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A CALIBRATION SYSTEM FOR GRAVIMETERS USING A SINUSOIDAL ACCELERATION RESULTING FROM A VERTICAL PERIODIC MOVEMENT

Abstract

An inertial force is induced by the oscillating vertical displacement $A \sin \omega t$ *of a platform supporting a feedback LaCoste Romberg gravimeter.*

The resulting acceleration dg *is proportional to the square of the frequency and allows to perform an absolute calibration of the instrument.*

The effects of the vertical gradient and of the residual tilt are eliminated by operating at different frequencies.

After the experiment with a first prototype, a new platform based on a different mechanical principle has been developed.

The relative precision is at least 0.25 % *as confirmed by the study of a* 6 *months tidal record. Some applications are described.*

1. Introduction

Defining the vertical position h of a gravimeter by

$$h = h_0 + A \sin \frac{2\pi t}{T} \tag{1}$$

with

h_0 , the initial position

A , the amplitude of a vertical sinusoidal movement

T , the period of the oscillation.

The amplitude of the resulting sinusoidal acceleration dg acting on the beam is the sum of the inertial acceleration *(Table 1)* and a perturbing term supposed sinusoidal with an amplitude R including the vertical gradient and the tilt effects. R is obviously in phase with the acceleration so that

$$dg = -\left(\frac{4\pi^2}{T^2}\right) A + R \tag{2}$$

Bull. Géod. 63 (1989) pp. 223–235.

Expressing A in centimeters and T in seconds, the resulting acceleration is given in gal (1 gal = 1 cm s^{-2}).

This principle is used to perform an absolute calibration of the gravimeter.

Table 1

Amplitudes of the inertial accelerations at low frequencies for an amplitude A of 5 mm.

Period T (s)	100	200	400	800	2000
dg (μgal)	2000	500	125	31	5

The accuracy of the determination of dg is a function of the precision in the measurement of the amplitude A and the period T and depends from the anharmonicity of the sinusoidal movement.

The noise in the frequency range from 0.1 Hz to 10 Hz is critical due to the reaction of the gravimeter's beam to large accelerations *(Table 2)*.

Table 2

Amplitudes of the inertial accelerations at high frequency for an amplitude of 1 μm.

Period T (s)	10	3	1
dg (μgal)	40	440	4000

The constant vertical gradient effect (about 0.3 μgal/mm^{-1}) and the effect of small tilts of the platform induced by the displacement (a few μgal), are eliminated by comparing the results obtained at different frequencies.

Also the difference between two measurements at periods T_1 and T_2 is free from these constant perturbations :

$$dg_2 - dg_1 = 4\pi^2 \left(\frac{1}{T_1^2} - \frac{1}{T_2^2} \right) A \tag{3}$$

For long period (for example 2 hours) the inertial acceleration becomes very small and a direct measurement of R becomes possible *(Figure 1)*.

2. Description of the new VRR 8601 oscillating platform

The mechanical problems met in the construction of the oscillating platform are complex due to the high sensitivity of the astatized gravimeters. Moreover, when we interrupt the platform oscillations to record the earth tides, the stability must be comparable to the stability of a pillar. In a first experiment (Van Ruymbeke, 1986) this condition was not satisfied, which decreased the quality of the tidal registrations.

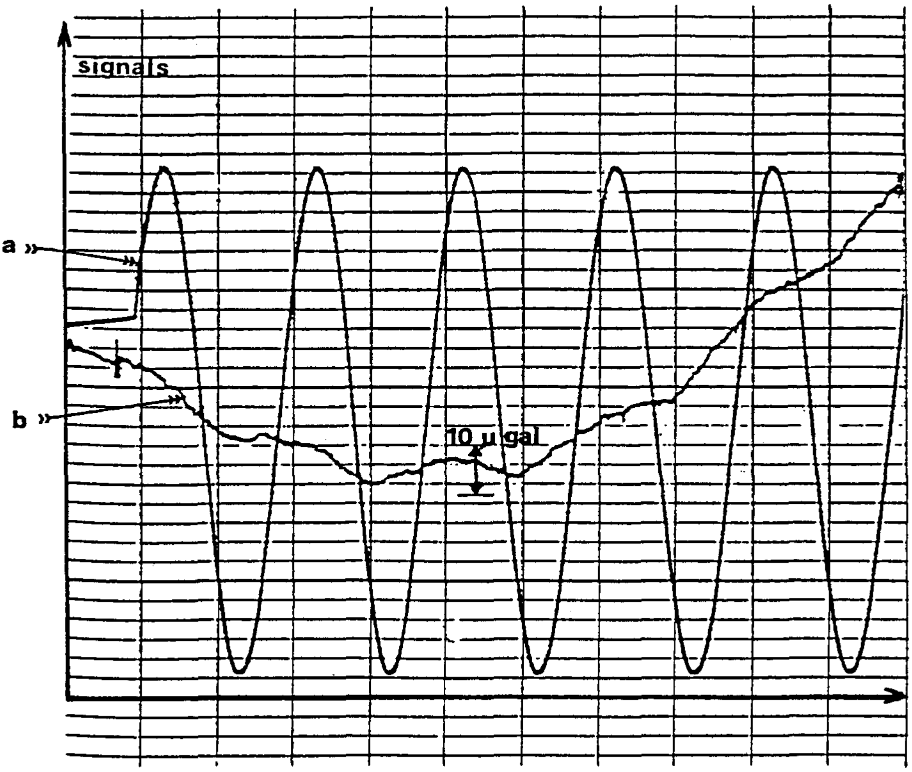

Fig. 1 — The curve (a) *represents the vertical oscillations of the platform recorded by an* L.V.D.T. *with an amplitude* A *of* 5 mm *and a period* T *of two hours. The curve* (b) *is the tidal signal from the gravimeter superimposed on a* 4 μgal *signal produced by the direct acceleration* (0.7 μgal) *and by the vertical gradient* (3.3 μgal).

Moreover the access to the gravimeter when micrometer rotations were needed was too difficult with the type of suspension. The new VRR 8601 platform has been designed to eliminate these difficulties.

The method allowing to induce a sinusoidal movement can be mechanical (eccentric wheel) or electrical (magnetic force on a mass fixed at a spring (Valliant, 1973)).

We have chosen a mechanical eccentric system for its long term stability and the low anharmonicity of the movement.

The principle *(Figure 2)* consists in using an horizontal square platform (1) with at each corner a vertical foot fixed at its lower extremity to two long flat springs (2) rolling upon three cylinders. The cylinder (3) and (4) have fixed axes while the axis of the cylinder (5) can be moved up and down by the action of an eccentric wheel (a principle similar to the Kelvin tidal predictor machine). When the cylinder (5) goes down, the springs extremities fixed at the feet of the platform move up. As the displacement of the feet and springs are identical at their contact points with the cylinders, the lateral forces produced to rigidly maintain the platform do not induce any friction.

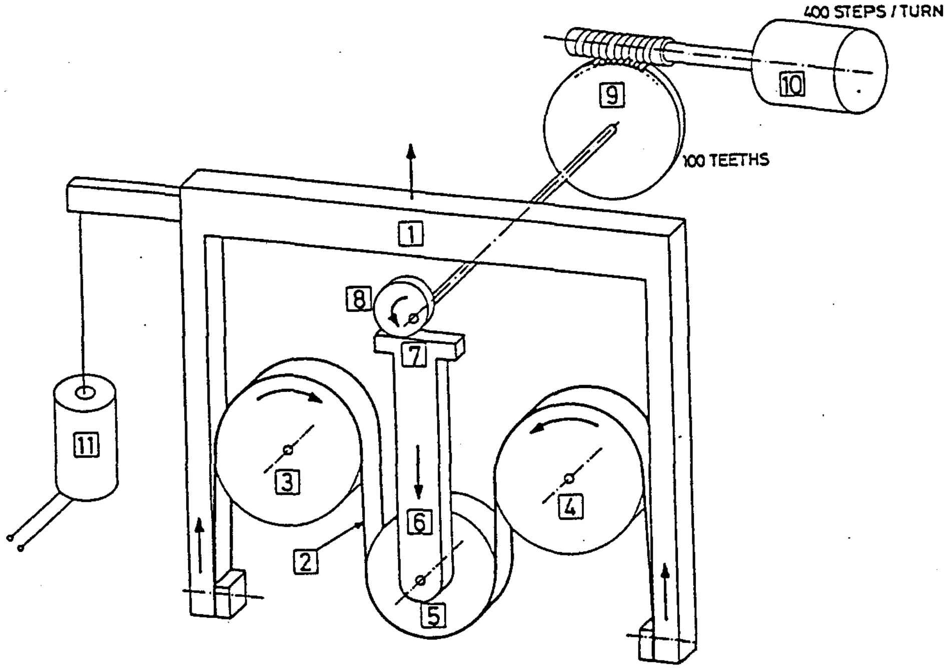

Fig. 2 – Principle of the VRR 8601 *platform.*

The eccentric (8) is placed inside a ball bearing which is rolling on a flat plate (7) fixed through (6) to the cylinder (5).

The rotation of the eccentric is driven by a large cog-wheel (9) driven itself by a 400 steps by turn motor (10) : a series of 40.000 steps thus corresponds to one turn of the wheel (8) and determines the period T of the platform sinusoidal movement.

The displacement of the system is measured with a Linear Variable Differential Transformer (L.V.D.T.) (11) with a precision of $0.1\,\mu m$. This allows to check the regularity and the vibration level of the lift motion. ***Figure 3*** is a sample of the signals recorded by the LVDT and by the gravimeter at a period of 420 seconds. ***Figure 4*** shows the evolution of the gravimeter signal with increasing speed.

3. The Data acquisition and treatment system

The amplitude of the acceleration is determined for a series of periods between 150 and 5000 seconds. The synchronization of the step motor is regulated by a 100 kHZ quartz crystal. The frequency is adjusted by means of a divider. The error on the determination of the period T is thus negligible. A series of 64 pulses per period synchronizes the data acquisition through a slave 6 digits volmeter connected to a mini computer. From these 64 measurements (X_i) we determine the amplitude V and the phase ϕ of the output voltage of the gravimeter.

We can write

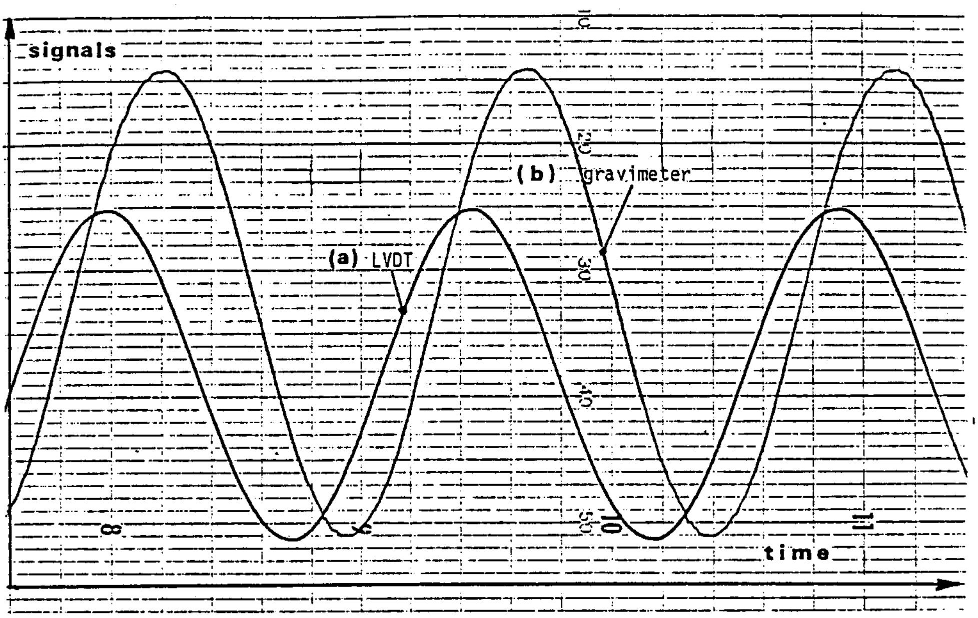

Fig. 3 — Registration of the signals of the feedback LaCoste Romberg gravimeter G 336 (b) *and of the* LVDT (a) *for a period* T = 420 *second. The horizontal offset between the two curves is produced by the overlapping of the recorder pens.*

$$X_i = V \sin(2\pi i/64 + \phi) + B , \quad 1 \leq i \leq 64 \tag{4}$$

B being the initial voltage and V the amplitude in volt.

The amplitude is estimated 16 times from independent groups of four measurements shifted by $\pi/32$:

$$V_j = \frac{1}{2}\sqrt{(X_j - X_{j+32})^2 + (X_{j+16} - X_{j+48}{}^2} \quad 1 \leq j \leq 16 \tag{5}$$

and the final determination of the amplitude V is the corresponding mean value

$$V = \frac{1}{16}\sum_{j=1}^{16} V_j \tag{6}$$

with the associated root mean square error

$$\text{M.S.E.} = \sqrt{\frac{\sum_{j=1}^{16} (V_j - V)^2}{240}} \tag{7}$$

This method of determination of V improves the rejection of the slight anharmonicities of the sinusoidal signal.

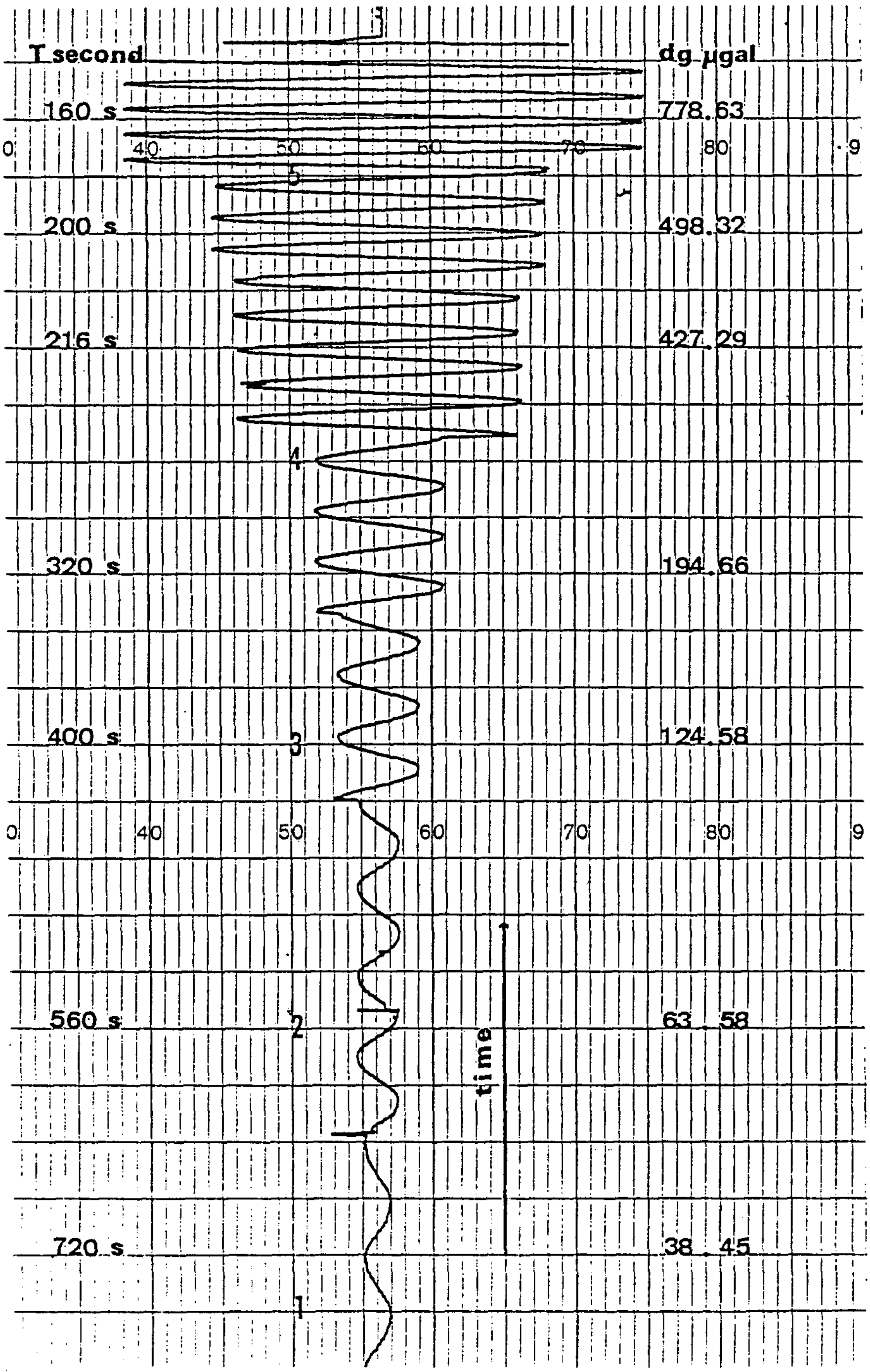

Fig. 4 — Registration of the feedback LaCoste Romberg gravimeter G 336 *placed upon the oscillating platform* VRR 8601.

An internal test of the quality of each group is possible by comparing the phases P_j defined by

$$P_j = arctan\left(\frac{X_j - X_{j+32}}{X_{j+16} - X_{j+48}}\right) - \frac{2\pi j}{64}\ , \qquad 1 \leq j \leq 16 \tag{8}$$

The phase shift ϕ is the mean of the individual phases P_j. It is increasing with the frequency, due to the effect of the filters which also attenuate the output voltage V.

4. Calibration of the VRR 8601 lift

Two parameters determine the acceleration :

- the period T which is directly measured by a frequency meter with a sufficient precision
- the amplitude A which has been calibrated with a Hewlett Packard Laser 5500 C interferometer, thanks to the support of the "Centre de Recherches Scientifiques et Techniques de l'Industrie des Fabrications Métalliques" (CRIF).

The reflecting corner cube of the interferometer was placed on the platform. The control unit was synchronized by the 64 pulses like the voltmeter. The amplitude has been determined for different periods *(Table 3)* by the same computation procedure and has been found to be

$$A = 5.050 \pm 0.002 \text{ mm}\ , \tag{9}$$

i.e. with a precision of 0.04 % .

For a period T, the amplitude of the acceleration is thus (eq. 2) :

$$dg = \frac{4\pi^2}{T^2} A = \frac{19{,}9287}{T^2} \text{ gal} \tag{10}$$

The LVDT has also been calibrated by comparison with the interferometer. Its sensitivity with a 15 volt supply voltage is

$$654.01 \pm 0.17 \text{ mV/mm} \tag{11}$$

i.e. with a precision of 0.03 % .

5. Measurements with the LaCoste Romberg G 336

This gravimeter is fixed on a VM horizontal pendulum baseplate (Melchior, 1983) to increase the levelling stability, it has been transformed in a feedback system by means of the VRL electronics (Van Ruymbeke, 1985).

The bubbles of the two standard level mounted on the instrument move by less than one tenth of a division during a complete sinusoidal displacement.

We adjust the gravimeter at the minimum of sensitivity to tilt so that the variations of the gravity forces acting on the beam due to the tilt become negligible (less than 2 μgal). This is confirmed by the data treatments (Equations (4) to (8)) which should detect any departure from a pure harmonic response.

Table 3

Measurements of the amplitude of the displacement A for different periods T by a LVDT and a LASER interferometer

Period T sec	Amplitude LVDT mV	Amplitude LASER μm	Ratio $mV/\mu m$
150		5052,6	
170	654,41	5053,1	0.12951
200	654,14	5048,3	0.12958
300	654,09	5053,4	0.12944
350	654,10	5048,7	0.12956
400	654,11	5050,6	0.12951
450	654,02	5048,8	0.12954
550	654,02	5049,4	0.12952
600	653,90	5050,5	0.12947
700	653,90	5049,1	0.12951
750	654,06	5048,9	0.12954
800	653,88	5049,6	0.12949
850	653,75	5046,0	0.12956
900	653,80	5049,4	0.12948
Average:	654.01 ± 0.17 mV	5049.89 ± 1.95 μm	0.12951 ± 0.00004 $\frac{mV}{\mu m}$

As a first experiment three series of calibrations were performed during the tidal records with periods of oscillation ranging between 200 and 720 seconds *(Table 4)* which correspond to the optimal range of accelerations.

Indeed at shorter periods the acceleration becomes very large and errors appear due to non linearity and damping of the gravimeter output.

At longer periods the signal to noise ratio becomes too small to give a good amplitude determination.

Table 4

Preliminary calibrations performed during the tidal registration

Period of oscillation	Amplitude of acceleration	Amplitudes recorded by the gravimeter at three epochs and corrected of the filter effect		
		86.07.30	86.10.10	86.12.15
T (s)	dg (μgal)	Z_1 (mV)	Z_2 (mV)	Z_3 (mV)
720	38.5	151.8	155.3	
480	86.5	328.1	327.6	324.4
400	124.6	478.3	476.6	474.8
360	153.8	587.4	584.6	587.7
320	194.6	737.4	747.7	742.5
200	498.3	1888.0	1900.0	1888.9

linear regressions

$Z_i = a_i + b_i$ dg

σ = standard deviation in mV

Epoch	a_i mV	b_i mV/μgal	σ_i mV	S mV/μgal	b/S
1 86.07.30	4.82	3.77837	2.85	3.77837	1.0000
2 86.10.16	3.33	3.80548	4.69	3.81463	0.9976
3 86.12.15	1.90	3.78979	4.05	3.79985	0.9974

S is the sensitivity obtained by the Nakai procedure (Nakai, 1979) and normalized on the inertial calibration of the 86.07.30.

The internal error on the individual acceleration measurements can be represented by the standard deviation σ of the regression. We obtain σ values of the order of one μgal *(Table 4).* We can also determine the evolution of the gravimeter sensitivity by the Nakai procedure (Nakai, 1979) applied to the tidal record. After a normalization on the inertial calibration of the 86.07.30 the ratios b/S is very close to unity for the

86.10.16 and 86.12.15 calibrations. It confirms that the calibrations follow the real changes of gravimeter sensitivity.

Later on we calibrated two times simultaneously every week to study the repeatability of the calibrations. So we obtain 26 double calibrations *(Table 5)*. The standard deviation of the differences between the two calibrations performed the same day reaches only 0.2 %.

6. Measurements with the LaCoste Romberg G003 equiped with a M.V.R. feedback system

This gravimeter is working with a new feedback system called Maximum Voltage Retroaction (Van Ruymbeke, 1988) with a gain close to infinity. A reaction time of less than one millisecond is obtained by replacing the original CPI electronics by the M.V.R. system.

The calibration presented at the *Table 6* confirmed the internal precision improvement (Standard deviation σ on the O–C residues is 0.27 μgal).

Table 5

Results of the linear regression for simultaneous calibrations

epoch	a (mV)	b (mV μgal^{-1})	σ (mV)	— —	a (mV)	b (mV μgal^{-1}	σ (mV)
87 01 09	5.715	3.674	4.97	—	3.527	3.683	1.46
87 01 19	1.875	3.687	2.32	—	3.243	3.676	1.23
87 02 04	4.090	3.660	4.81	—	3.219	3.661	3.23
87 02 18	4.701	3.686	2.43	—	4.285	3.691	4.25
87 02 24	4.096	3.630	1.33	—	4.813	3.619	1.26
87 03 06	-1.906	3.708	2.34	—	0.614	3.720	9.40
87 03 13	5.382	3.690	6.16	—	7.133	3.687	5.28
87 03 20	2.433	3.664	3.49	—	0.653	3.665	3.51
87 03 30	5.549	3.683	4.63	—	2.305	3.679	7.26
87 04 06	1.687	3.669	3.49	—	3.601	3.657	1.73
87 04 17	3.515	3.640	2.63	—	4.001	3.641	3.19
87 04 28	7.336	3.678	2.62	—	5.799	3.679	3.05
87 05 05	6.210	3.636	1.57	—	2.985	3.645	1.24
87 05 15	7.889	3.659	3.04	—	7.031	3.663	4.63
87 05 22	4.783	3.653	3.03	—	5.502	3.644	1.54
87 05 26	1.517	3.675	4.45	—	2.313	3.673	3.25
87 06 05	3.911	3.677	2.55	—	4.117	3.678	2.34
87 06 11	4.557	3.696	3.25	—	2.705	3.702	6.15
87 07 03	4.876	3.669	2.20	—	5.560	3.665	2.55
87 07 10	6.718	3.689	5.29	—	2.825	3.701	3.93
87 07 17	4.956	3.686	6.22	—	6.574	3.688	2.16
87 07 24	6.042	3.682	3.23	—	4.657	3.686	4.44
87 07 31	6.875	3.664	2.45	—	5.438	3.668	3.13
87 08 24	4.770	3.655	2.59	—	4.966	3.656	5.29
87 09 09	7.140	3.656	1.72	—	7.432	3.657	4.51
87 09 30	-2.959	3.683	5.94	—	2.552	3.676	5.81

Table 6

Calibration performed with MVR Feedback system

Period of Oscillation T (s)	Amplitude of acceleration dg_{Theor} (μgal)	Amplitude recorded by the gravimeter Z (mV)	Amplitude computed from Z dg_{Meas} (μgal)	Residual dg_{Meas}- dg_{Theor} (μgal)
720	38.45	97.3	38.23	-0.22
560	63.56	156.5	63.20	-0.36
400	124.58	302.5	124.78	+0.20
320	194.66	469.4	195.18	+0.52
216	427.29	1019.4	427.18	-0.11
200	498.32	1188.3	498.42	+0.10
160	778.63	1852.3	778.50	-0.13

linear regression
$Z = a + b\, dg_{Theor}$
$dg_{Meas} = 6.6677 \text{ mV} + 2.37073\, dg_{Theor}$
standard deviation
$\sigma = 0.27\ \mu\text{gal}$

A first analysis of the tidal records obtained in Brussels with this improved system gives for the wave O_1 an amplitude factor

$$\delta O_1 = 1.1518 \pm 0.0032$$

This is corresponding to a correction of about 0.8 % to the Brussels Standard Model (Ducarme, 1975) defined by

$$\delta_{O_1} = 1.1610$$

For the wave M_2 the diminution of the δ factor reaches 1.0 %.

7. Applications of the VRR 8601 lift

It is important to check the linearity of the LaCoste Romberg gravimeter transformed into a fedback system. The Sato-Harrison method is based on the comparison of the output signal.

We adjust the linearity without the limitation of the precision due to the micrometer errors *(Figure 5)* using the same acceleration for different values of the feedback voltage.

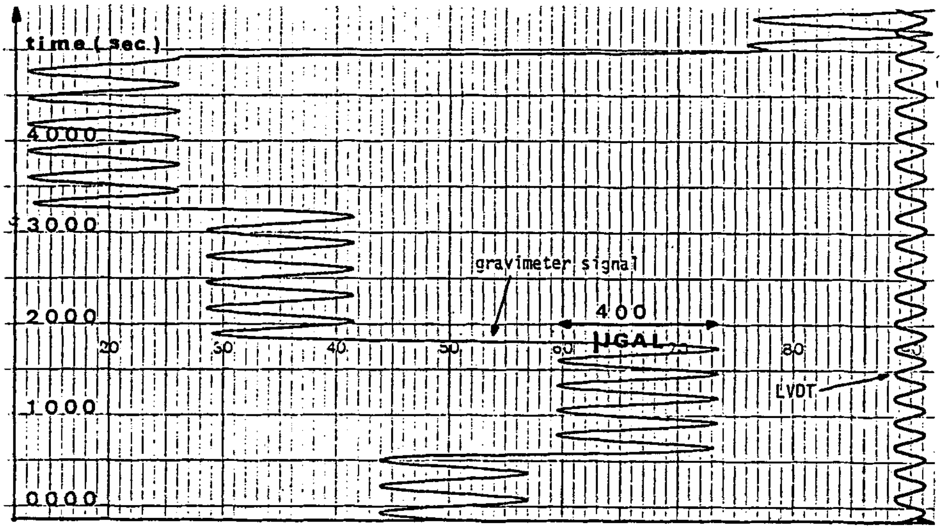

Fig. 5 — Linearity adjustment by determinations of the sensitivity of the gravimeter G 487 *for five different positions of the micrometer. The five* 200 µgal *amplitudes must be equal if the linearity is well adjusted.*

It is also possible to determine the amplitudes and phases transfer function of the adjustable damping system used in some LaCoste Romberg gravimeters. As an example, a feedback electronics installed on the LCR G600, equipped with such a system, was tested with and without the maker damping. The sensitivity was exactly the same. However the maximum possible gain becomes much higher, typically 100 instead of 30. The oscillations showing up at large micrometer rotations (2 milligal) seem to be suppressed by the damping of the gravimeter.

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Mr. F. Renders and Mr. R. Laurent have participated to the technical part of this research while Mrs. J. Jean performed the calibrations of the gravimeter and the digitization of the records. Miss L. Vandercoilden was responsible for the data processing.

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