Journal of Radioanalytical Chemistry, Vol. 61, No. 1–2 (1981) 345–359

COUNTING STATISTICS OF SHORT-LIVED NUCLIDES*

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(Received August 6, 1980)

The well-known Poisson formula for counting statistics is generalized to the situation where the radioactive source studied, with mean lifetime $1/\lambda$, decays appreciably during the total time of observation T. A general expression is given for the modified probability $_\lambda P(k)$ of observing k events in a short time interval $t_0 = T/n$, where the results are averaged over the period of observation T. This corresponds to the experimental distribution which is obtained by pooling together all the $n \gg 1$ individual measurements of k made with a given source. The deviation from the simple Poisson law, which neglects decay, depends essentially on the quantity $\vartheta = \lambda \cdot T$. If ϑ is of the order of unity, the deformation is strong enough that it can serve as the basis of a new method for measuring the half-life of the nuclide involved.

Introduction

For many applications in the field of radioactivity mesurements, the well-known Poisson formula

$$P_\mu(k) = \frac{\mu^k}{k!} \cdot e^{-\mu}, \qquad \mu > 0, \tag{1}$$

for the probability of observing exactly $k = 0, 1, 2, \ldots$ events within a fixed time interval t, when $\mu = \rho t$ is the corresponding expectation value, gives a most useful and sufficiently accurate description of the random emissions originating from a radioactive source. For metrological or other purposes where one has to meet the most exacting standards, refinements of Eq. (1) may be needed, however. The reasons can be broadly classified as experimental or theoretical. Among the experimental causes for observing deviations from Eq. (1) – assuming a correct functioning of the measuring equipment – there are in particular the counting losses. In fact, the probability density for the time interval $\delta = t_{j+1} - t_j$ between the arrival times t_j of consecutive events belonging to a Poisson process is given by the exponential

$$f(\delta) = \rho \cdot e^{-\rho\delta}, \qquad \text{for } \delta \geq 0,$$

*Invited paper.

which reaches its maximum at $\delta = 0$. Hence, as a consequence of the finite length of a pulse and the limited resolving time of a counter, such losses can never be completely avoided. For high count rates, dead-time losses may become a serious problem; their treatment will be deferred to the section "Dead-time corrections".

What about the "theoretical" reasons for violating the Poisson law which, it would seem, is often chosen mainly for its simplicity? If all known influences due to the counting equipment and the (slow) decay are properly taken into account, then the most recent and accurate experimental checks (Ref.[1], also for earlier references) show no significant deviation from Eq. (1). Thus, the basic properties needed for an integer-valued random process to be of the Poisson type (essentially independent increments and singly-occurring events; for details see e.g. Refs[2,3]) seem to be sufficiently well realized in the nuclear systems available for observation. Earlier claims for the detection of real deviations were no doubt caused by incomplete dead-time corrections.

However, there is an important feature common to any source and which is neglected in the derivation of Eq. (1), namely its finite lifetime. It is the purpose of this paper to show how the effect of decay modifies the Poisson law.

Although one cannot expect such a basic and obvious influence to be an entirely new subject of study, the relevant literature is scarce. An early attempt by RUARK and DEVOL[4] to tackle this problem is still quite interesting, although it does not go sufficiently far to be of real use. The main result obtained for a decaying source is given in their Eq. (16). For a source which consists at $t = 0$ of N atoms, each of which has a probability $p = 1 - \exp(-\lambda t_0)$ of decaying within a time t_0, the probability of observing k counts in the time interval from T_1 to $T_1 + T_2$ is denoted by $P_k(T_1, T_1 + T_2)$. The overall efficiency of the detector is written as gA. Although the expression indicated by the authors is correct, it is given in a form which hides some of its essential features. However, the result can be obtained readily in the equivalent form (still using their notation)

$$P_k(T_1, T_1 + T_2) = \binom{N}{k} (1 - \pi)^{N-k} \cdot \pi^k, \qquad (2)$$

with $\pi = gA\, e^{-\lambda T_1} (1 - e^{-\lambda T_2})$,

where the binomial structure is now clearly visible.[5] Unfortunately, this does not yet provide a generalization of the Poisson law since the result given above assumes a single sampling interval T_2. What we would like to have, instead, is an expression, applicable to a single source, where T_2 covers successively the whole period of observation. The necessary averaging process, although clearly recognized by the authors (Ref.[4], p. 358), could not be performed. The RUARK and DEVOL approach, especially in some more developed form, is likely to be useful for measurements with microscopic sources where N is no longer a very large number and, as a consequence, the usually supposed exponential diminution of activity with time becomes questionable.

In a previous attempt to generalize the Poisson law we started by deriving the probability densities for the arrival times of pulses for the case of an exponential decay of the source.[6,7] However, this approach soon became too complicated and it had to be abandoned for mathematical reasons before useful results concerning the modified counting statistics could be obtained.

The approach presented in what follows avoids these difficulties by restricting to larger sources where $N \gg 1$. In order to emphasize the main ideas, we shall omit most of the mathematical details, some of which, however, can be found in an earlier publication.[8]

As is well known, the development in time of the activity stemming from a given radionuclide may be quite complicated, especially if the decay is an intermediate step in a longer chain process. These more intricate situations, although they may be of practical interest, will not be considered here. In a majority of cases the decay can be described by a simple exponential law. However, it may be difficult or impossible (for instance when betas are observed) to pick out one single nuclide when the source available contains other isotopes or impurities as well. Then, one may have to deal with the case where the count rate of the source available for experimentation is given by

$$\rho(t) = \sum_{s=1}^{S} \rho_{os} \cdot e^{-\lambda_s t} + \beta, \tag{3}$$

where S denotes the number of decay branches considered, with respective mean lifetimes $1/\lambda_s$, and β is a constant background rate. Unfortunately, it turns out that a closed form of the probability distribution can only be derived for $S = 1$.

Evaluation of the modified distribution law

It will be assumed in what follows that the total measuring time T has been subdivided into a large number n of counting intervals which cover the whole range T and are of equal length $t_0 = T/n$. Our goal is to obtain an expression for the probability of observing exactly k events in t_0 (independent of its location within T). Assuming Poisson statistics, this probability is given, according to Eqs

(1) and (3), for the interval number j (i.e. for $(j-1)\,t_0 \leqslant t < jt_0$), by

$$P_j(k) = \frac{(\rho_j \cdot t_0)^k}{k!} \cdot e^{-\rho_j \cdot t_0}, \tag{4}$$

$$\text{where } \rho_j = \beta + \sum_{s=1}^{S} \rho_{os} \cdot e^{-j \cdot \lambda t_0}$$

and* $0 \leqslant j < n$.

If all the n results obtained within T are pooled together, we have

$$_\lambda P(k) = \frac{1}{n} \sum_{j=0}^{n-1} P_j(k) = \frac{t_0 \cdot e^{-\beta t_0}}{T \cdot k!} \sum_j (\beta t_0 + \sum_s M_s)^k \cdot \exp(-\sum_s M_s),$$

with $M_s = \rho_{os} t_0 \cdot \exp(-j\lambda_s t_0)$, $\qquad s = 1, 2, \ldots, S$.

Since $n \gg 1$ this is, to a good approximation (putting $jt_0 = t$),

$$_\lambda P(k) = \frac{e^{-g}}{T \cdot k!} \int_0^T (g + \sum_s M_s)^k \cdot \exp(-\sum_s M_s)\, dt,$$

with $g = \beta t_0$.

Expansion of the multinomial gives

$$_\lambda P(k) = \frac{e^{-g}}{Tk!} \sum_{(r_s)} \frac{k!\, g^{r_0}}{\sum_{s=0}^{S} r_s} \int_0^T \prod_{s=1}^{S} M_s^{r_s} \cdot \exp(-\sum_{s=1}^{S} M_s)\, dt, \tag{5}$$

where (r_s) includes all summations for which

$$\sum_{s=0}^{S} r_s = k, \qquad \text{with } r_s = 0, 1, 2, \ldots, k.$$

*Use of the mean activity instead of the one corresponding to the beginning of t_0 would amount to replacing j by $j + (1 - e^{-\lambda t_0})/(\lambda t_0)$, a correction which is negligible for $n \gg 1$.

Unfortunately, this requires for $S > 1$ the evaluation of integrals which are unknown. Thus, for instance even the case $S = 2$ leads to an integral of the type

$$\int_0^T \exp[-(r_1\lambda_1 + r_2\lambda_2)t - \mu_{01} e^{-\lambda_1 t} - \mu_{02} e^{-\lambda_2 t}]\, dt,$$

which we cannot solve.

We therefore must restrict ourselves to $S = 1$, dropping the subscript s in what follows. Putting $\rho_0 t_0 = \mu_0$, we have for a single branch

$$_\lambda P(k) = \frac{e^{-g}}{Tk!} \sum_{r=0}^{k} \frac{k!}{(k-r)!\, r!}\, g^{k-r} \int_0^T M_1^r \cdot e^{-M_1}\, dt =$$

$$= \frac{e^{-g}}{Tk!} \sum_{r=0}^{k} \binom{k}{r} g^{k-r} \mu_0^r \int_0^T \exp(-r\lambda t - \mu_0 e^{-\lambda t})\, dt. \tag{6}$$

In the absence of background ($g = 0$), this can be further simplified to

$$_\lambda P(k) = {}_0P(k) \cdot \frac{1}{T} \int_0^T \exp[-k\lambda t + \mu_0(1 - e^{-\lambda t})]\, dt, \tag{7}$$

where $_0P(k) = (\mu_0/k!) \cdot e^{-\mu_0}$ is the Poisson probability without decay ($\lambda = 0$).

It should be noted that the general case [Eq. (5)], although at present not amenable to a closed mathematical form, may be treated numerically by a computer program (at least for moderate values of S), but this approach will not be followed here any more.

On the other hand, the integral appearing in Eq. (6) can be evaluated further and it is this situation with a single branch which will be described in what follows. In the evaluation of Eq. (6) use can be made of two special mathematical devices, namely the exponential integral function $E_1(\mu)$ and the incomplete gamma function $\gamma(k, \mu)$, which are usually defined[9] by the integrals

$$E_1(\mu) = \int_\mu^\infty \frac{e^{-x}}{x}\, dx, \qquad \text{for } \mu > 0,$$

$$\gamma(k, \mu) = \int_0^\mu e^{-x} \cdot x^{k-1}, \qquad \text{for } k > 0. \tag{8}$$

As has been shown in Ref.[8], use of these definitions allows us to bring Eq. (6) into the form

$$\text{– for } k = 0: \qquad {}_\lambda P(0) = \frac{e^{-g}}{\vartheta} \left\{ E_1(\mu_1) - E_1(\mu_0) \right\}, \tag{9}$$

$$\text{– for } k \geqslant 1: \; {}_\lambda P(k) = \frac{e^{-g}}{\vartheta k!} \left\{ g^k \left[E_1(\mu_1) - E_1(\mu_0)\right] + \sum_{r=1}^{k} \binom{k}{r} g^{k-r} \left[\gamma(r,\mu_0) - \gamma(r,\mu_1)\right] \right\},$$

where $\vartheta = \lambda T$ and $\mu_1 = \mu_0 \cdot e^{-\vartheta}$.

In the absence of background the general expression [Eq. (9)] reduces to

$${}_\lambda P(k = 0) = \frac{1}{\vartheta} \left[E_1(\mu_1) - E_1(\mu_0)\right] \text{ and}$$

$${}_\lambda P(k \geqslant 1) = \frac{1}{\vartheta k!} \left[\gamma(k,\mu_0) - \gamma(k,\mu_1)\right]. \tag{10}$$

To show more directly the deviation from the corresponding Poisson distribution ${}_0P(k)$ without decay ($\vartheta = 0$), the result [Eq. (10)] can be written in the form of the product

$${}_\lambda P(k) = {}_0P(k) \cdot C_k,$$

where the correction factor C_k is given by

$$C_k = \frac{e^{\mu_0}}{\vartheta} \cdot \begin{cases} \left[E_1(\mu_1) - E_1(\mu_0)\right], & \text{for } k = 0, \\ \mu_0^{-k}\left[\gamma(k,\mu_0) - \gamma(k,\mu_1)\right] & \text{for } k \geqslant 1. \end{cases} \tag{11}$$

It is interesting to note that an expression for the modified probability distribution for the case without background was found at much the same time and quite independently by a group from Columbia University, New York.[10] It is completely equivalent to Eq. (10). As the motivation for the research and the approach chosen in Ref.[10] to tackle the problem differ in many respects from Ref.[8], it is worth while to study both papers.

Some examples of deformed distributions, as described by Eq. (10), are plotted in Fig. 1 for illustration. Two other examples (for $\mu_0 = 10$ and 50) can be found in Ref.[8]

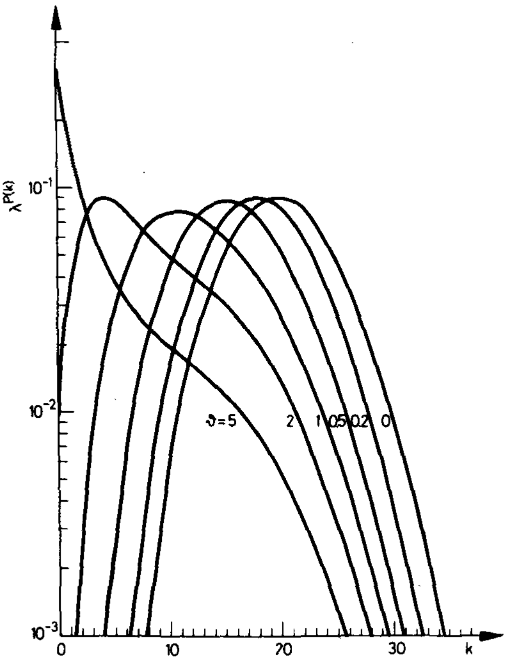

Fig. 1. Decay-modified Poisson probabilities $_{\lambda}P(k)$ for $\mu_0 = 20$ and some values of $\vartheta = \lambda T$. Background is assumed to be negligible. The case $\vartheta = 0$ corresponds to the normal Poisson distribution. For clarity, the discrete distributions are indicated by continuous lines.

For very small perturbations, i.e. $0 < \vartheta \ll 1$, approximate formulae may be useful and sufficient. A particularly simple expression for the correction factor C_k is given by

$$C_k \cong \frac{1 - e^{-\vartheta(k-\mu_0)}}{\vartheta(k-\mu_0)} \,. \tag{12}$$

A second-order expansion in ϑ can also be derived which reads

$$C_k \cong 1 - \frac{\vartheta}{2}(k-\mu_0) + \frac{\vartheta^2}{6}\,[(k-\mu_0)^2 - \mu_0\,]. \tag{13}$$

For details again see Ref.[8]

The moments of the modified distribution

Direct evaluation of the moments of k, defined for order r = 0, 1, 2, ... by

$$E(k^r) = \sum_{k=0}^{\infty} k^r \cdot {}_\lambda P(k), \tag{14}$$

first looks like a rather hopeless enterprise, in view of the complicated form of the probabilities ${}_\lambda P(k)$ as given in Eqs (6) or (7). The situation changes completely, however, upon realizing that Eq. (14) involves two summations (namely over the number k of events and the observation time T) the order of which may be interchanged.

If we define a "momentary" expectation value of k by

$$E_t(k) = \rho(t) \cdot t_0 = g + \mu_0 \cdot e^{-\lambda t}, \tag{15}$$

then the process is at any given time t of the Poisson type, with expectation $E_t(k)$.

The first moment (or expectation value) is now obtained by an averaging over the total measuring time T, thus

$${}_\lambda E(k) \equiv {}_\lambda[E_t(k)] = \frac{1}{T} \int_0^T E_t(k)\, dt. \tag{16}$$

Use of Eq. (15) gives readily

$${}_\lambda E(k) = g + \frac{\mu_0}{\vartheta} (1 - e^{-\vartheta}), \qquad \text{for } \vartheta \neq 0. \tag{17}$$

This result is in agreement with the corresponding expression given by TEICH et al. (Eq. (14) in Ref.[10]).

The variance of k is obtained in a similar way. Since for any given time t we have

$$E_t(k^2) = V_t(k) + E_t^2(k),$$

time averaging yields

$${}_\lambda V(k) = {}_\lambda[E_t(k^2)] - {}_\lambda[E_t(k)]^2 .$$

This, when written more explicitly as

$${}_\lambda V(k) = \frac{1}{T} \int_0^T [V_t(k) + E_t^2(k)]\, dt - \left[\frac{1}{T} \int_0^T E_t(k)\, dt\right]^2, \tag{18}$$

corresponds to an equation given previously by LEWIS et al.[11] in a somewhat different form. Some rearrangements then lead with Eq. (15) to the expression

$$ {}_{\lambda}V(k) = {}_{\lambda}E(k) - {}_{\lambda}E^2(k) + g^2 + \frac{2g\mu_0}{\vartheta}(1 - e^{-\vartheta}) + \frac{\mu_0^2}{2\vartheta}(1 - e^{-2\vartheta}) = $$

$$ = {}_{\lambda}E(k) + [{}_{\lambda}E(k) - g]^2 \left\{ \frac{\vartheta}{2} \left(\frac{1 + e^{-\vartheta}}{1 - e^{-\vartheta}} \right) - 1 \right\}, \quad \text{for } \vartheta \neq 0. \tag{19} $$

For $g = 0$ this agrees with a result given in Ref.[11] Eq. (19) appears to disagree with the corresponding expression (Eq. (16)) in Ref.[10], but the missing contribution ${}_{\lambda}E(k)$ turns up in the "count number domain", although the exact reason for this remains obscure to the present writer.

It is interesting to note that the ratio

$$ R \equiv \frac{{}_{\lambda}V(k) - {}_{\lambda}E(k)}{[{}_{\lambda}E(k) - g]^2} = \frac{\vartheta}{2} \left(\frac{1 + e^{-\vartheta}}{1 - e^{-\vartheta}} \right) - 1 \tag{20} $$

is a function of ϑ alone. For small values of ϑ one has the simple approximate relation $R \cong \vartheta^2/12$. Therefore, if the experimental values for E and V are inserted in Eq. (20), we can determine numerically the corresponding (positive) value of ϑ which, in turn, permits the evaluation of the half-life of the nuclide under study as

$$ T_{1/2} = (1/\lambda) \cdot \ln 2 = (T/\vartheta) \cdot \ln 2. \tag{21} $$

Obviously, for a situation such as the one described above where not only the moments but the whole probability distribution ${}_{\lambda}P(k)$ can be calculated. it will be preferable to deduce the half-life from a least-squares fit of the theoretical distributions to the observed data points. To simplify the numerical adjustment, we restrict the choice to curves the mean of which agrees with the experimental value while treating the lifetime as an unknown parameter, but more general strategies are possible and may be preferred by others. The present approach circumvents the need to know explicitly the initial count rate ρ_0 (or alternatively μ_0). A computer program performing the necessary numerical work has been written and successfully used. Fig. 2 shows as an example a fit to measurements[12] made with a decaying source of $^{116}In^m$. The half-life corresponding to this adjustment is $T = (3245 \pm 30)$ s which agrees very well with the currently accepted value. Since ϑ was only about 0.9, a longer observation of the decay would no doubt

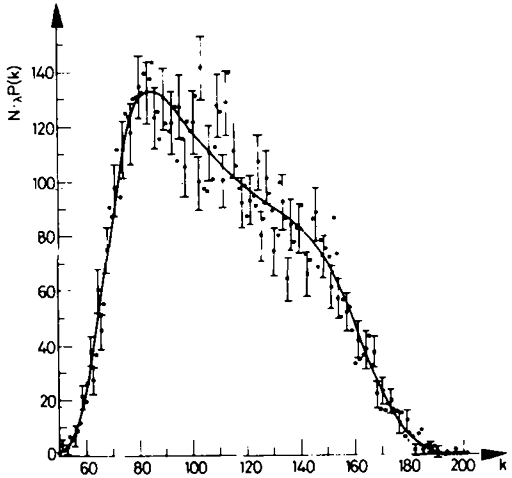

Fig. 2. Experimental measurement of a modified counting distribution for a $^{116}In^m$ source.[12] The theoretical curve giving the best fit assumes a half-life of 3245 s. About N = 10 000 measurements, each of duration $t_0 = 0.2$ s, have been made. This figure is taken from Ref.[13]

have resulted in a smaller uncertainty. The pooling of the individual counts can be performed during the measurement, using for instance an electronic device such as the one described in Ref.[14] The accumulation of the data can be followed on the screen of a multichannel analyzer.

Dead time corrections

As mentioned before, the experimental data obtained in such measurements are often somewhat distorted by counting losses due to the dead time of the measuring device, especially if a source with relatively strong initial activity is used in order to get better counting statistics. No such correction has been included in the approach described above.

Some attempts to derive explicit expressions for $_\lambda P(k)$ which take care of dead-time effects soon became too complicated and had to be abandoned. On the other hand, it turned out to be possible to take these distortions into account for the evaluation of the moments. The approach used will be sketched in what follows. Technical details of the derivation and more general results are given in two separate reports.[15,16]

To simplify the presentation, let us assume that background can be neglected. The count rate due to the source then follows as a function of time a simple exponential law

$$r_t = \rho_0 \cdot e^{-\lambda t}. \tag{22}$$

The distortion due to a dead time τ is well understood. Since in this type of experiment the beginning of a counting interval of length t_0 is chosen independently of the arrival of pulses, we are in the situation which corresponds to an equilibrium process (see e.g. Ref.[17] for this classification). Therefore, the count rate, after passage of the dead time, is given by the expressions

$$\rho(t) = \frac{r_t}{1 + r_t \tau}, \qquad \text{for } \tau \text{ non-extended,}$$

or (23)

$$\rho(t) = r_t \cdot \exp(-r_t \tau), \qquad \text{for } \tau \text{ extended.}$$

For both types of dead time we have further assumed that $\lambda\tau \ll 1$, a condition which is normally very well fulfilled. In any case the parameter ρ_0 stands for the unperturbed count rate at the beginning of the experiment ($t = 0$). As previously, with $E_t(k) = \rho(t) \cdot t_0$, the expectation value of k for the total observation time T is given by

$$_\lambda E(k) = \frac{1}{T} \int_0^T E_t(k)\, dt.$$

With the abbreviations $b = \rho_0 \tau$ and $B = b \cdot e^{-\vartheta}$ this leads to

– for a non-extended dead time:

$$_\lambda E(k) = \frac{\mu_0}{T} \int_0^T \frac{e^{-\lambda t}\, dt}{1 + b\, e^{-\lambda t}} = \frac{\mu_0}{b\vartheta} [\ln(1 + b) - \ln(1 + B)], \tag{24a}$$

– for an extended dead time:

$$_\lambda E(k) = \frac{\mu_0}{T} \int_0^T e^{-\lambda t} \exp(-b\, e^{-\lambda t})\, dt = \frac{\mu_0}{b\vartheta} [e^{-B} - e^{-b}], \tag{24b}$$

always assuming that neither τ nor λ vanish.

It may be interesting to mention that quite recently I found, to my surprise, that a formula given long ago by SCHIFF,[18] when properly interpreted, is in fact identical with Eq. (24b), or more exactly with the corresponding more general expression indicated in Ref.[16] where background has also been taken into account.

The evaluation of the variance is a bit more cumbersome. As before, the starting point is LEWIS' formula Eq. (18). The variance for a dead-time perturbed Poisson process is well known from previous studies (see e.g. Ref.[17]). For the present purpose the "exact" expressions may be somewhat simplified by dropping a small contribution which is independent of time and which differs for the various counting processes. We then have, – still to a very good approximation, since $b \ll t_0/\tau$ – the following expressions for the variance of k

$$V_t(k) = \frac{r_t\, t_0}{(1 + r_t)^3} \qquad \text{or} \qquad V_t(k) = r_t\, t_0 \left[\frac{e^{r_t \tau} - 2r_t \tau}{\exp(2r_t\, \tau)} \right], \tag{25}$$

which are valid for a dead-time of the non-extended or the extended type, respectively.

Substitution of $E_t(k)$ and $V_t(k)$, as given in Eqs (23) and (25), into Eq. (18) leads to some integrals which will not be given here explicitly; they are all of an elementary form and can be readily evaluated. The final result for the time-averaged variance of k can be stated as follows

– for a non-extended dead time:

$$ {}_\lambda V(k) = \frac{\mu_0}{b} \left\{ {}_\lambda E(k) + \frac{\mu_0}{b\vartheta} \left[\frac{1}{1+b} - \frac{1}{1+B} \right] - \frac{1}{2\vartheta} \left[\frac{1}{(1+b)^2} - \frac{1}{(1+B)^2} \right] \right\} - {}_\lambda E^2(k), \tag{26a}$$

– for an extended dead time:

$$ {}_\lambda V(k) = {}_\lambda E(k) - E^2(k) + \frac{\mu_0(\mu_0 - 2b)}{4b^2\, \vartheta} \left[(1 + 2B) \cdot e^{-2B} - (1 + 2b) \cdot e^{-2b} \right]. \tag{26b}$$

The expressions [Eq. (24)] for the first moment and Eq. (26) for the second moment generalize the previous formulae [Eqs (17) and (19)] for the presence of a dead time, covering thereby the majority of practical situations.

A quantity which can be easily obtained from a measurement of the distribution of k is the so-called Lexian ratio which is defined by

$$L \equiv {}_{\lambda}V(k)/{}_{\lambda}E(k). \tag{27}$$

It will be of interest to know how sensitive this ratio is to dead times. Whereas for $\tau = 0$ and no background we have simply

$$L = 1 + {}_{\lambda}E(k) \cdot R,$$

where R is a function of ϑ, as indicated in Eq. (20), the dependence on τ is more complicated. It has been calculated numerically for some values of ϑ, assuming ${}_{\lambda}E(k) = 10$. As can be seen from Fig. 3, the neglect of the effect of a dead time would resulting in attributing to an experimental Lexian ratio too low a value of ϑ. This influence augments rapidly with increasing $\tau_0 = \tau/t_0$, especially for a dead time of the extended type.

Conclusion

A method has been described for modifying the Poisson law in order to take account of the decay of a radioactive source during the measurement. This results in a modified probability distribution ${}_{\lambda}P(k)$. For small values of the characteristic parameter $\vartheta = \lambda T$, simple approximations are available for estimating the necessary corrections.

If the distortion of the Poisson distribution by decay is strong enough, the observed effect can be used for an evaluation of the half-life of the nuclide involved by fititng theoretical curves to the experimental data. In this way we can avoid a number of awkward problems connected with the adjustment of an exponential or of a straight line, if the logarithmic form is used. The difficulty arises particularly for results such as k = 0 or 1, as may occur towards the end of a decay. These data, which are therefore frequently eliminated, turn out in the new approach to contain valuable information which can be readily used.

If the influence of dead-time on the measurements cannot be neglected, the evaluation of ${}_{\lambda}P(k)$ is no longer possible in an analytical way and numerical methods have to be used. However, the moments of the distorted distribution can still be evaluated exactly. By comparison with the experimental values, for instance in the form of the Lexian ratio, a preliminary value for the half-life can then be readily obtained.

Further refinements of the method are possible and new practical applications are being worked out.

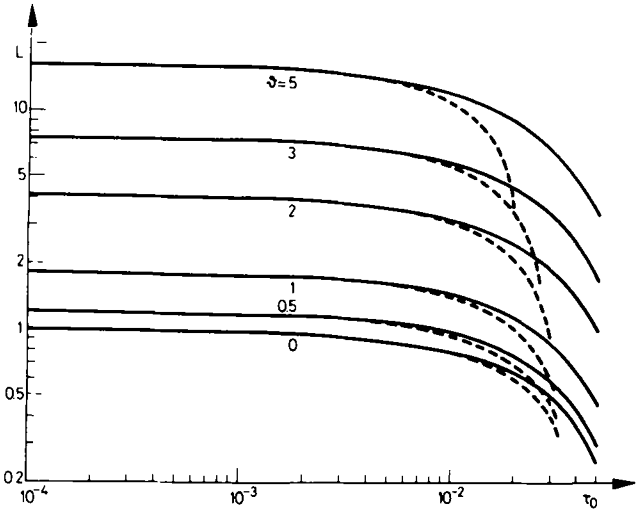

Fig. 3. Effect of the dead time τ on the Lexian ratio $L = {}_\lambda V(k)/{}_\lambda E(k)$ as a function of $\tau_0 = \tau/t_0$, for some values of $\vartheta = \lambda T$. All data assume an experimental mean value ${}_\lambda E(k) = 10$. The dashed curves are for an extended dead time

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It is a pleasure to thank Dr. G. W. McBETH of Brighton Polytechnic for sending me additional information on the measurement of $^{116}In^m$ reported in Ref.[12], and I also gratefully acknowledge an interesting exchange of information with Prof. M. C. TEICH of Columbia University, New York.

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