SCIENTIFIC AND TECHNICAL SECTION

EFFICIENCY OF THE METHOD OF SPECTRAL VIBRODIAGNOSTICS FOR FATIGUE DAMAGE OF STRUCTURAL ELEMENTS. PART 4. ANALYSIS OF DISTORTION OF HARMONICITY OF VIBRATION CYCLE OF BEAMS WITH CLOSING TRANSVERSE CRACKS

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Using an asymptotic method of nonlinear mechanics and concepts of linear fracture mechanics, we derive analytical expressions that relate the constant component and amplitudes of even harmonics, and the ratio of maximum deviations and half-periods of displacements of opposite signs in a vibration cycle of prismatic beams to the length of a closing transverse crack. The analytical solution is compared to the data of experimental determination of these parameters in bending vibrations of rectangular cantilever beams with an edge transverse crack. Analysis of the results of analytic and experimental investigations has shown that harmonic analysis of the strain waveform of beam-type elements in resonance vibrations can be used as an efficient diagnostic method for detecting fatigue damage such as closing cracks.

Notation

ω – natural frequency of an intact beam

ω_0 – natural frequency of a beam with a closing crack

ω_m – natural frequency of a beam with an open crack

β' – relative change of the natural frequency of a beam with a closing crack

L – beam length

h – height of the cross section

α – coefficient of relative changes in the stiffness of a cracked beam

l – crack length

γ – relative crack length

x_c – coordinate of the cracked section

Introduction. The previous reports [1, 2] give an analytical solution of a problem of longitudinal and bending vibrations of prismatic beam-like elements with transverse cracks, which is based on using an asymptotic method of nonlinear mechanics and concepts of linear fracture mechanics. In [3] we present relationships for the determination of changes in natural frequencies in longitudinal and bending vibrations of prismatic beams with transverse cracks of different type, size, and location, under various boundary conditions and vibration modes.

The objective of this work was to determine analytically the relative values of the constant component and amplitudes of even harmonics as well as the ratios of maximum deviations and half-periods of displacements of opposite signs in the cycle of vibrations of beams with closing transverse cracks. The analytical solution is compared with the

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data of the experimental determination of the above parameters in bending vibrations of a cantilever beam rectangular in cross section with an edge transverse crack.

Evaluation of the Error of the Approximate Analytical Solution. In [1], using an asymptotic method of nonlinear mechanics, we obtained the following second-approximation solution of the nonlinear differential equation, which represents free and resonance vibrations of beam-like elements with a closing crack for the case where the first harmonic of displacements of a generalized mass varies along a cosine curve:

$$u = a\left[\cos\varphi + \bar{a}_0 + \sum_{n=2,4,\ldots} (-1)^{n/2}\, \bar{a}_n \cos n\varphi\right], \tag{1}$$

where

$$\bar{a}_0 = \frac{\alpha}{\pi}, \quad \bar{a}_n = \frac{2\alpha}{\pi(n^2 - 1)^2}. \tag{2}$$

In the case of a sine variation of the first harmonic, the solution is rearranged to give

$$u = a\left[\sin\varphi + \bar{a}_0 + \sum_{n=2,4,\ldots} \bar{a}_n \cos n\varphi\right]. \tag{3}$$

The above solutions are simple and convenient for the approximate evaluation of the dependence of the most representative parameters of vibration-cycle harmonic components a_0 and a_2 on the extent of damage of a beam, which is given by the relative change α of the beam stiffness K. The coefficient α is determined as the difference in the longitudinal or bending stiffness of a cracked beam in half-cycles of its vibrations of different signs:

$$\alpha = \frac{K_- - K_+}{K_-}, \quad K_+ < K_-. \tag{4}$$

In Part 3 [3] we give expressions for calculating the coefficient α in longitudinal and bending vibrations of prismatic beams with transverse cracks of various types under various boundary conditions.

The Fourier expansion [4] of the form

$$u = \frac{A_0}{2} + \sum_{n=1,2,3,\ldots} (A_n \cos n\varphi + B_n \sin n\varphi) \tag{5}$$

of the solution to the differential equation of free vibrations of a single-mass piecewise-linear system which simulates a body with a crack and has no formal constraints as to the level of nonlinearity, yields more complicated (from the analysis standpoint) expressions for the coefficients of harmonic components:

$$A_n = \frac{2v_0\omega_a^2\left(1 + \cos \pi n \dfrac{\omega_0}{\omega_m}\right)}{T(\omega^2 - n^2\omega_0^2)(\omega_m^2 - n^2\omega_0^2)}, \quad n = 0, 1, 2, \ldots, \tag{6}$$

$$B_n = \frac{2v_0\omega_a^2 \sin \pi n \dfrac{\omega_0}{\omega_m}}{T(\omega^2 - n^2\omega_0^2)(\omega_m^2 - n^2\omega_0^2)}, \quad n = 0, 1, 2, \ldots, \tag{7}$$

where v_0 is the velocity of motion of a generalized mass m at an initial moment of time with a displacement equal to zero, T is the period of vibration of the system, $\omega_m^2 = K_+/m$, $\omega^2 = K_-/m$, $\omega_a^2 = \omega^2 - \omega_m^2 = (K_- - K_+)/m$, and $\omega_0 = 2\omega\omega_m/(\omega + \omega_m)$.

TABLE 1. Relative Values of Amplitudes of Some Higher Harmonics

α	A_1/B_1	B_2/A_2	B_3/A_2	A_3/A_2	B_4/A_2	A_4/A_2	$\bar{a}_0/\bar{A}_0$	$\bar{a}_2/\bar{A}_2$
0.05	0.020	0.040	0.008	0.0005	0.003	0.040	0.972	0.973
0.10	0.041	0.083	0.017	0.0020	0.006	0.039	0.947	0.951
0.15	0.064	0.128	0.027	0.0050	0.010	0.038	0.921	0.929
0.20	0.088	0.177	0.036	0.0100	0.013	0.036	0.893	0.908
0.25	0.113	0.229	0.046	0.0160	0.016	0.034	0.864	0.889
0.30	0.141	0.287	0.056	0.0250	0.019	0.031	0.834	0.871

To compare solutions (3) and (5), let us represent the latter as

$$u = B_1\left[\sin\varphi + \bar{A}_1\cos\varphi + \bar{A}_0 + \sum_{n=2,3,\dots}(\bar{A}_n\cos n\varphi + \bar{B}_n\sin n\varphi)\right]. \tag{8}$$

In doing so, we express the coefficients $\bar{A}_0$, $\bar{A}_n$, and $\bar{B}_n$ in terms of the parameter α:

$$\bar{A}_0 = \frac{(Q^2-1)(R^2-1)}{Q^2R^2\sin(\pi/Q)}, \quad \bar{A}_n = \frac{(Q^2-1)(R^2-1)[1+\cos(\pi n/Q)]}{(Q^2-n^2)(R^2-n^2)\sin(\pi/Q)}, \quad \bar{B}_n = \bar{A}_n\frac{\sin(\pi n/Q)}{1+\cos(\pi n/Q)}, \tag{9}$$

where

$$Q = \frac{1+\sqrt{1-\alpha}}{2}, \quad R = \frac{Q}{\sqrt{1-\alpha}}.$$

Table 1 summarizes the calculated values of the coefficients A_n and B_n with the values $\alpha \le 0.3$, which are the most important from the practical standpoint, i.e., which are of interest for diagnostics of early stages of damage. The data given in Table 1 demonstrate the following. First, for the first harmonic the determining amplitude is that at a sine function B_1. Second, among higher harmonics the most significant is the second one with the determining amplitude at a cosine function A_2. Third, the approximate solution (3) provides reasonable accuracy of the determination of the most significant parameters ($\bar{a}_0$, $\bar{a}_2$) of harmonic components of the displacement (strain) wave cycle.

However, it should be mentioned that solution (3), like solution (5), does not include the difference in vibration modes of a system in half-cycles of opposite signs. This restricts also the use of solution (5) in the case of systems with distributed parameters with a small nonlinearity.

Results of Analytical Solution. Let us consider the constant component $\bar{a}_0$, the second-harmonic amplitude $\bar{a}_2$, and the sum of amplitudes of all higher harmonics $\sum_{n=2,4,\dots}\bar{a}_n \approx 0.264\frac{\alpha}{\pi}$, whose dependence on the parameter α is linear (Fig. 1), as vibration damage indicators. However, it should be pointed out that their linear dependence on α does not dictate a linear dependence of these indicators on the crack size because the parameter α depends nonlinearly on the relative crack length, the crack relative location along the length of a beam, and the relative height of the cross section, and on the vibration mode of the latter [2, 3]. To see if a possible relationship exists between the parameter α and a particular damage, we give in Fig. 2 the dependence of the parameter α on the relative length of a transverse crack γ ($\gamma = l/h$) located at a different distance x_c from the fixture, for three modes of bending vibrations of a cantilever beam rectangular in cross section with the ratio of the height of the cross section to the beam length $h/L = 0.1$.

We can also consider the relative differences in half-periods $\Delta\bar{T}$ and maximum deviations $\Delta\bar{U}$ with opposite signs as damage indicators:

$$\Delta\bar{T} = \frac{T_+ - T_-}{T_+ + T_-} = \frac{1-\sqrt{1-\alpha}}{1+\sqrt{1-\alpha}}, \tag{10}$$

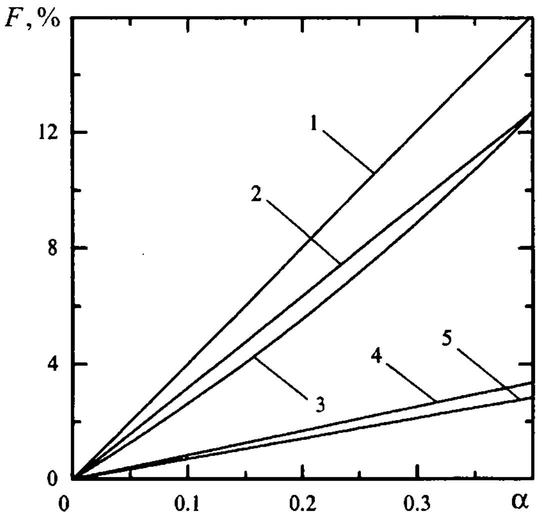

Fig. 1

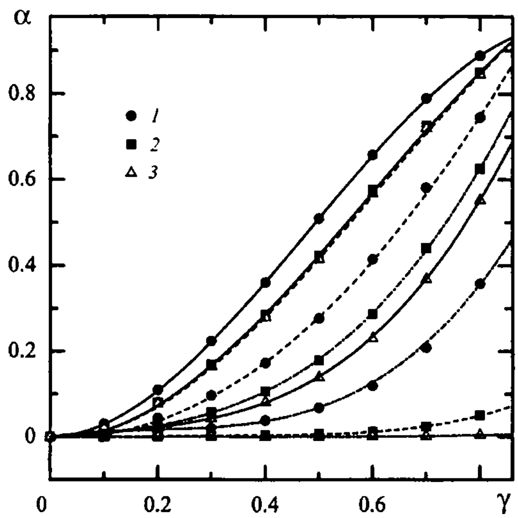

Fig. 2

Fig. 1. Dependence of various damage indicators on the parameter F: *1)* $F = \Delta\bar{U}$; *2)* $F = \bar{a}_0$; *3)* $F = \Delta\bar{T}$; *4)* $F = \sum\limits_{n=2,4,...} \bar{a}_n$; *5)* $F = \bar{a}_2$.

Fig. 2. Parameter α vs the relative crack length γ for different locations of the crack for the first (solid lines), second (dashed lines), and third (dot-and-dash lines) modes of vibration: *1)* $L_c / L = 0.1$; *2)* $L_c / L = 0.2$; *3)* $L_c / L = 0.5$.

$$\Delta\bar{U} = \frac{U_+ - U_-}{U_+ + U_-} = \frac{\alpha}{\pi}\left[1 + \sum_{n=2,4,...} \frac{2}{(n^2-1)^2}\right]. \qquad (11)$$

Note that the value of the relative difference in half-periods $\Delta\bar{T}$ correlates with the value of the relative change of the natural frequency of vibration of a cracked beam (β'), which was discussed as a damage indicator [3], i.e., $\Delta\bar{T} = \beta'$, while the value of the relative difference in maximum deviations $\Delta\bar{U}$ is proportional to the constant component ($\Delta\bar{U} \approx 1.264\bar{a}_0$).

Figure 1 shows also the functions $\Delta\bar{T}(\alpha)$ and $\Delta\bar{U}(\alpha)$. It is obvious that the most representative crack indicators, in decreasing order of their relative values, are $\Delta\bar{U}$, $\Delta\bar{T} = \beta'$, $\bar{a}_0$, $\sum\limits_{n=2,4,...} \bar{a}_n$, and $\bar{a}_2$. However, the practical significance of a particular indicator will depend on the resolution of the respective instruments used for measuring the determining parameters of this indicator. Furthermore, it should be taken into account that in the case of crack detection or crack length determination by measuring the natural frequency of vibrations of a beam, one must know an initial value, while a noticeable value of any other indicator can even indicate not only the presence of a crack but, to a certain extent, the crack size as well.

Notice that the dependence of the parameters $\bar{a}_0$, $\bar{a}_2$, and $\Delta\bar{U}$ on the relative crack length γ, crack location x_c, and the vibration mode number i is qualitatively similar to the $\beta'(\gamma)$, $\beta'(x_c / L)$, and $\beta'(i)$.

Results of Experimental Investigation. The experiment was carried out using an KD-1M setup [5]. We studied bending vibrations of cantilever specimens of 15Kh2NMFA and 08Kh18N10 steels; their test portion was rectangular in cross section. The relative height of the cross section h / L was 0.0627 in the first case and 0.133 in the second case. A heavy mass was fixed to a free end of the 08Kh18N10 steel specimens.

The geometrical characteristics of the specimens and the mechanical properties of the steels were described in detail in [6].

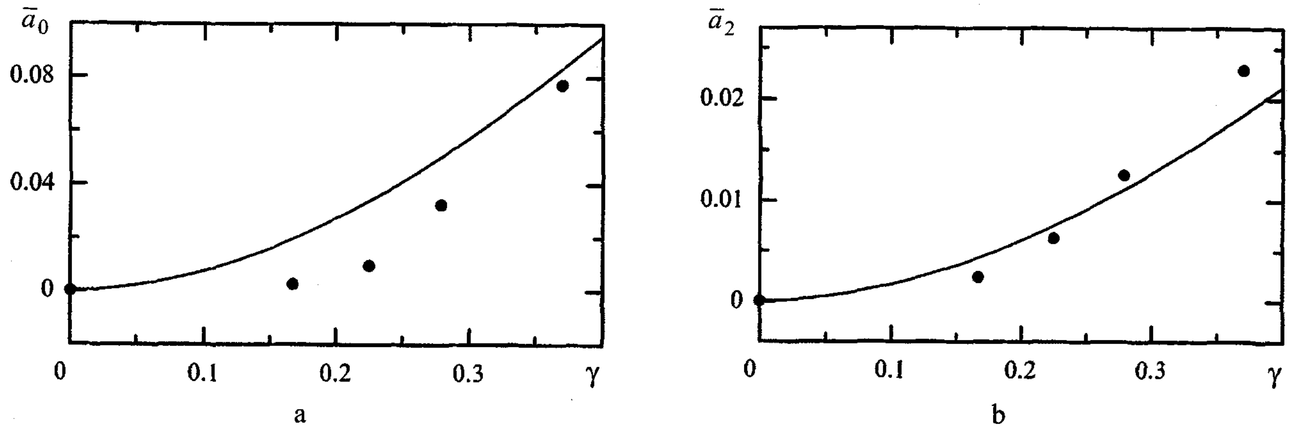

Fig. 3. Amplitudes of the zero (a) and second (b) harmonics vs the relative crack length γ for a 15Kh2NMFA steel specimen (dots – experimental data, line – calculated results).

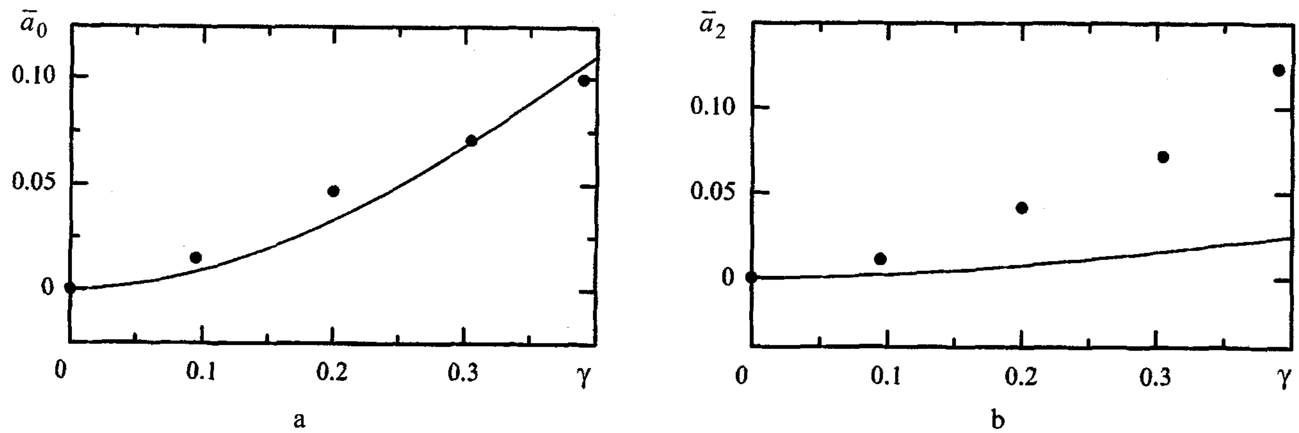

Fig. 4. Amplitudes of the zero (a) and second (b) harmonics vs the relative crack length γ for a 08Kh18N10 steel specimen (dots – experimental data, line – calculated results).

A test specimen was fixed rigidly in a massive frame, which was hung on thin steel strings for the purpose of vibration insulation. Vibrations of specimens were excited by a resonance method through the use of an electromagnetic system, which consisted of a waveform generator, an amplifier, and electromagnets. Bending vibrations of the specimens were excited in the plane of their maximum stiffness. The strain recording system was made up of strain gauges glued to the specimen test surface, a strain amplifier, a digital voltmeter, an oscillograph, a frequency meter, an analog-digital converter, and a personal computer.

The fatigue cracks were initiated from a preliminarily produced stress concentrator in the given section of the specimen test portion. For the harmonic analysis of the recorded strain wave we used the procedure and software developed earlier [7].

The results of the investigations support essentially the approximate analytical solution. The most pronounced indicators of the crack initiation in a specimen are the constant component and the amplitude of the second harmonic at a cosine curve, which are detected quite reliably by using the proposed experimental procedure of the harmonic analysis even for rather small cracks with $\gamma \approx 0.1$. The crack growth leads to a considerable increase in these parameters. As an illustration, Fig. 3 gives the functions $\bar{a}_0(\gamma)$ and $\bar{a}_2(\gamma)$ calculated for a 15Kh2NMFA steel specimen by formulas (2) using the expressions for the coefficient α derived earlier [3].

For a 08Kh18N10 steel specimen with a mass at its end we observed a good agreement between the results of analytical and experimental determination of the zero harmonic amplitude (Fig. 4a). However, it should be mentioned that a substantial difference exists between the calculation and experimental results (Fig. 4b). We have also established a significant dependence of the experimental data on the strain gauge location with respect to the crack, which requires further investigation.

Conclusions. The results of the analytical and experimental studies of the distortion of the strain wave harmonicity in the vibration of prismatic beams with transverse cracks suggest that harmonic analysis of the wave of the real strain cycle of beam-like elements in their resonance vibrations can be used as an efficient diagnostic method for the detection of fatigue damage such as closing cracks.

The proposed experimental procedure of the harmonic analysis of an experimentally recorded strain signal allows a reliable detection of distortions of harmonicity of a strain wave due to rather small cracks. The most significant indicators of the presence of a closing crack are the constant component of the strain wave and the second harmonic amplitude at a cosine curve.

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