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ANALYSIS OF AMPLITUDE AND PHASE CHARACTERISTICS OF TWO-DIMENSIONAL OPTICAL FIELDS USING THE MODULATION–SPECTRUM METHOD

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Abstract

A solution to the phase problem in optics is considered within the context of the registration and analysis of two-dimensional stationary optical fields transformed by an object under study or fields forming an image. The modulation-spectrum method put forward by the authors is used for obtaining information on the amplitude and phase distributions of a light field. To solve the problem, the intensity distribution is directly detected for the spatial spectrum or the image of a signal and for those additionally modulated in a special way. The modulation should provide a visualization of the phase information. The intensity distributions obtained make it possible to calculate the two-dimensional structure of the initial signal. It is essential that the method require no iteration procedures in solving the problem. This allows one to expect speeding up of the processing and analyzing of the information. Three variants of optical schemes for the analysis of light fields are considered in the paper. The first one uses an additional spatial modulation in the plane of the investigated field, the spectrum of spatial frequencies being recorded. In the second case, the spatial modulation is performed at the input of the processing scheme, the spatial spectrum being registered likewise. In the third variant of the scheme, the spatial modulator is placed at the plane of spatial frequencies, and the image is registered.

1. Introduction

Problems of obtaining information on the amplitude and phase structure of a two-dimensional light field are of constant interest, even though only the intensity distribution or the amplitude structure of a signal is registered in any optical investigation. These problems arise in solving fundamental problems of physical and coherent optics as well as in applied problems.

Different interference methods of analyzing the field structure taking into account the phase have been well elaborated [1–5] and substantially developed after the invention of lasers [6–9]. Holographic methods are also used to register, restore, and analyze the amplitude and phase structure of light fields [10–12]. We would also like to list here the new branches of interferometry connected with the use of lasers, such as holographic interferometry [9, 12–17] and speckle interferometry [9, 16–21]. However, any practical realization of the holographic and speckle interferometry methods is rather complicated. In both cases, it is necessary to create an additional field of a known reproducible structure. All phases are measured with respect to the phase of this field. The presence of uncontrolled phase nonuniformities in the additional reference wave causes errors in the determined phase of the analyzed field. In particular, the condition of uniformity of the field means, in practice, the reduction of the size of the analyzed object or the useful area of the field. The interference and holographic methods require extreme rigidity of the set-up. In addition, the registering materials must be, as a rule, of high resolution, which leads to their low sensibility. These circumstances have forced investigators to focus their attention on other methods of obtaining phase and amplitude information on the field structure.

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The contemporary development of laser devices and physical and coherent optics has provided us with the possibility of establishing and solving fundamental phase problems connected with detection, processing, and analysis of the amplitude and phase characteristics of light fields formed by objects or fields producing images [22–24]. These problems are of particular interest for fundamental optical investigations and technical applications. In particular, as far as spatial optical signals are concerned, it is worth mentioning such applications as the study of the structure of objects changing their optical characteristics as a result of different physical, chemical, and biological processes, the analysis of the nonstationary interaction of light and matter, etc. The solution of these problems is based on measurements of the amplitude and phase structure of optical signals and determining the change of this structure resulting from testing the object. In addition, in solving certain problems it is required to simultaneously determinate the amplitude and phase structure of the optical signal and the structure of the complex transfer function or the spread function responsible for the action of the optical system, object, or medium on the signal [25].

In this paper, in order to measure the spatial amplitude and phase characteristics of a two-dimensional optical field we consider the approach proposed by the authors in [26, 27] for the analysis of phase objects, i.e., the modulation-spectrum method of analysis and solution of the phase problem in optics. The method is based on the registration of amplitude distributions of signals, which are specially formed by spatial modulators. Such an approach for analysis of one-dimensional optical signals depending only on time was considered in [28–30]. The distinguishing features and, naturally, further complications encountered in the present case are related to the two-dimensional structure of signals, to the transmission of two-dimensional information over a certain distance with regard for diffraction effects, to the processing of a two-dimensional image, and to the need for recording several two-dimensional distributions of intensity, which is technically more complicated. On the other hand, some simplification is accomplished by forming the spatial spectrum in the focal plane of the optical system without using any spectral device. This provides an additional flexibility of the scheme and the possibility of using different set-up variations. The modulation-spectral method includes the use of spatial light modulators with known characteristics providing in one form or another the visualization of the phase information and the formation (as a rule) of the spectrum of spatial frequencies or of the image [31–33].

Note that existing methods [22–24] are based on the use of iteration methods in signal processing and, therefore, require considerable resources of computers, making it practically impossible to operate in real time. The method used in the paper does not require any iteration procedures. That gives the possibility of processing the data almost in real time. In contrast to [26, 27], here we consider and analyze schemes using only one optical system forming either the spectrum of spatial frequencies or the image of the field under investigation.

In the paper, three variants of optical schemes for analysis of the structure of light fields are considered. The first one uses an additional spatial modulation in the plane of the studied field, the spectrum of spatial frequencies being recorded. In the second case, the spatial modulation is performed at the input of the processing scheme, and the spatial spectrum is registered likewise. In the third variant of the scheme, the spatial modulator is placed at the plane of spatial frequencies, and the image is registered. In principle, some other variants of the processing scheme can be realized and used according to the variation of the problem, an additional modulation having been applied in one plane or another and either the spatial spectrum or the image having been formed by the optical system. The variants considered in the paper are rather convenient in practice.

The tools of physical optics are used in the calculation of optical fields [2, 34, 35].

To describe the optical fields the following complex functions are used:

$$E_i(x,y,t) = \mathcal{E}_i(x,y)\exp(-i\omega t) = a_i(x,y)\exp[i\phi_i(x,y)]\exp(-i\omega t)\,, \tag{1}$$

where the complex amplitude of the field $\mathcal{E}_i(x,y)$ contains the amplitude $a_i(x,y)$ and the phase $\phi_i(x,y)$

components. The spectrum of spatial frequencies is described by the complex amplitude

$$F_i(\omega_\xi,\omega_\eta) = A_i(\omega_\xi,\omega_\eta)\exp[i\Phi_i(\omega_\xi,\omega_\eta)] = \int \mathcal{E}_i(x,y)\exp[-i\omega_\xi x - i\omega_\eta y]\,dx\,dy\,, \tag{2}$$

also containing the amplitude $A_i(\omega_\xi,\omega_\eta)$ and phase $\Phi_i(\omega_\xi,\omega_\eta)$ components. The indices i in (1) and (2) correspond to the analyzed signal (index s) or to the action of the additional modulation (index m) or to the analyzed signal with the additional modulation (index sm). The propagation of light in free space is described by the Kirchhoff integral. Some assumptions are implied to simplify the matter considerably, in fact, without any loss of generality of the analysis [2, 4, 35]. Scalar fields are considered. All effects caused by a change in polarization are described by a factor of order one. All calculations are written down for the electric part of the light wave, since the electric field is the main factor in recording the light. Light waves are taken to propagate in vacuum when outside the investigated object. This enables us not to take into account the dielectric constant of the medium. The effects caused by the action of the edges of an object or other shields are negligible, i.e., the size of the analyzed part of the field is much larger than the wavelength. The light fields are studied at distances that are considerably larger than the size of the optical elements used in the set-up.

In analytic calculations, Fresnel's approximation is used. The validity of the use of this approximation and of the results is based on the following reasons [34]. In the course of analysis of the field formed at a certain point by the central part of the initial field, Fresnel's approximation is valid. In outlying parts of the initial field, the phase changes quickly with distance from the center. Therewith, the amplitude changes slowly as a rule. As a result, the contribution of the parts of the field for which Fresnel's approximation is not valid is rather small.

It should be noted additionally that we analyze the two-dimensional structure of a light field though in practice the three-dimensional structure of the field is often of interest as well. However, it is not a great complication in most cases. In fact, if a field is determined at some surface, it can be calculated in the rest of space using the Kirchhoff integral.

2. The Scheme with Two-Dimensional Modulation in the Plane of the Analyzed Signal and with Registration of the Spectrum of Spatial Frequencies

The schematic optical diagram of processing and registration using the modulation-spectral method with placement of the spatial modulator **M** in the plane xy of the analyzed field with the formation of the spectrum of spatial frequencies ω_ξ and ω_η in the registration plane $\xi\eta$ by the optical system **OS** placed in the plane uv, is given in Fig. 1.

The distribution of the field of frequency ω [see (1)] produced by the coherent laser radiation

$$E_s(x,y,t) = \begin{cases} \mathcal{E}_s(x,y)\exp(-i\omega t)\,, & -x_0 \le x \le +x_0\,,\quad -y_0 \le y \le +y_0\,, \\ 0\,, & x < -x_0\,,\quad x > +x_0\,;\quad y < -y_0\,,\quad y > +y_0 \end{cases} \tag{3}$$

is described by the complex amplitude $\mathcal{E}_s(x,y)$; $2x_0 \times 2y_0$ is the area where the field is analyzed.

The scheme includes the two-dimensional spatial modulator **M** placed in the studied plane xy. The transmission of the modulator is described, in the general case, by a known complex function $M(x,y)$ [see (1)] characterizing the effect on the amplitude [component $a_m(x,y)$] of the field and on its phase [component

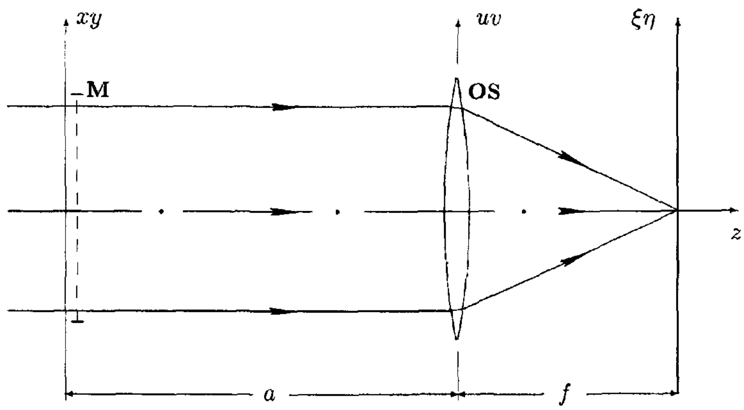

Fig. 1. Schematic optical diagram of formation of the spatial spectrum with the use of an additional spatial modulation in the investigated plane.

$\phi_m(x,y)]$:

$$M(x,y) = \begin{cases} a_m(x,y)\exp[i\phi_m(x,y)]\,, & -x_0 \le x \le +x_0\,,\ \ -y_0 \le y \le +y_0\,, \\ 0\,, & x < -x_0\,,\ \ x > +x_0\,;\ \ y < -y_0\,,\ \ y > +y_0\,. \end{cases} \tag{4}$$

The area $2x_0 \times 2y_0$ of the modulator action should coincide or be larger than the area where the field is studied. From now on, the areas are taken to be equal for simplicity. As a result, the field formed after the modulator is

$$E_{sm}(x,y,t) = \mathcal{E}_{sm}(x,y)\exp(-i\omega t) = \mathcal{E}_s(x,y)\,M(x,y)\exp(-i\omega t)\,. \tag{5}$$

Within the framework of physical optics [2, 34, 35], the field in front of the optical system **OS** in the plane uv at a distance a from the analyzed plane xy has the complex amplitude $\mathcal{E}_{sm}(u,v)$ and is characterized by the following expression:

$$E_{sm}(u,v,t) = \mathcal{E}_{sm}(u,v)\exp(-i\omega t) = -\frac{i}{\lambda a}\int \mathcal{E}_{sm}(x,y)\exp[-i\omega t_a]\Big|_{t_a=t-r_a/c} dx\,dy\,, \tag{6}$$

where

$$r_a = a + \frac{x^2+y^2}{2a} + \frac{u^2+v^2}{2a} - \frac{xu+yv}{a}\,.$$

The effect of the optical system **OS** with focus distance f placed in the uv plane is described in the paraxial approximation [2, 34, 35] by the formula

$$O(u,v) = \exp\left[-i\,\frac{\omega}{c}\,\frac{u^2+v^2}{2f}\right]. \tag{7}$$

It is assumed that the aperture of the optical system of dimension $2u_0 \times 2v_0$ does not restrict the informative area of the field necessary for analysis. The spectrum of spatial frequencies is formed in the detection plane

$\xi\eta$ at a distance f from the optical system. It is described within the framework of physical optics [2, 34, 35] as [see (6) and (7)]

$$E_{sm}(\xi,\eta,t) = \mathcal{E}_{sm}(\xi,\eta)\exp(-i\omega t) = -\frac{i}{\lambda f}\int \mathcal{E}_{sm}(u,v)\, O(u,v)\exp[-i\omega t_f]\Big|_{t_f=t-r_f/c} du\, dv\,, \tag{8}$$

where

$$r_f = f + \frac{u^2+v^2}{2f} + \frac{\xi^2+\eta^2}{2f} - \frac{u\xi+v\eta}{f}\,.$$

Then, taking into account (5)–(8), $E_{sm}(\xi,\eta,t)$ is calculated. The integrals over u and v can be taken over the whole space, because as is pointed out above, the optical system does not limit the informative area of the field. In this case, the integrals are reduced to the tabulated integrals (see 3.352.7 in [36])

$$\int \exp\left[i\,\frac{\omega}{c}\,\frac{u^2+v^2}{2a}\right]\exp\left[-i\,\frac{\omega}{c}\,u\left(\frac{x}{a}+\frac{\xi}{f}\right)-i\,\frac{\omega}{c}\,v\left(\frac{y}{a}+\frac{\eta}{f}\right)\right] du\, dv$$
$$= \lambda a\exp\left[-i\,\frac{\omega}{c}\left(\frac{x^2+y^2}{2a}+\frac{\xi^2+\eta^2}{2f}\,\frac{a}{f}+\frac{x\xi+y\eta}{f}\right)\right].$$

Taking the spatial frequencies

$$\omega_\xi = \frac{\omega\xi}{cf} \quad\text{and}\quad \omega_\eta = \frac{\omega\eta}{cf} \tag{9}$$

and calculating the integrals over x and y, one gets the Fourier transform of the complex amplitudes of the spectrum (8) as

$$\begin{aligned} F_{sm}(\omega_\xi,\omega_\eta) &= \int \mathcal{E}_{sm}(x,y)\exp[-i\omega_\xi x - i\omega_\eta y]\, dx\, dy \\ &= \int \mathcal{E}_s(x,y)\, M(x,y)\exp\left[-i\,\frac{\omega}{c}\,\frac{x\xi+y\eta}{f}\right] dx\, dy \\ &= \frac{1}{4\pi^2}\int F_s(\omega'_\xi,\omega'_\eta)\, F_m(\omega_\xi-\omega'_\xi,\omega_\eta-\omega'_\eta)\, d\omega'_\xi\, d\omega'_\eta, \end{aligned} \tag{10}$$

where $F_s(\omega'_\xi,\omega'_\eta)$ and $F_m(\omega_\xi-\omega'_\xi,\omega_\eta-\omega'_\eta)$ are the spectra of spatial frequencies of the analyzed signal (3) and of the modulation function (4), which are determined in accordance with (2). Finally, the field (8) is

$$E_{sm}(\xi,\eta,t) = -\frac{1}{\lambda f}\, C(\xi,\eta)\, F_{sm}(\omega_\xi,\omega_\eta)\exp(-i\omega t)\,. \tag{11$'$}$$

For the sake of brevity, the term inessential for the analysis is denoted as

$$C(\xi,\eta) = \exp\left[i\,\frac{\omega}{c}\left(a+f+\frac{\xi^2+\eta^2}{2f}\,\frac{f-a}{f}\right)\right]. \tag{12}$$

In further calculations, we will also use the expression for the analyzed field in the detection plane in absence of modulation, i.e., when $M(x,y)=1$. Taking into account (9)–(12), it is

$$E_s(\xi,\eta,t) = -\frac{1}{\lambda f}\, C(\xi,\eta)\, F_s(\omega_\xi,\omega_\eta)\exp(-i\omega t)\,. \tag{11$''$}$$

Similarly to other optical measuring systems, the detected parameter is the intensity distribution. With allowance made for the amplitude and phase structure of the spectrum of the spatial frequencies [see (2)], the two intensity distributions registered, one just for the analyzed field and the other for the field with an additional spatial modulation, are described by the following expressions [see (10) and (11)]:

$$\begin{aligned} I_s(\omega_\xi,\omega_\eta) &= \left(\frac{1}{\lambda f}\right)^2 F_s(\omega_\xi,\omega_\eta)\, F_s^*(\omega_\xi,\omega_\eta) = \left(\frac{1}{\lambda f}\right)^2 A_s^2(\omega_\xi,\omega_\eta)\,, \\ I_{sm}(\omega_\xi,\omega_\eta) &= \left(\frac{1}{\lambda f}\right)^2 F_{sm}(\omega_\xi,\omega_\eta)\, F_{sm}^*(\omega_\xi,\omega_\eta) \\ &= \left(\frac{1}{\lambda f}\right)^2 \left[\frac{1}{4\pi^2}\int A_s(\omega_\xi',\omega_\eta')\, \exp[i\Phi_s(\omega_\xi',\omega_\eta')]\, F_m(\omega_\xi-\omega_\xi',\omega_\eta-\omega_\eta')\, d\omega_\xi'\, d\omega_\eta'\right] [\text{c.c.}]\,. \end{aligned} \tag{13}$$

The detected intensity distributions $I_s(\omega_\xi,\omega_\eta)$ and $I_{sm}(\omega_\xi,\omega_\eta)$ [see (13)] and the known spatial spectrum $F_m(\omega_\xi-\omega_\xi',\omega_\eta-\omega_\eta')$ of the modulator **M** make it possible to determine the amplitude $A_s(\omega_\xi,\omega_\eta)$ and the phase $\Phi_s(\omega_\xi,\omega_\eta)$ structure of the spectrum of spatial frequencies of the signal under study. The amplitude spectrum is determined from the expression describing the intensity distribution $I_s(\omega_\xi,\omega_\eta)$:

$$A_s(\omega_\xi,\omega_\eta) = (\lambda f)\,\Big[I_s(\omega_\xi,\omega_\eta)\Big]^{1/2}. \tag{14}$$

From the expression for $I_{sm}(\omega_\xi,\omega_\eta)$ that describes the intensity distribution for the modulated field, one can determine the phase structure $\Phi_s(\omega_\xi,\omega_\eta)$ of the spatial spectrum of the field by solving the integral equation taking into account (14):

$$\Phi_s(\omega_\xi,\omega_\eta) = f_{s\phi}\Big(I_{sm}(\omega_\xi,\omega_\eta), I_s(\omega_\xi,\omega_\eta), F_m(\omega_\xi-\omega_\xi',\omega_\eta-\omega_\eta')\Big)\,. \tag{15}$$

The amplitude-phase structure of the analyzed two-dimensional field can be obtained from the distributions in spatial spectra (14) and (15) by means of the inverse Fourier transform:

$$\mathcal{E}_s(x,y) = \frac{1}{4\pi^2}\int A_s(\omega_\xi,\omega_\eta)\exp[i\Phi_s(\omega_\xi,\omega_\eta)]\exp[i\omega_\xi x + i\omega_\eta y]\, d\omega_\xi\, d\omega_\eta\,. \tag{16}$$

The most simple and illustrative way of analyzing the two-dimensional amplitude-phase structure of the light field, which shows the reality of the approach considered, is to use a spatial filter that provides a linear modulation of the amplitude along one of the coordinates, for example along x. The amplitude transmission of the modulator, in this case, is given by the amplitude transmittance [see (4)]

$$M(x,y) = \begin{cases} a_m + b_m \frac{x}{x_0}\,, & -x_0 \le x \le +x_0\,, \quad -y_0 \le y \le +y_0\,; \\ 0, & x < -x_0\,, \quad x > +x_0\,; \quad y < -y_0\,, \quad y > +y_0\,. \end{cases} \tag{17}$$

The amplitude transmission of the modulator can only be positive and less than unity, which corresponds, if $a_m,\, b_m > 0$ and $a_m,\, b_m < 1$, to the following constraints:

$$\begin{aligned} x = -x_0\,, &\quad -y_0 \le y \le +y_0\,, \quad M(x,y) = a_m - b_m \ge 0\,, \\ x = 0\,, &\quad -y_0 \le y \le +y_0\,, \quad M(x,y) = a_m \ge 0\,, \\ x = +x_0\,, &\quad -y_0 \le y \le +y_0\,, \quad M(x,y) = a_m + b_m \le 1\,. \end{aligned}$$

Actually, the modulator transmittance varies within the limits from $a_m - b_m$ to $a_m + b_m$.

For the type of modulation under consideration, the complex amplitudes of the spectra of spatial frequencies of the fields not modulated and of the fields subjected to spatial modulation, up to unimportant factors, are characterized by the following factors [see (11)]:

$$F_s(\omega_\xi,\omega_\eta) \quad \text{and} \quad F_{sm}(\omega_\xi,\omega_\eta) = a_m\, F_s(\omega_\xi,\omega_\eta) + i\,\frac{b_m}{x_0}\,\frac{dF_s(\omega_\xi,\omega_\eta)}{d\omega_\xi}\,. \tag{18}$$

The intensity distributions of the spectral components recorded in the system are described by expressions (13) [see (2)]:

$$\begin{aligned} I_s(\omega_\xi,\omega_\eta) &= \left(\frac{1}{\lambda f}\right)^2 A_s^2(\omega_\xi,\omega_\eta)\,, \\ I_{sm}(\omega_\xi,\omega_\eta) &= \left(\frac{1}{\lambda f}\right)^2 \left\{\left[A_s(\omega_\xi,\omega_\eta)\left(a_m - \frac{b_m}{x_0}\,\frac{d\Phi_s(\omega_\xi,\omega_\eta)}{d\omega_\xi}\right)\right]^2 + \left[\frac{b_m}{x_0}\,\frac{dA_s(\omega_\xi,\omega_\eta)}{d\omega_\xi}\right]^2\right\}. \end{aligned} \tag{19}$$

The distributions

$$A_s(\omega_\xi,\omega_\eta) = (\lambda f)\,\Big[I_s(\omega_\xi,\omega_\eta)\Big]^{1/2} \qquad \text{and} \qquad \frac{dA_s(\omega_\xi,\omega_\eta)}{d\omega_\xi} \tag{20}$$

are determined by the intensity $I_s(\omega_\xi,\omega_\eta)$ in the further processing of the data [see (14)]. Then, using the intensity distribution $I_{sm}(\omega_\xi,\omega_\eta)$ obtained, we can find

$$\begin{aligned} \frac{d\Phi_s(\omega_\xi,\omega_\eta)}{d\omega_\xi} &= \frac{x_0}{b_m}\left\{a_m - \frac{1}{A_s(\omega_\xi,\omega_\eta)}\left[(\lambda f)^2 I_{sm}(\omega_\xi,\omega_\eta) - \left(\frac{b_m}{x_0}\,\frac{dA_s(\omega_\xi,\omega_\eta)}{d\omega_\xi}\right)^2\right]^{1/2}\right\}, \\ \Phi_s(\omega_\xi,\omega_\eta) &= \frac{a_m}{b_m}x_0\omega_\xi - \frac{x_0}{b_m}\int \frac{1}{A_s(\omega_\xi,\omega_\eta)}\left[(\lambda f)^2 I_{sm}(\omega_\xi,\omega_\eta) - \left(\frac{b_m}{x_0}\,\frac{dA_s(\omega_\xi,\omega_\eta)}{d\omega_\xi}\right)^2\right]^{1/2} d\omega_\xi\,. \end{aligned} \tag{21}$$

Finally, using the amplitudes $A_s(\omega_\xi,\omega_\eta)$ and phases $\Phi_s(\omega_\xi,\omega_\eta)$ [see(20) and (21)] obtained for the spectrum of spatial frequencies, we can calculate the two-dimensional structure $F_s(\omega_\xi,\omega_\eta)$ of the Fourier transform of the complex amplitude $\mathcal{E}_s(x,y)$ of the analyzed field with regard for the amplitudes and phases [see (1)]. The complex amplitude of the field is determined according to (16):

$$\mathcal{E}_s(x,y) = \frac{1}{4\pi^2}\int F_s(\omega_\xi,\omega_\eta)\; exp[i\omega_\xi x + i\omega_\eta y]\; d\omega_\xi\, d\omega_\eta\,. \tag{22}$$

Thus, the modulation-spectral method of registration and processing of the spatial spectrum formed by the optical system makes it possible to obtain the amplitude and phase structure of the two-dimensional distribution of the analyzed field in a certain plane defined by the position of the modulator.

3. Analysis with Two-Dimensional Modulation at the Input of the Processing System and with Registration of the Spectrum of Spatial Frequencies

Here we consider the variant of the scheme where the field in the analyzed plane is not affected, which is essential in the case where the investigated plane cannot be achieved for some reason. In course of the registration and processing, the amplitude and phase structure of the field are determined at the input of

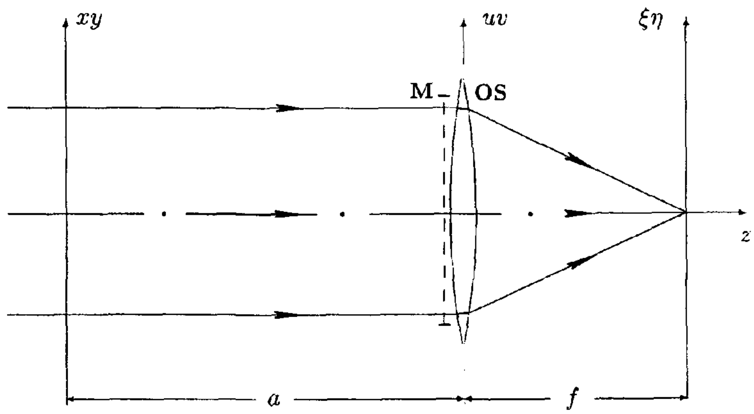

Fig. 2. Schematic optical diagram of formation of the spatial spectrum with the use of an additional spatial modulation at the input of the registration system.

the optical system in use. For the analyzed plane, the field is calculated by the methods of physical optics using the Kirchhoff integral in Fresnel's approximation [2, 34, 35] (taking into account the conditions of its validity).

The schematic optical diagram of processing and registration using the modulation-spectral method with placement of the spatial modulator **M** at the input of the processing system in the vicinity of the plane uv with formation of the spectrum of spatial frequencies ω_ξ and ω_η in the registration plane $\xi\eta$ by the optical system **OS** also placed in the vicinity of the plane uv is given in Fig. 2.

The distribution of the field $E_s(x, y, t)$ [see (3)] is considered in a certain plane xy. The field is produced by coherent laser radiation of frequency ω in the area of the plane

$$-x_0 \leq x \leq +x_0 , \qquad -y_0 \leq y \leq +y_0 ,$$

and is described by the complex amplitude $\mathcal{E}_s(x, y)$.

At a distance a from the xy plane at the input of the optical scheme in the vicinity of the plane uv the field structure is formed, which is described as is done in (6) by the methods of physical optics [2, 35, 36]:

$$E_s(u, v, t) = \mathcal{E}_s(u, v) \exp(-i\omega t) = -\frac{i}{\lambda a} \int \mathcal{E}_s(x, y) \exp[-i\omega t_a]\Big|_{t_a = t - r_a/c} dx\, dy , \tag{23}$$

where r_a is similar to that in (6).

Further processing and registration of the fields correspond to the analysis of the structure of the field in the plane uv and are performed similarly to the analysis in Sec. 2.

The two-dimensional spatial modulator **M** is placed in the vicinity of the plane uv. Its effect, in the general case, is described as above by a known complex function $M(u, v)$ [see (4)]:

$$M(u, v) = \begin{cases} a_m(u, v) \exp[i\phi_m(u, v)] , & -u_0 \leq u \leq +u_0 , \quad -v_0 \leq v \leq +v_0 , \\ 0 , & u < -u_0 , \quad u > +u_0 ; \quad v < -v_0 , \quad v > +v_0 . \end{cases} \tag{24}$$

The modulator affects the amplitude [the component $a_m(u,v)$] of the field and its phase [the component $\phi_m(u,v)$] inside the area $2u_0 \times 2v_0$, which should cover all that is essential for the analysis of the informative structure of the field formed by the signal under study at the input of the scheme.

Then the field is transformed by the optical system **OS** with focus distance f, which is also placed in the vicinity of the uv plane. The effect of the optical system is considered, as above, in the paraxial approximation [2, 34, 35] and is described by the expression for $O(u,v)$ [see (7)]. As a result, after modulating at the output of the optical system, the following field is formed:

$$E_{sm}(u,v,t) = \mathcal{E}_{sm}(u,v)\exp(-i\omega t) = \mathcal{E}_s(u,v)\, M(u,v)\, O(u,v)\exp(-i\omega t)\,.$$

In the registration plane $\xi\eta$ at a distance f from the optical system, the field $E_{sm}(\xi,\eta,t)$ is formed. It is described within the framework of physical optics [2, 34, 35] by expression (8). Calculations similar to (11′) lead to

$$E_{sm}(\xi,\eta,t) = -\frac{i}{\lambda f}\, C'(\xi,\eta)\, F_{sm}(\omega_\xi,\omega_\eta)\exp(-i\omega t)\,. \tag{25′}$$

In this expression, the spatial frequencies ω_ξ and ω_η are taken similarly to (9), and for the sake of brevity [see (12)] the term that is inessential for the analysis is denoted as $C'(\xi,\eta)$:

$$C'(\xi,\eta) = \exp\left[i\,\frac{\omega}{c}\left(f + \frac{\xi^2+\eta^2}{2f}\right)\right].$$

The spectrum of spatial frequencies is described by the integral [see (10)]

$$\begin{aligned} F_{sm}(\omega_\xi,\omega_\eta) &= \int \mathcal{E}_{sm}(u,v)\exp[-i\omega_\xi u - i\omega_\eta v]\, du\, dv \\ &= \int \mathcal{E}_s(u,v)\, M(u,v)\exp\left[-i\,\frac{\omega}{c}\,\frac{u\xi+v\eta}{f}\right] du\, dv \\ &= \frac{1}{4\pi^2}\int F_s(\omega'_\xi,\omega'_\eta)\, F_m(\omega_\xi-\omega'_\xi,\omega_\eta-\omega'_\eta)\, d\omega'_\xi\, d\omega'_\eta. \end{aligned} \tag{26}$$

In the further analysis, similarly to (11″) we will also use the expression for the analyzed field in the form of the spectrum of spatial frequencies in the detection plane in the absence of the modulation, i.e., when $M(x,y)=1$. Taking into account (9) and (25′), it is

$$E_s(\xi,\eta,t) = -\frac{i}{\lambda f}\, C'(\xi,\eta)\, F_s(\omega_\xi,\omega_\eta)\exp(-i\omega t)\,. \tag{25″}$$

Similarly to other optical measuring systems, the detected parameter is the intensity distribution. With allowance made for the amplitude and phase structure of the spectrum of spatial frequencies [see (2)], the two intensity distributions registered, one just for the analyzed field and the other for the field with an additional spatial modulation, are described by expressions similar to (13) [see (25)].

The detected intensity distributions $I_s(\omega_\xi,\omega_\eta)$ and $I_{sm}(\omega_\xi,\omega_\eta)$ and the known spatial spectrum $F_m(\omega_\xi-\omega'_\xi,\omega_\eta-\omega'_\eta)$ describing the effect of the modulator **M** make it possible to determine the amplitude $A_s(\omega_\xi,\omega_\eta)$ and the phase $\Phi_s(\omega_\xi,\omega_\eta)$ [see (15)] structure of complex amplitudes of the spatial spectrum of the signal under study (see Sec. 2).

The amplitude-phase structure of the complex amplitude $\mathcal{E}_s(u,v)$ of the two-dimensional field in the input plane uv of the optical system can be obtained from the distributions in spatial spectra by means of the inverse Fourier transformation. Similarly to (16), the field in the uv plane is described by the function

$$\begin{aligned} E_s(u,v,t) &= \mathcal{E}_s(u,v)\, \exp[-i\omega t] \\ &= \frac{1}{4\pi^2}\int A_s(\omega_\xi,\omega_\eta)\exp[i\Phi_s(\omega_\xi,\omega_\eta)]\exp[i\omega_\xi u + i\omega_\eta v]\, d\omega_\xi\, d\omega_\eta\,. \end{aligned} \tag{27}$$

The field in the analyzed plane xy can be calculated based on expression (23) in the following way.

Let the complex amplitude of the field in the investigated plane xy be (we take into account the quadratic phase term)

$$\mathcal{E}_{se}(x,y) = \mathcal{E}_s(x,y)\exp\left[i\,\frac{\omega}{c}\,\frac{x^2+y^2}{2a}\right].$$

Then the complex amplitude of the analyzed field at the input of the optical system in accordance with (23) is described as

$$\mathcal{E}_s(u,v) = -\frac{i}{\lambda a}\,\exp\left[i\,\frac{\omega}{c}\left(a+\frac{u^2+v^2}{2a}\right)\right]\int \mathcal{E}_{se}(x,y)\exp\left[-i\,\frac{\omega}{c}\,\frac{xu+vy}{a}\right]dx\,dy\,.$$

To save some notations, we consider the spectrum $F_{se}(\omega_u,\omega_v)$ of spatial frequencies represented in the form

$$\omega_u = \frac{\omega u}{ca} \qquad \text{and} \qquad \omega_v = \frac{\omega v}{ca}\,.$$

Then the Fourier transform of the analyzed field taking into account the quadratic phase term is

$$F_{se}(\omega_u,\omega_v) = i\lambda a\,\exp\left[-i\,\frac{\omega}{c}\left(a+\frac{u^2+v^2}{2a}\right)\right]\mathcal{E}_s(u,v)\,.$$

The inverse Fourier transformation with regard for $\mathcal{E}_{se}(x,y)$ gives the resulting expression for the complex amplitude of the analyzed field in the plane under investigation:

$$\mathcal{E}_s(x,y) = \frac{i\lambda a}{4\pi^2}\exp\left[-i\,\frac{\omega}{c}\,\frac{x^2+y^2}{2a}\right]\int \mathcal{E}_s(u,v)\exp\left[-i\,\frac{\omega}{c}\left(a+\frac{u^2+v^2}{2a}\right)\right]\exp\left[i\,\frac{\omega}{c}\,\frac{ux+vy}{a}\right]du\,dv\,, \qquad (28)$$

where $\mathcal{E}_s(u,v)$ is defined in accordance with (27).

To obtain illustrative results, we consider, as in Sec. 2, one of the simplest variants of analysis of the two-dimensional amplitude-phase structure of the light field. It is realized with the use of a spatial filter that provides a linear modulation of the amplitude along one of the coordinates, for example along x. The transmission of the modulator [see (24)] placed in the uv plane in this case is given by the amplitude transmittance [see (17)]

$$M(u,v) = \begin{cases} a_m + b_m\frac{u}{u_0}\,, & -u_0 \le u \le +u_0\,,\quad -v_0 \le v \le +v_0\,,\\ 0\,, & u < -u_0\,,\quad u > +u_0\,;\quad v < -v_0\,,\quad v > +v_0\,. \end{cases}$$

The Fourier transforms of the complex amplitudes of the spectra of spatial frequencies for the modulated and nonmodulated fields in the $\xi\eta$ plane, up to inessential terms, are characterized by the following factors [see (18)]:

$$F_s(\omega_\xi,\omega_\eta) \quad \text{and} \quad F_{sm}(\omega_\xi,\omega_\eta) = a_m F_s(\omega_\xi,\omega_\eta) + i\,\frac{b_m}{u_0}\,\frac{dF(\omega_\xi,\omega_\eta)}{d\omega_\xi}\,.$$

The registered intensity distributions $I_s(\omega_\xi,\omega_\eta)$ and $I_{sm}(\omega_\xi,\omega_\eta)$ of the spectral components are described similarly to (13) by expressions (19). The distributions

$$A_s(\omega_\xi,\omega_\eta) \quad \text{and} \quad \frac{dA_s(\omega_\xi,\omega_\eta)}{d\omega_\xi}$$

are determined by the intensity $I_s(\omega_\xi,\omega_\eta)$ in further processing of the data [see (20)]. Then, in view of the intensity distribution $I_{sm}(\omega_\xi,\omega_\eta)$ obtained, we can find [see (21)]

$$\frac{d\Phi_s(\omega_\xi,\omega_\eta)}{d\omega_\xi} \quad \text{and} \quad \Phi_s(\omega_\xi,\omega_\eta)\,.$$

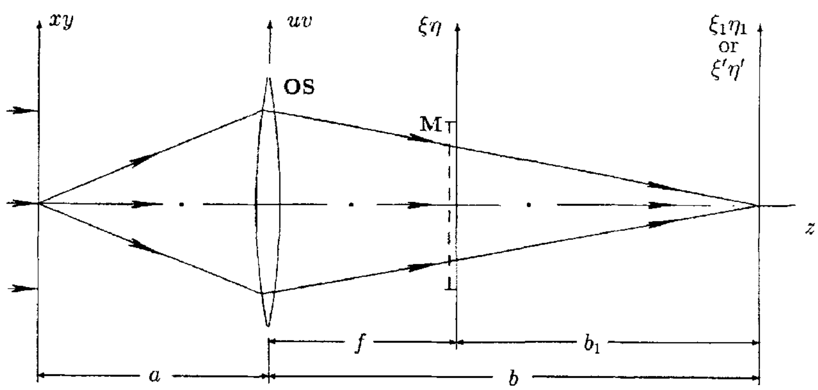

Fig. 3. Schematic optical diagram of formation of the image of the investigated plane with the use of an additional spatial modulation in the plane of spatial frequencies.

Finally, using the amplitudes $A_s(\omega_\xi, \omega_\eta)$ and phases $\Phi_s(\omega_\xi, \omega_\eta)$ [see (16) and (22)] obtained for the spectrum of spatial frequencies, we can calculate in accordance with (27) the two-dimensional structure of the complex amplitude of the field formed at the input of the processing system in the vicinity of the uv plane. The field in the investigated plane $E_s(x, y, t)$ is calculated in accordance with expression (28) describing the complex amplitude $\mathcal{E}_s(x, y)$ of the analyzed field.

Thus, modulation at the input of the optical system and registration of the spatial spectrum allow one to obtain the amplitude-phase structure of the investigated field in a selected plane, the tools of physical optics being used within the validity range of Fresnel's approximation for the Kirchhoff integral. The results are certainly valid only under the conditions used in deriving the Kirchhoff integral. In most practical cases, they are fulfilled.

4. Analysis with Two-Dimensional Modulation in the Plane of Spatial Frequencies and with Registration of the Image

Here we consider the variant of processing with an additional filtration and with registration of the image. As was considered in Sec. 3, there is no modulation in the analyzed plane xy. At the same time, the field in this plane is determined without any additional calculations based on Fresnel's approximation of the Kirchhoff integral. The modulator **M** is placed next to the optical system in the $\xi\eta$ plane where the spatial frequencies ω_ξ and ω_η are formed. The optical system **OS** placed in the uv plane forms the image of the studied plane in the registration plane $\xi_1\eta_1$. This is the condition determining the position of the analyzed plane xy.

This variant of the method differs from the above modulation-spectral methods (see [31–33] and Secs. 2 and 3), since the spectrum of spatial frequencies is not registered. But, in a sense, the method can also be regarded as a modulation-spectral one, because it is connected with the additional modulation in the plane of spatial frequencies.

Figure 3 shows the optical scheme of the considered variant.

The analyzed field $E_s(x, y, t)$ [see (3)], as in the variants considered above (see Secs. 2 and 3), is produced by coherent laser radiation of frequency ω in the plane xy. It has the complex amplitude $\mathcal{E}_s(x, y)$ and is defined inside the area $2x_0 \times 2y_0$. At a distance a from the initial plane, in the input plane uv of the processing system the field $E_s(u, v, t)$ is formed without any additional modulation [see (23)].

The optical system **OS** with focal distance f placed in the same plane uv forms the image of the analyzed plane xy. The effect $O(u, v)$ of the optical system is considered in the paraxial approximation [2, 34, 35] [see (7)]. We assume that it does not restrict the informative area of the field in the uv plane:

$$-u_0 \le u \le +u_0 \quad \text{and} \quad -v_0 \le v \le +v_0 .$$

In the absence of modulation, the field $E_s(\xi, \eta, t)$ is formed [see (11″)] at a distance f from the optical system in the plane $\xi\eta$ of spatial frequencies $\omega_\xi\omega_\eta$ determined by (9).

In this plane $\omega_\xi\omega_\eta$, a two-dimensional modulator **M** of size $2\omega_{\xi 0} \times 2\omega_{\eta 0}$ is placed. The area of action of the modulator should not restrict the essential informative structure of the field. The effect of the modulator, in the general case, is described by a known complex function $M(\omega_\xi, \omega_\eta)$ [see (4) and (24)]:

$$M(\omega_\xi, \omega_\eta) = \begin{cases} A_m(\omega_\xi, \omega_\eta) \exp[i\Phi_m(\omega_\xi, \omega_\eta)] , & -\omega_{\xi_0} \le \omega_\xi \le +\omega_{\xi_0} , \quad -\omega_{\eta 0} \le \omega_\eta \le +\omega_{\eta 0}, \\ 0 , & \omega_\xi < -\omega_{\xi 0} , \quad \omega_\xi > +\omega_{\xi 0} , \\ 0 , & \omega_\eta < -\omega_{\eta 0} , \quad \omega_\eta > +\omega_{\eta 0} . \end{cases} \tag{29}$$

With the additional modulation in the plane of spatial frequencies $\omega_\xi\omega_\eta$ taken into account, the field, similar to (11′), is

$$E_{sm}(\xi, \eta, t) = \mathcal{E}_{sm}(\xi, \eta) \exp[-i\omega t] = -\frac{1}{\lambda f} C(\xi, \eta)\, F_s(\omega_\xi, \omega_\eta)\, M(\omega_\xi, \omega_\eta) \exp[-i\omega t] , \tag{30}$$

where $C(\xi, \eta)$ is described by (12). The field in the registration plane $\xi_1\eta_1$ (it is also the plane of the formed image), at a distance b from the optical system and at a distance $b_1 = b - f$ from the plane of spatial frequencies, is described by the expression [see (6) and (8)]

$$E_{sm}(\xi_1, \eta_1, t) = \mathcal{E}_{sm}(\xi_1, \eta_1) \exp[-i\omega t] = -\frac{i}{\lambda b_1} \int \mathcal{E}_{sm}(\xi, \eta) \exp[-i\omega t_b]\Big|_{t_b = t - r_b/c} d\xi\, d\eta ,$$

where

$$r_b = b_1 + \frac{\xi^2 + \eta^2}{2b_1} + \frac{\xi_1^2 + \eta_1^2}{2b_1} - \frac{\xi\xi_1 + \eta\eta_1}{b_1} .$$

With the complex amplitude of the field $\mathcal{E}_{sm}(\xi, \eta)$ [see (30)] and the conditions of forming the image

$$\frac{1}{a} + \frac{1}{b} = \frac{1}{f}$$

taken into account, the complex amplitude of the field in the registration plane $\xi_1\eta_1$ is

$$\mathcal{E}_{sm}(\xi_1, \eta_1) = \frac{if}{4\pi^2 b_1} C''(\xi_1, \eta_1) \int F_s(\omega_\xi, \omega_\eta)\, M(\omega_\xi, \omega_\eta) \exp\left[-i\frac{f}{b_1}(\omega_\xi\xi_1 + \omega_\eta\eta_1)\right] d\omega_\xi\, d\omega_\eta ,$$

where the inessential term is denoted as

$$C''(\xi_1, \eta_1) = \exp\left[i\,\frac{\omega}{c}\left(a + f + b_1 + \frac{\xi_1^2 + \eta_1^2}{2b_1}\right)\right] .$$

For the sake of brevity, the new variables ξ' and η' are used in the registration plane $\xi_1\eta_1$:

$$\xi' = -\frac{f}{b_1}\,\xi_1 = -\frac{a}{b}\,\xi_1\,, \quad \eta' = -\frac{f}{b_1}\,\eta_1 = -\frac{a}{b}\,\eta_1\,.$$

Then, in the absence of modulation $[M(\omega_\xi,\omega_\eta) = 1]$ the field in the registration plane $\xi'\eta'$ is

$$E_s(\xi',\eta',t) = i\,\frac{a}{b}\,C''(\xi',\eta')\,\mathcal{E}_s(\xi',\eta')\,\exp[-i\omega t]\,. \tag{31$'$}$$

The modulated field is described by

$$E_{sm}(\xi',\eta',t) = i\,\frac{a}{b}\,C''(\xi',\eta')\,\mathcal{E}_{sm}(\xi',\eta')\,\exp[-i\omega t]\,, \tag{31$''$}$$

where

$$\mathcal{E}_{sm}(\xi',\eta') = \int \mathcal{E}_s(\xi'',\eta'')\,F_m(\xi'-\xi'',\eta'-\eta'')\,d\xi''\,d\eta''. \tag{32}$$

The function

$$F_m(\xi'-\xi'',\eta'-\eta'') = \frac{1}{4\pi^2}\int M(\omega_\xi,\omega_\eta)\exp[i\omega_\xi(\xi'-\xi'')+i\omega_\eta(\eta'-\eta'')]\,d\omega_\xi\,d\omega_\eta$$

describes the Fourier transform of the known function of action of the spatial modulator **M**.

With the amplitude and phase structure [see (1)] of the field under study taken into account, the two intensity distributions registered, one just for the analyzed field and the other for the field with the additional spatial modulation, are described by the expressions [see (13)]

$$\begin{aligned} I_s(\xi',\eta') &= \left(\frac{a}{b}\right)^2 a_s^2(\xi',\eta')\,,\\ I_{sm}(\xi',\eta') &= \left(\frac{a}{b}\right)^2\left[\int a_s(\xi'',\eta'')\exp[i\phi_s(\xi'',\eta'')]\,F_m(\xi'-\xi'',\eta'-\eta'')\,d\xi''\,d\eta''\right][\text{c.c.}]\,. \end{aligned} \tag{33}$$

From the measured intensity distributions $I_s(\xi',\eta')$ and $I_{sm}(\xi',\eta')$ [see (33)] and from the Fourier transform $F_m(\xi'-\xi'',\eta'-\eta'')$ of the known function $M(\omega_\xi,\omega_\eta)$ of the modulator **M**, one can obtain the amplitude [see (14)]

$$a_s(\xi',\eta') = \frac{b}{a}\left[I_s(\xi',\eta')\right]^{1/2}$$

and the phase [see (15)]

$$\phi_s(\xi',\eta') = f_{s\phi}\Big(I_{sm}(\xi',\eta'),I_s(\xi',\eta'),F_m(\xi'-\xi'',\eta'-\eta'')\Big)$$

structure of the analyzed field in the registration plane $\xi'\eta'$.

In the studied plane xy for the variables

$$\xi' = x\,, \quad \eta' = y\,,$$

the complex amplitude of the studied structure of the field is [see (1)]

$$\mathcal{E}_s(x,y) = a_s(x,y)\exp[i\phi_s(x,y)] = a_s(\xi',\eta')\exp[i\phi_s(\xi',\eta')]\,, \tag{34}$$

the optical magnification being taken into account.

To illustrate the results, we consider, as in the above sections, one of the simplest variants of analysis of the two-dimensional amplitude-phase structure of the light field. It is realized with the use of a spatial filter that provides a linear modulation of the amplitude along one of the coordinates, for example along ω_ξ. The amplitude transmission of the modulator in this case is given by the amplitude transmittance [see (29)]

$$M(\omega_\xi,\omega_\eta)=\begin{cases} a_m+b_m\dfrac{\omega_\xi}{\omega_{\xi 0}}\,, & -\omega_{\xi_0}\le\omega_\xi\le+\omega_{\xi_0}\,,\quad -\omega_{\eta 0}\le\omega_\eta\le+\omega_{\eta 0}\,,\\ 0\,, & \omega_\xi<-\omega_{\xi 0}\,,\ \ \omega_\xi>+\omega_{\xi 0}\,;\ \ \omega_\eta<-\omega_{\eta 0}\,,\ \ \omega_\eta>+\omega_{\eta 0}\,.\end{cases}$$

With the above results taken into account, the light field in the registration plane $\xi'\eta'$ is:
without modulation [see (31′)]

$$E_s(\xi',\eta',t)\ =\ i\,\frac{a}{b}\,C''(\xi',\eta')\ \mathcal{E}_s(\xi',\eta')\ \exp[-i\omega t]\,,$$

with the additional modulation specified above [see (31″) and (32)] similarly to (18)

$$E_{sm}(\xi',\eta',t)\ =\ i\,\frac{a}{b}\,C''(\xi',\eta')\ \left[a_m\,\mathcal{E}_s(\xi',\eta')+i\frac{b_m}{\omega_{\xi 0}}\frac{d\mathcal{E}_s(\xi',\eta')}{d\xi'}\right]\ \exp[-i\omega t]\,.$$

The registered intensity distributions in the output plane are characterized like (33) by expressions similar to (19). With regard for (1),

$$\begin{aligned} I_s(\xi',\eta') &= \left(\frac{a}{b}\right)^2 a_s^2(\xi',\eta')\,,\\ I_{sm}(\xi',\eta') &= \left(\frac{a}{b}\right)^2\left\{\left[a_s(\xi',\eta')\left(a_m-\frac{b_m}{\omega_{\xi 0}}\frac{d\phi_s(\xi',\eta')}{d\xi'}\right)\right]^2+\left[\frac{b_m}{\omega_{\xi 0}}\frac{da_s(\xi',\eta')}{d\xi'}\right]^2\right\}.\end{aligned}\tag{35}$$

In further processing [see (35)] we obtain the following values from the intensity distribution $I_s(\xi',\eta')$ [see (20)]:

$$a_s(\xi',\eta')\quad\text{and}\quad\frac{da_s(\xi',\eta')}{d\xi'}\,.$$

Then, taking into account the obtained intensity distributions $I_{sm}(\xi',\eta')$ we get [similarly to (21)]

$$\frac{d\phi_s(\xi',\eta')}{d\xi'}\quad\text{and}\quad\phi_s(\xi',\eta')\,.$$

Finally, based on the calculated distributions of the amplitudes $a_s(\xi',\eta')$ and the phases $\phi_s(\xi',\eta')$, the two-dimensional complex amplitudes $\mathcal{E}_s(\xi',\eta')$ and $\mathcal{E}_s(x,y)$ of the analyzed field are calculated in the registration $\xi'\eta'$ and the analyzed xy planes in correspondence with (34), taking into account the amplitudes and the phases.

Thus, the amplitude-phase structure of a two-dimensional optical field in a certain plane, determined by the parameters of the optical system, can be obtained by using the amplitude-phase method of registration and processing of the image of the analyzed field.

5. Estimation of Characteristics of Optical Schemes

Here we estimate the spatial characteristics for the scheme of registration of the spatial spectrum of the analyzed field (see Sec. 2). We consider the spatial resolution in directions x and y and the whole dimension $2x_0\times 2y_0$ of the investigated field in the analyzed plane.

To perform the analysis, we assume that some one-dimensional harmonic structure oriented, for definiteness, along the x axis is placed in the investigated plane xy. The structure has linear size $2x_0$ and is described by the complex function

$$\exp\left[ik_x x\right] = \exp\left[i\,\frac{2\pi}{\Delta x}\,x\right],$$

where Δx is the period of the structure. For the structure oriented along the y axis, the analysis can be done in a similar way. The field produced by the structure in the investigated plane xy inside the limited analyzed area of size

$$-x_0 \leq x \leq +x_0\,, \quad -y_0 \leq y \leq +y_0\,,$$

is, in accordance with (1) and (3),

$$E_s(x,y,t) = \mathcal{E}_s(x,y)\exp[-i\omega t] = \mathcal{E}_s\exp\left[ik_x x\right]\exp[-i\omega t] = \exp\left[i\,\frac{2\pi}{\Delta x}\,x\right]\exp[-i\omega t]\,. \tag{36}$$

The field $E_s(\xi,\eta,t)$ in the registration plane $\xi\eta$ without any additional modulation $[M(x,y) = 1]$ is described by expression (11$''$). For the field of structure (36), the Fourier transform of the complex amplitude inside the limited area $2x_0 \times 2y_0$ is [see (2)]

$$F_s(\omega_\xi,\omega_\eta) = \mathcal{E}_s\int \exp[-i(\omega_\xi - k_x)x - i\omega_\eta y]\,dx\,dy = 4\mathcal{E}_s x_0 y_0\,\frac{\sin[(\omega_\xi - k_x)x_0]}{(\omega_\xi - k_x)x_0}\,\frac{\sin[\omega_\eta y_0]}{\omega_\eta y_0}\,. \tag{37}$$

Expression (37) describes the spread function of the scheme, which is similar to the δ-function.

In accordance with (37), the resolution, defined as the distance between the first two zeros of the spread function, is characterized by the relations

$$\omega_\xi x_0 = \pm\pi\,, \quad \omega_\eta y_0 = \pm\pi\,,$$

or, in accordance with the definition of spatial frequencies [see (9)], by the expressions

$$\frac{\omega}{cf}\,\delta\xi = \frac{2\pi}{x_0}\,, \quad \frac{\omega}{cf}\,\delta\eta = \frac{2\pi}{y_0}\,.$$

Thus, the resolution intervals $\delta\xi$ and $\delta\eta$ in the registration plane $\xi\eta$ determine the size of the investigated field in the analyzed plane xy:

$$2x_0 = 2\,\frac{2\pi c}{\omega}\,\frac{f}{\delta\xi} = 2\,\frac{\lambda f}{\delta\xi}\,, \quad 2y_0 = 2\,\frac{2\pi c}{\omega}\,\frac{f}{\delta\eta} = 2\,\frac{\lambda f}{\delta\eta}\,.$$

On the other hand, the harmonic structure of period Δx in the investigated field [see (36)] in the xy-plane gives rise to the spatial frequency $\omega_{\Delta\xi}$ in the registration plane $\xi\eta$, which is determined by relation (9) for the maximum of the spread function (37) and is characterized by the coordinate $\Delta\xi$:

$$\omega_{\Delta\xi} - k_x = 0\,, \quad \omega_{\Delta\xi} = \frac{\omega\Delta\xi}{cf} = \frac{2\pi}{\Delta x}\,, \quad \text{and} \quad \Delta x = \frac{2\pi}{\omega_{\Delta\xi}} = \frac{\lambda f}{\Delta\xi}\,.$$

It follows that for the interval of the registered spatial frequencies $\omega_{\xi\,\max}$, one can obtain the spatial resolution in the studied field

$$\delta x = \frac{2\pi}{\omega_{\xi\,\max}} = \frac{\lambda f}{\xi_{\max}}\,.$$

Similarly, for the axes y and ω_η, the spatial resolution is

$$\delta y = \frac{2\pi}{\omega_{\eta\,\max}} = \frac{\lambda f}{\eta_{\max}}.$$

Thus, the dimension of the area of the distribution registered in the plane of spatial frequencies ω_ξ and ω_η characterizes the spatial resolution in the investigated plane xy.

In a similar way, one can estimate the spatial characteristics (resolution δx, δy and the dimension of the investigated area $2x_0 \times 2y_0$) of the scheme forming the image of the analyzed plane (see Sec. 4).

To compare the pictures in the analyzed plane xy and in the registration plane $\xi_1\eta_1$, we assume, as above, that the field in the analyzed plane (see Fig. 3) contains some one-dimensional harmonic structure of period Δx and linear size $2x_0 \times 2y_0$ oriented along the x axis. The field formed in the investigated plane xy is described by (36).

In absence of the modulation $[M(\xi, \eta) = 1]$ the field in the registration plane is described by an expression analogous to (31′). In this case, we should take into account the final resolution in the registration plane $\xi_1\eta_1$, which is determined by the diffraction on the aperture of the optical system **OS**. In this case, the field formed in the plane of the image registration $\xi_1\eta_1$ is [see (8)]

$$E_s(\xi_1, \eta_1, t) = \mathcal{E}_s(\xi_1, \eta_1)\exp[-i\omega t] = -\frac{i}{\lambda b}\int \mathcal{E}_s(u, v)\, O(u, v)\exp[-i\omega t_b]\Big|_{t_b = t - r_b/c}\, du\, dv\,,$$

where

$$r_b = b + \frac{u^2 + v^2}{2b} + \frac{\xi_1^2 + \eta_1^2}{2b} - \frac{u\xi_1 + v\eta_1}{b}.$$

In this expression, the term $O(u, v)$ is responsible for the effect of the optical system [see (7)], and $\mathcal{E}_s(u, v)$ is the complex amplitude of the field in front of the optical system, which is described by formula (23).

Let us calculate the investigated field $E_s(\xi_1, \eta_1, t)$ in the registration plane $\xi_1\eta_1$, taking into account the condition of forming the image

$$\frac{1}{a} + \frac{1}{b} = \frac{1}{f}.$$

The integrals over u and v in the limits of the working area of the optical system

$$-u_0 \le u \le +u_0\,, \quad -v_0 \le v \le +v_0$$

give the spread function of the optical system **OS** as

$$4u_0v_0\, \frac{\sin\left[\frac{\omega}{c}u_0\left(\dfrac{x}{a} + \dfrac{\xi_1}{b}\right)\right]}{\frac{\omega}{c}u_0\left(\dfrac{x}{a} + \dfrac{\xi_1}{b}\right)}\, \frac{\sin\left[\frac{\omega}{c}v_0\left(\frac{y}{a} + \frac{\eta_1}{b}\right)\right]}{\frac{\omega}{c}v_0\left(\frac{y}{a} + \frac{\eta_1}{b}\right)}. \tag{38}$$

As above, the spread function is similar to the δ-function. With this in mind, the integrals over x and y are calculated for the maximum of the spread function, i.e., under the conditions

$$x = -\frac{a}{b}\,\xi_1\,, \quad y = -\frac{a}{b}\,\eta_1\,. \tag{39}$$

As a result, the field in the registration plane $\xi_1\eta_1$ is

$$E_s(\xi_1, \eta_1, t) = -\frac{4u_0v_0}{\lambda^2 ab}\exp\left[-i\omega t + i\frac{\omega}{c}\left(a + b + \frac{\xi_1^2 + \eta_1^2}{2b}\left(1 + \frac{a}{b}\right)\right)\right]\, \mathcal{E}_s\left(-\frac{a}{b}\,\xi_1, -\frac{a}{b}\,\eta_1\right).$$

In accordance with (39), for the analyzed field (36) in the registration plane we have

$$\mathcal{E}_s \exp\left[-i\,\frac{2\pi}{\Delta x}\,\frac{a}{b}\,\xi_1\right] .$$

This means that the harmonic structure of period Δx in the investigated plane xy corresponds to that of period $\Delta\xi_1 = (b/a)\Delta x$ in the registration plane $\xi_1\eta_1$, i.e., the harmonic structure with the scale determined by the magnification of the optical system.

With the magnification of the optical system taken into account, the width of the spread function [see (38)] defined by the distance between the first two zeros is determined from the relations

$$\frac{\omega}{c}\,u_0\,\frac{\xi_1}{b} = \pm\pi\,, \quad \frac{\omega}{c}\,v_0\,\frac{\eta_1}{b} = \pm\pi$$

and is

$$\delta\xi_1 = \frac{2\pi c}{\omega}\frac{b}{u_0} = \frac{\lambda b}{u_0}, \quad \delta\eta_1 = \frac{2\pi c}{\omega}\frac{b}{v_0} = \frac{\lambda b}{v_0}.$$

Hence, the smallest resolvable structure of the investigated field $E_s(x,y,t)$ in the analyzed plane xy is defined as

$$\delta x = \frac{a}{b}\,\delta\xi_1 = \frac{\lambda a}{u_0}, \quad \delta y = \frac{a}{b}\,\delta\eta_1 = \frac{\lambda a}{v_0},$$

i.e., is determined by the diffraction on the aperture of the optical system.

The size of the analyzed field $2x_0\times 2y_0$ is also determined by the size of the registered field $2\xi_{1\,\max}\times 2\eta_{1\,\max}$, taking into account the magnification of the optical system [see (39)]:

$$2x_0 \times 2y_0 = \frac{a}{b}\,(2\xi_{1\,\max} \times 2\eta_{1\,\max})\,.$$

Thus, the choice of the characteristics of the optical scheme (the focal distance f and the investigated area $2x_0\times 2y_0$) allows one to change the resolution δx, δy and the size of the analyzed field $2x_0\times 2y_0$ in the investigated plane xy.

6. Conclusions

In this paper, we demonstrated the possibility of determining the amplitude and phase spatial structure of two-dimensional optical fields. To solve the problem, schemes forming a spatial spectrum can be used, as well as those forming an image. In principle, in addition to the variants specified in the paper, other schematic decisions and spatial modulators can also be used. The specific variant of the scheme and the modulator is determined by the task at hand.

The spatial resolution in the scheme with registration of the spatial spectrum is determined by the entire range of registered spatial frequencies. The dimension of the analyzed field is defined by the resolution in the plane of spatial frequencies, depending on the resolution of the optical system. As for the scheme with registration of the image, the spatial resolution and the dimension of the investigated area are determined only by the characteristics of the optical system.

To solve the problem, no additional information or special conditions are required, except for the condition of validity of Fresnel's approximation for the Kirchhoff integral. It is fulfilled for most practical problems in physical and coherent optics. Note that the considered variants of solving the phase problem use no iteration procedures. This allows one to look forward to a high speed of processing of the information and to performing measurements in quasi-real time with the use of TV equipment with matrix recorders and computer processing of the data.

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