gravity*** systems we apply (9) to the Swedish absolute gravity station Mårtsbo ($\varphi = 60°.6$). We find the discrepancy between mean gravity and zero gravity to be $g_m - g_z = 39\ \mu\text{gals}$. This is nearly ten times the standard error of the gravity value of Mårtsbo. Obviously this is an important effect when using absolute gravity measurements in gravity networks (cf. Haller & Ekman, 1988), or when comparing, say, the world system IGSN 71 with the new Swedish system RG 82 ***(Table 2).***

As a numerical example for ***height*** systems we apply (14) to the Swedish-Finnish border from Åland to Lapland ($\varphi_N = 69°.1$, $\varphi_S = 59°.8$). We find the discrepancy between the height difference above the mean geoid and the height difference above the non-tidal geoid to be $\Delta H_m - \Delta H_n = 3.0$ cm. This is about equal to the Finnish standard error of the height difference. Consequently this effect should be taken into account when comparing the Swedish and Finnish height systems *(Table 1)* with each other.

A similar problem arises within Denmark and Sweden when computing the *land uplift.* In these countries the land uplift has been computed by comparing the second high precision levelling referring to a non-tidal geoid *(Table 1)* with the first high precision levelling referring to a mean geoid (no tidal correction). Strictly speaking, the first levelling should have been corrected by (14) before being compared with the second one. In Denmark one has been aware of this problem but chosen to neglect it because of the smallness of the country (Bedsted Andersen et al., 1974). In Sweden the problem happened to be circumvented by the fact that sea level recordings were used extensively in the land uplift computation, and given a considerably higher weight than the levellings (Ussisoo, 1977). Otherwise a systematic error in the land uplift would have occurred, which can be determined in the following way : (14) yields, across the whole of Sweden ($\varphi_N = 69°.1$, $\varphi_S = 55°.3$), $\Delta H_m - \Delta H_n = 4.7$ cm. Dividing this by 68 years, which is the time interval between the levellings, we obtain 0.7 mm/year. This error in the land uplift is about three times the standard error of the land uplift at a sea level station. Accordingly it has to be avoided when making new land uplift computations based on e.g. the principles in section 2.3.

1.4. Mean sea level in the Baltic Sea

An undisturbed sea level would coincide with the mean geoid. In reality, however, the mean sea level is disturbed by oceanographic (including meteorological) effects. The main ones are salinity, temperature, currents, air pressure and winds. The oceanographic deviation of mean sea level can be determined by high precision levelling between sea level stations. In order to obtain this quantity we must refer the height of mean sea level to the mean geoid.

The height of mean sea level in the Baltic Sea and adjacent waters has been determined for twelve stations along the Swedish coast by Ussisoo (1977). The heights published are given in the Swedish height system RH 70, which refers to a non-tidal geoid *(Table 1).* Consequently the heights are somewhat misleading for the study of the oceanographic deviation. They should be transformed to heights above the mean geoid instead. Let us take a look at the effect of this along the Swedish coast.

In the height system RH 70 the mean sea level in the northern part of the Gulf of Bothnia ($\varphi = 65°.8$) is situated 10 cm higher than in the southern part of the Baltic proper ($\varphi = 55°.3$). To correct this to a height difference above the mean geoid we add (14), i.e.

$$\Delta H_m - \Delta H_n = 29.6\,\gamma\,(\sin^2 \varphi_N - \sin^2 \varphi_S) \quad \text{cm}$$

This makes the height difference increase by 4 cm to 14 cm. Furthermore, this brings the difference somewhat closer to the one estimated by oceanographic models. According to such model calculations by Lisitzin (1974) the difference should be 23 cm.

1.5. Heights from satellite positioning (GPS)

Heights determined by satellite positioning (GPS) are heights above the ellipsoid. To obtain a height corresponding to (6) we subtract

$$h' = h\frac{W}{g} \tag{18}$$

where the Love number $h \approx 0.62$. The result could be described as the ***height of the non-tidal crust above the ellipsoid.*** The heights corresponding to (7) and (8) are identical; to obtain them we subtract

$$h'' = h''' = h\frac{W}{g} - h\frac{\overline{W}}{g} \tag{19}$$

The result of this could be described as the ***height of the mean crust or zero crust above the ellipsoid.*** To transform a GPS height difference of the non-tidal crust to a GPS height difference of the mean crust or zero crust we should add

$$\Delta h_m - \Delta h_n = \Delta h_z - \Delta h_n = 29.6\ h\,(sin^2\varphi_N - sin^2\varphi_S)\ \text{cm} \tag{20}$$

In the future it is expected that GPS, in combination with accurate knowledge of the geoid, will be used for determining height differences above the geoid over long distances. This method may serve as a check of high precision levelling networks. (Alternatively, GPS in combination with levelling may be used for determining geoid height differences.) As a consequence, it will be necessary to treat the permanent tide in one and the same way in gravimetry, levelling and GPS. Let us give an example of what might happen if we do not.

We want to check a height difference between northernmost Norway ($\varphi = 71°.2$) and southernmost Denmark ($\varphi = 54°.8$) determined by motorized high precision levelling. The levelling is tidally corrected according to (6) with $\gamma = 0.68$, giving heights of the non-tidal crust above the non-tidal geoid. For the sake of simplicity we suppose that this crust happens to coincide with the non-tidal geoid at both end points so that the resultant height difference becomes $\Delta H = 0$ cm. We further suppose that the ellipsoid coincides with the mean geoid at both ends. The heights determined by GPS are tidally corrected according to (18) with $h = 0.62$, giving heights of the non-tidal crust above the ellipsoid. In our case, then, the GPS height difference Δh becomes equal to the height difference of the non-tidal geoid above the mean geoid. From (17) we obtain, using $1 + k = \gamma + h$,

$$\Delta h = 29.6\,(\gamma + h)\,(sin^2\varphi_N - sin^2\varphi_S)\ \text{cm} \tag{21}$$

The reader may compare (21) with (14) and (20). The GPS height difference becomes $\Delta h = 9$ cm. The geoid heights above the ellipsoid should be those of the non-tidal geoid, but here we make the mistake of using those of the mean geoid. Then the geoid height difference is $\Delta N = 0$ cm. We now have $\Delta H = 0$ cm from levelling, but $\Delta H = \Delta h - \Delta N = 9 - 0 = 9$ cm from GPS and geoid. Conclusion : Mixing different geoids, i.e. not bothering about the permanent tide, may cause a decimeter error in a height difference across the Nordic countries.

Let us now assume that we want to use GPS combined with geoid determination to check the levelled height difference above the mean geoid instead. We use mean crust, mean geoid and mean gravity throughout. The mean gravity is put into Stokes' formula to find the heights of the mean geoid above the ellipsoid. Then, according to section 1.1, an error in the geoid height difference will occur, amounting to $(72/26) \cdot 9\,\mathrm{cm} = 25\,\mathrm{cm}$. Thus $\Delta H = \Delta h - \Delta N$ from GPS and geoid will be 25 cm smaller than ΔH from levelling. Conclusion : Treating the permanent tide in the wrong way in gravimetry may cause an error of more than two decimeters in a height difference across the Nordic countries.

How to do : Use zero gravity, zero geoid and zero crust (= mean crust) . This is already done in the fundamental gravity networks of Sweden and Denmark, and should be extended to all gravity networks, levelling networks, and GPS . The exception that proves the rule : The mean sea level must be related to the mean geoid to give oceanographically relevant information.

2. Postglacial Land Uplift and Sea Level Changes

2.1. Introduction : Apparent, levelled and absolute uplift

In Fennoscandia as well as in Canada we have, due to the deloading of the large ice sheets, a postglacial uplift of the crust. So far only the Fennoscandian uplift has been fairly well determined by geodetic observations; hence we will concentrate on this one.

The first geodetic land uplift map was constructed by Blomqvist & Renqvist (1914). A modern Fennoscandian land uplift map compiled by Ekman (1988a) is shown in *Figure 2*. A determination of parts of the Canadian uplift has been published by Vaniček & Nagy (1981).

Figure 2 shows the uplift of the crust relative to mean sea level. We call this the ***apparent land uplift***, $\dot{H}_a$. The maximum apparent uplift, in the northern part of the Gulf of Bothnia, amounts to $\dot{H}_a = 9.2$ mm/yr .

Due to more or less global changes of climate there is a eustatic change of sea level. Since the end of the last century a rise of the sea level has occurred, first estimated by Gutenberg (1941). Modern analyses like those of Lisitzin (1974), Barnett (1984) and Lambeck & Nakiboglu (1984) confirm that the eustatic change of sea level during our century has been $\dot{H}_e \approx 1.0$ mm/yr . According to Ekman (1988) the eustatic sea level change due to northern hemisphere climatic variations since 550 A.D. has varied within -1 and $+2$ mm/yr .

Correcting the apparent land uplift for the eustatic change of sea level we obtain the uplift of the crust relative to the geoid; see, however, also section 2.3 . We call this the ***levelled land uplift***, $\dot{H}$. Thus

$$\dot{H} = \dot{H}_a + \dot{H}_e \tag{22}$$

Assuming the uplift of the crust to be associated with a corresponding viscous flow of mantle, the resultant addition of mass will cause a rise of the geoid. Maps of the geoid rise have been computed by Ekman (1977), Sjöberg (1983) and Zeman (1984); see also Dietrich (1979). The maximum geoid rise, in the centre of the land uplift area, is $\dot{N} = 0.7$ mm/yr , while in the outskirts the geoid rise seems to be about half of that.

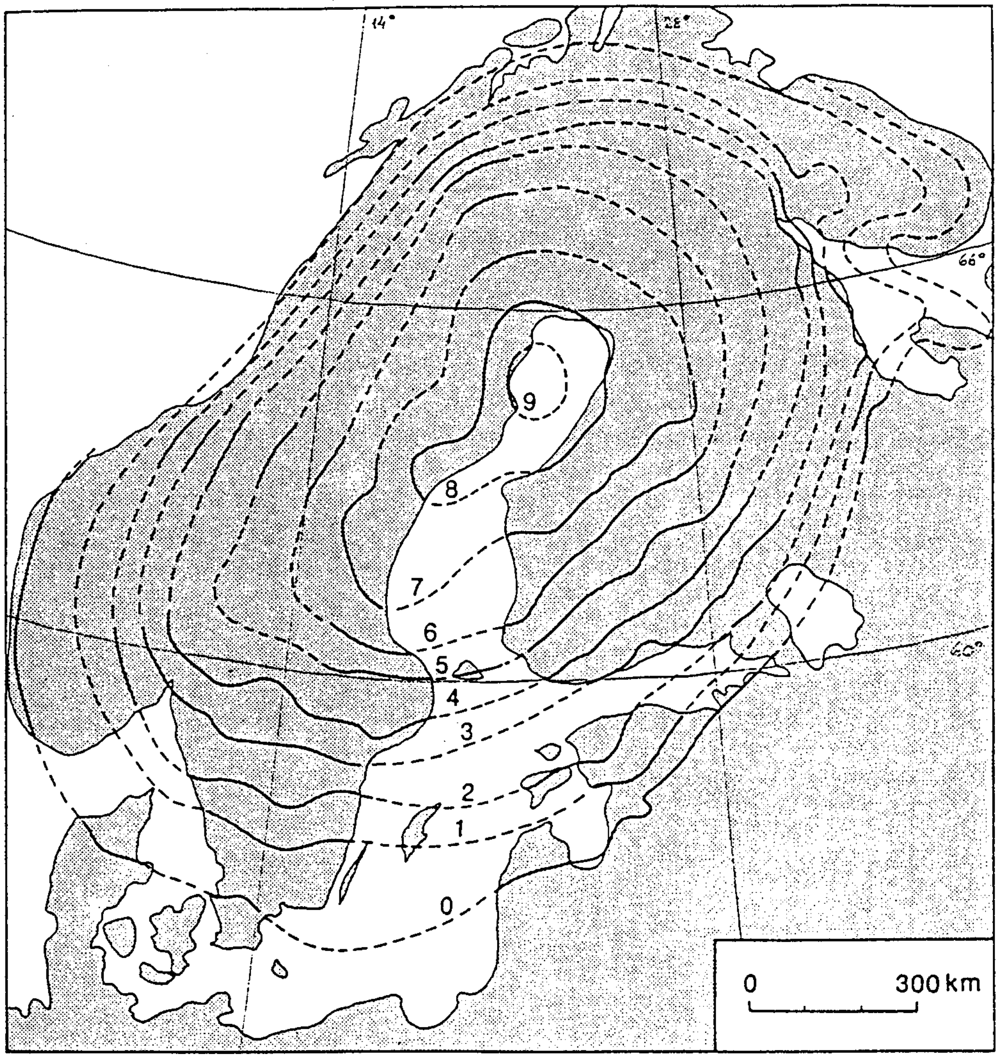

Fig. 2 — Apparent land uplift of Fennoscandia c. 1890 — c. 1970 in mm/yr.

In all the mentioned maps the effect of mass changes outside the uplift area is neglected; a zone of small subsidence around the uplift area would reduce or eliminate the geoid rise in the outskirts.

Correcting the levelled land uplift for the rise of the geoid we obtain the uplift of the crust relative to the ellipsoid. We call this the ***absolute land uplift,*** $\dot{h}$. Thus

$$\dot{h} = \dot{H}_a + \dot{H}_e + \dot{N} \tag{23}$$

Above we assume that the land uplift is associated with a full addition of mass from the upper mantle with the density $\rho = 3.3\ \mathrm{g/cm^3}$. Provided this model is correct, the land uplift will cause a ***gravity decrease*** on the surface of the crust according to the

Bouguer formula, $\dot{g}/\dot{h} = -0.31 + 0.14 = -0.17\,\mu gal/mm$. The repeated gravity measurements made on the Fennoscandian land uplift gravity lines since 1966 (Mäkinen et al., 1986) vaguely indicate that $\dot{g}/\dot{h}$ is close to $-0.2\,\mu gal/mm$; see Becker & Groten (1983), Ekman et al. (1987) and Sjöberg (1988).

The three kinds of postglacial land uplift presented in this section are needed in the following different situations : 1. The apparent land uplift is needed when calculating old shore levels. 2. The levelled land uplift is needed when reducing levelled heights to some given year. 3. The absolute land uplift is needed when dealing with gravity decrease due to the land uplift and, in the future, when handling GPS heights. These items will be commented upon in the following sections.

2.2. On height and gravity systems in the Nordic countries

The height systems of the Nordic countries refer to different epochs, and are associated with different kinds of land uplift. Due to the considerably increased accuracy of modern gravity networks the problems of epoch and land uplift are now getting essential for gravity systems as well. A summary of the epochs and land uplifts used in the present height systems of the Nordic countries (except Iceland) is given in ***Table 3.*** A corresponding summary for the gravity systems is shown in ***Table 4.*** For details see Ekman (1988a), and also Mäkinen (1987).

Table 3

Epochs and land uplifts in the Nordic height systems

Country	Height system	Epoch	Land uplift
Denmark	DNN GI 1950.5	1950.5	Apparent–levelled
Norway	NN 1954	–	–
Sweden	RH 70	1970	Apparent
Finland	N 60	1960	Levelled ($H_e = 0.8$ mm/yr)

Table 4

Epochs and land uplifts in the Nordic gravity systems

Country	Gravity system	Epoch	Land uplift
Denmark	(No name yet)	1980?	?
Norway	IGSN 71	1971	–
Sweden	RG 82	1982	Absolute ($\dot{H}_e = 1.0$ mm/yr, $\dot{N} \leq 0.5$ mm/yr)*
Finland	FOGN	1962	–

* $\dot{g}/\dot{h} = -0.2\,\mu gal/mm$

Let us give a numerical example of the treatment of ***heights.*** When extrapolating heights in a high precision levelling network to a later epoch, the levelled land uplift should be used. This has not been done in Sweden where the apparent uplift has been used instead *(Table 3)*. As the discrepancy is about 1.0 mm/yr, and as the original epoch of the fundamental bench mark used for the height system RH 70 is 26 years earlier than the epoch of the system itself, all RH 70 heights are nearly 3 cm to small. This then applies to the heights of the mean sea level in section 1.4 as well.

We now turn to ***gravity.*** Here we take an example from the Swedish system RG 82 *(Table 4)* and the absolute gravity station Mårtsbo. Its absolute land uplift is $\dot{h} = 6.5 + 1.0 + 0.5 = 8.0$ mm/yr. With the approximate factor $\dot{g}/\dot{h} = -0.2\,\mu$gal/mm we have a gravity decrease of $\dot{g} = -1.6\,\mu$gal/yr. Suppose we want to compare a new absolute gravity measurement made there in 1995 with the RG 82 value. During the 13 years in between, then, the land uplift has caused gravity to decrease by 21 μgals. This is five times the standard error of the gravity value of Mårtsbo and, consequently, an important effect (cf. Haller & Ekman, 1988). We may not even neglect the eustatic rise of sea level and the rise of the geoid, i.e. we may not use the apparent uplift instead of the absolute one. If we do, we will make an error of 4 μgals, equal to the standard error.

2.3. The role of sea level in land uplift determination

Hitherto we have looked upon the eustatic change of sea level as the only sea level change besides the rise of the geoid. However, in section 1.4 we noted that the mean sea level deviates from the mean geoid due to oceanographic factors : salinity, temperature, currents, air pressure, winds. A secular change in one or several of these quantities will evidently produce a corresponding change in the oceanographic deviation of mean sea level. Let us call this effect the deviational change of sea level, $\dot{H}_d$. If it is large enough to influence land uplift determination, equations (22) and (23) should be modified to

$$\dot{H} = \dot{H}_a + \dot{H}_e + \dot{H}_d \tag{24}$$

and

$$\dot{h} = \dot{H}_a + \dot{H}_e + \dot{H}_d + \dot{N} \tag{25}$$

Sea level data can, in principle, be combined with levelling data in two different ways when determining the land uplift. One way is to let the land uplift values from the sea level stations only supply an additive constant to the land uplift differences from relevellings. Such methods has been applied in Finland (Suutarinen, 1983) and Denmark (Bedsted Andersen et al., 1974). The other way is to let the sea level stations contribute with their land uplift values in a common adjustment with the relevelled uplift differences. Such a method has been applied in Sweden. Recently Sjöberg (1987) and Vaniček & Sjöberg (1987) have proposed a new version of this method : Differences of land uplift between sea level stations, calculated from the differential sea level data, should be included together with the relevelled land uplift differences in the common adjustment.

However, if this method is to work we have to assume that the oceanographic deviation of mean sea level is time-independent, i.e. that $\dot{H}_d$ is zero (or a constant) all over. There is nothing to show that such an assumption is justified. If it is not, the method leads to a mixture of apparent and levelled land uplift in the adjustment. The deviation of the mean sea level along the Swedish east coast is 14 cm according to geodesy and 23 cm according to oceanography (section 1.4). A change of, say, 3 – 4 cm during the 68 years between the two Swedish levellings would cause an effect of 0.5 mm/yr in the apparent land uplift difference along the coast, but not in the levelled land uplift difference. Thus the two differences may not be treated as interchangeable with each other.

The author would like to recommend the following procedure : First, determine the apparent land uplift from c. 100 years of sea level observations of one fundamental station, e.g. Stockholm (which has the longest continued series of sea level observations in the world). Second, estimate and add the eustatic change of sea level for the time period in question to obtain the (approximate) levelled land uplift of the fundamental station. Third, use this together with the relevelling data in the network adjustment to obtain the levelled land uplift in the whole network of Fennoscandia. Where two fairly close sea level stations are connected by an extremely long levelling line only, one might also use the differential sea level data (without applying the very small standard error). A good example would be the mareographs of Stockholm and Hanko (Hangö), where the present levelling distance around the whole Gulf of Bothnia is about ten times the distance across the sea south of Åland. Fourth, compare the obtained levelled land uplift differences between the other sea level stations in the network with the apparent land uplift differences to see if any oceanographic or other conclusions can be drawn. Fifth, for gravity purposes (and GPS) estimate and add the rise of the geoid to obtain the absolute land uplift.

2.4. Remaining land uplift

The remaining uplift of the crust Δh, until equilibrium is restored, should be possible to compute from the remaining rise of the geoid ΔN. Considering the land uplift to have the shape of a cosine surface one can show that, r being the radius of the land uplift area,

$$\Delta h_o = \frac{g}{4G\rho r}\,\Delta N_o = 1.1 \cdot 10^7\,\frac{\Delta N_o}{r} \quad \mathrm{m} \tag{26}$$

(cf. Ekman, 1977). This formula holds for the centre of the land uplift (indicated by the index o); outside this the remaining uplift may then be estimated by proportionality to the present uplift rate.

In the practical application of (26) one faces the problem of determining the remaining rise ΔN_o of the geoid or, in other words, of determining the present depression of the geoid corresponding to the mass deficiency related to the land uplift process. One attempt to do this has been made by Bjerhammar (1980), using a relevant part of the series expansion of the Earth's gravity field. A similar idea was presented already by Takeuchi & Yamashina (1973). Another attempt has been made by Kakkuri (1985) using a geometrical way of reasoning.

However, both attempts suffer from the same uncertainty : One does not really know to what extent the obtained values of ΔN_o reflect the postglacial mass deficiency, and to what extent they reflect mass deficiencies of other origins. Anderson (1984) pointed out that the Moho depth under Fennoscandia is so large that a considerable part of the geoid depression might be due to the thickness of the crust here.

Bjerhammar's and Kakkuri's values of ΔN_0 are about $10\,m$ which, inserted in (26) together with $r = 700\,km$, would yield $\Delta h_0 \approx 150\,m$. Correcting for the crustal structure according to Anderson ΔN_0 would be reduced to only about $2\,m$, resulting in $\Delta h_0 \approx 30\,m$. This illustrates the difficulties in the numerical determination of the remaining uplift; cf. also Balling (1980), Wolf (1986) and Mörner (1988).

3. Polar Drift

The mean pole moves slowly, at a speed of $0''.3\,(10\,m)$ per century, in the direction towards eastern Canada. The origin of this polar drift was unknown until Nakiboglu & Lambeck (1980) discovered that the drift could be accounted for by redistributions of mass in connection with the postglacial rebounds of Canada and Fennoscandia; see also Wu & Peltier (1984).

A change in the position of the pole causes a change in the centrifugal force of the Earth and, thereby, in gravity; cf. Burša (1972), Lambeck (1973) and Wahr (1985) :

$$dg = 16.3\,\delta\,(x \cos\lambda - y \sin\lambda) \sin 2\theta \ \mu\text{gals} \qquad (27)$$

Here again $\delta \approx 1.16$; x and y are coordinates of the pole in seconds of arc, λ is longitude and θ colatitude $(90° - \varphi)$.

Absolute gravity measurements have to be corrected for the effect of polar motion according to (27). Normally the correction is then made to the CIO. This means that we also make a correction for the polar drift since about 1900. If we want the gravity value to refer to some recent epoch, close to or equal to the year of the measurement, we should not make a reduction to the CIO but to the mean pole of the year in question. Let us, therefore, calculate the maximum effect of the polar drift since about 1900 on gravity. The mean pole of today has the approximate coordinates $x = 0''.1$, $y = 0''.3$ in the CIO system. Inserting this into (27) we find, for the point $\theta = \varphi = 45°$ and $\lambda = -72°$, $dg = 6\,\mu\text{gals}$. This is about equal to the gravity standard error. Consequently one has to bother about the polar drift when defining an accurate gravity system, at least in e.g. North America. In most of Europe, on the other hand, the effect becomes more or less negligible (cf. Haller & Ekman, 1988).

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