THE MEAN DIAMETER OF THE FRAGMENTS FORMED BY BLASTING ROCK

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The formula of Rozin and Rammler has been suggested [1, 2] as an analytical representation of the fragment-size composition of blasted rock:

$$R = \frac{V(x)}{V_0} = \exp\left[-\left(\frac{x}{x_0}\right)^n\right]. \quad (1)$$

Here V_0 is the total volume of blasted rock; $V(x)$ is the volume of all the fractions with linear dimensions greater than x; x_0 and n are the distribution parameters. The distribution function (the total relative volume of fractions not longer than x) is

$$\Phi(x) = 1 - R(x) = 1 - \exp\left[-\left(\frac{x}{x_0}\right)^n\right]. \quad (2)$$

The probability that the dimension of a fraction will lie in the range (x, x, + dx) is thus

$$dp = \Phi'(x)\,dx = \frac{n}{x_0}\left(\frac{x}{x_0}\right)^{n-1} \exp\left[-\left(\frac{x}{x_0}\right)^n\right]. \quad (3)$$

The number of fragments in this range will be

$$dm = \frac{V_0}{m}\,dp, \quad (4)$$

where V is the mean volume of a particle with dimensions in the range (x, x + dx).

From Eq. (4) it follows that the idea of probability thus introduced coincides with the method usually accepted in mining science for representing fragment-size composition:

$$dp = \frac{V\,dm}{V_0} = \frac{dV}{V_0}$$

or, in finite quantities,

$$\Delta p_i = \frac{\Delta V_i}{V_0}, \quad (5)$$

where ΔV_i is the volume of the i-th fraction.

Thus the probability that a fragment has a dimension lying in the range from x_i to $x_i + \Delta x_i$ is simply the relative content of the given fraction. The mean dimension of a fragment in an experiment is usually given as

$$\langle x \rangle = \sum_{i=1}^{k} x_i \frac{\Delta V_i}{V_0}, \quad (6)$$

where k is the number of groups of fractions.

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In our case, similarly

$$<x> = \int_0^\infty x dp. \tag{7}$$

Substituting the value from (3) and integrating, we get

$$<x> = x_0 \Gamma\left(1 + \frac{1}{n}\right), \tag{8}$$

where $\Gamma\left(1 + \frac{1}{n}\right) = \int_0^\infty e^{-t} t^{1/n} dt$ is a gamma function.

The dispersion of x is defined as

$$D = \int_0^\infty (x - <x>)^2 dp = <x>^2 \left[\frac{1 + \frac{2}{n}}{\Gamma\left(1 + \frac{1}{n}\right)} - 1 \right]. \tag{9}$$

Usually n > 1; therefore approximately we have

$$<x> \approx x_0, \quad D \approx \frac{x_0^2}{n}. \tag{10}$$

Thus the distribution parameter of x_0 in Eq. (1) is approximately equal to the mean fragment dimension, and n characterizes the root-mean-square deviation from the mean or in other words the uniformity of crushing.

Another interpretation of the meaning of the parameter n can be derived from the following considerations. Let us suppose that all the fragments formed by the blast are geometrically similar. Then a fragment with characteristic dimension x has a surface area and a volume given respectively by

$$S = K_S x^2, \quad V = K_V x^3,$$

where K_S and K_V are constant coefficients.

The total surface area of all the fragments is

$$S = \int_0^\infty S dm = V_0 \frac{K_S}{K_V} \int_0^\infty \frac{dp}{x} = \frac{V_0}{x_0} \frac{K_S}{K_V} \Gamma\left(1 - \frac{1}{n}\right).$$

Clearly the theoretically optimum variant of crushing is crushing of the whole bulk into equal pieces of dimension < x >. Then the surface area of all the fragments is

$$<S> = \frac{V_0}{<x>} \frac{K_S}{K_V}.$$

The relative "excess" surface is

$$\frac{\Delta S}{<S>} = \frac{S - <S>}{<S>} = \Gamma\left(1 + \frac{1}{n}\right) \Gamma\left(1 - \frac{1}{n}\right) - 1. \tag{11}$$

When n = 1, Eq. (2) is the Poisson distribution law, well-known from the theory of probability. From Eq. (11) it follows that this is the most disadvantageous case, because $\Gamma(0) \to \infty$, and hence the total surface area of the fragments is infinitely great.

Koshelev et al. [2] experimentally investigated the Rozin-Rammler law over a very wide range of materials, structures, and shapes. They blasted metal rings, Plexiglas blocks, and limestone specimens. All these field and laboratory experiments agreed with the theoretical data in the overwhelming majority of cases. The experiments

TABLE 1

Series of blasts		1	3	5	7	9	17
Specific explosives consumption, kg/m³		0,37	0,38	0,33	0,47	0,36	0,30
Charge mass, kg		390	400	480	350	320	410
Av. fragment length	Expt.	37	40	55	46	59	45
	Theor. (13)	42	41	48	34	41	38

TABLE 2

Series of blasts		1	2	3	4	5	7
Specific explosives consumption, kg/m³		0,31	0,34	0,37	0,39	0,39	0,32
Charge mass, kg		260	285	352	320	270	293
Av. fragment length	Expt.	34,7	58,4	57,6	62,9	55,5	63,9
	Theor. (13)	60	56	59	56,6	55	64,5

TABLE 3

Series of blasts		1	3	5	7	9	11	13
Specific explosives consumption, kg/m³		0,74	0,49	0,59	0,55	0,49	0,62	0,63
Charge mass, kg		675	625	557	600	710	730	533
Av. fragment length	Expt.	59	50	48	47	64	53	58
	Theor. (13)	38	51,5	44,5	47	48	44	41

with limestone are of special interest. These were performed in the Toguchin quarry (Novosibirsk region). The hardness was $f = 8$. The weights of the blasted blocks ranged from 0.8 g to 5000 kg, and the weights of explosives from 0.5 g to 0.5 kg. The explosive was Geksogen. After processing, the experimental data confirmed the Rozin-Rammler law very precisely; a semiempirical formula was found for the mean fragment diameter of the blasted rock. It is as follows:

$$\langle x \rangle = 7\left(\frac{V_0}{Q}\right)^{4/5} Q^{1/6}\theta^{-2/3}, \tag{12}$$

where $\langle x \rangle$ is the mean dimension in centimeters, V_0 is the volume of the blasted rock in cubic meters, Q is the weight of explosives in kilograms, and θ is the TNT equivalent of the charge.

Equation (12) was obtained by blasting separate blocks. According to general considerations, the mean fragment dimension from blasting the solid rock should be larger than this; therefore, for assessment and comparison with the results of large-scale blasting we take the expression

$$\langle x \rangle = 10\left(\frac{V_0}{Q}\right)^{4/5} Q^{1/6}, \tag{13}$$

TABLE 4

Series of blasts		1	2	3	4	5	6	7
Specific explosives consumption, kg/m³		0,62	0,59	0,62	0,60	0,65	0,66	0,51
Charge mass, kg		440	465	440	540	530	485	560
Av. fragment length	Expt.	46,2	43,9	38,1	32,7	36	59	51,4
	Theor. (13)	44,5	42,3	44,5	43	40	39	49

TABLE 5

Fragment size, cm	150–90	90–15	15–2,5	2,5–0
Percent of tot. volume	3	34	42	21

TABLE 6

Screen size	183	122	91	61	30,5	15,2	10,2	5,1	0
Percentage passing through screen	100	88	75	60	40	30	25	20	0

where Q is the weight of TNT equivalent to the energy of the charge. The main experimental results for analysis of the fragment-size composition were taken from a book by Marchenko [3]. We analyzed large-scale blasts in quarries of the Karakub Ore Administration, the Sorsk Mining and Beneficiation Combine, the A. P. Zavenyagin Noril'sk Combine, and the Olenegorsk Mining and Beneficiation Combine.

As Q we took the weight of explosives in one blast hole. The quantity $Q/V_0 = q$ is the specific explosives consumption in kg/m^3. We mainly analyzed blasts without air gaps. The Karakub deposit consists of badly fissured light-gray limestones with hardness $f = 8$. Results calculated by Eq. (12) with $\theta = 1$, together with experimental data, are listed in Table 1. The numbering of the series of experiments corresponds to that in [3]. The mean deviation of the experimental data from the theoretical data is ±15%. The Sorsk deposit consists of medium-hard rocks (for the ores $f = 10$-15, for the rocks $f = 8$-12). To estimate the mean fragment diameter, in this case we can use Eq. (13). The results and the experimental data are listed in Table 2.

In this case the mean relative error is again ±15%. The quarries of the Noril'sk Mining-Metallurgical Combine and the Olenegorsk Mining and Beneficiation Combine are composed of hard and very hard rocks, $f = 12$-16.

The experimental data for these quarries are given respectively in Tables 2 and 3. According to Table 2, on average they are 30% greater than the mean fragment lengths given by Eq. (13) with a mean deviation of ± 12%. For the Olenegorsk Mining and Beneficiation Combine the experimental data differ by ±15% from the theoretical [given by Eq. (13)]. Apparently there is no precise analytical relation between the hardness of the rock and the mean fragment size. We also cannot find a quantitative effect of the fissuring of the blasted rock, because the concept itself is very vague. Generalizing Eqs. (12) and (13) and the experimental data in Tables 1-4, we can adopt the following formula for the mean fragment size:

$$\langle x \rangle = A\left(\frac{V_0}{Q}\right)^{4/5} Q^{1/6}\ \text{cm}, \tag{14}$$

where

$$A = \begin{cases} 7 \text{ — for medium hard rocks, } f = 8\text{-}10; \\ 10 \text{ — for hard but highly fissured rocks, } f = 10\text{-}14; \\ 13 \text{ — for very hard, weakly fissured rocks, } f = 12\text{-}16. \end{cases}$$

Here Q is the weight of TNT in kilograms equivalent in energy to the explosive charge in one borehole, and $Q/V_0 = q$ is the specific explosives consumption in kg/m^3.

Let us consider two more blasts on a very large scale. Rodionov [4] gives American data obtained in an underground nuclear explosion of a charge with an energy of 60 kton at a depth of 400 m in granodiorite. The cavity formed partly caved from above, and fragment-size analysis of the caved rock gave the fragment distribution shown in Table 5.

The mean fragment size calculated from these data is 25 cm. The total volume of crushed rock was estimated to be 10^7 m^3. Substituting in Eq. (14) the values $V_0 = 10^7$ m^3, $Q = 6 \cdot 10^7$ kg, $A = 7$, we get $\langle x \rangle = 33$ cm, which is not too bad an agreement if we consider the very rough nature of the measurements. V. A. Adushkin has reported the data on fragment-size composition obtained from the detonation of 20,000 kg of TNT (Table 6). The mean fragment size calculated from these data is 56.4 cm. If we assume that the specific consumption in this blast was 1 kg/m^3 (there are no exact figures), formula (13) gives the value 52 cm.

Thus we can reckon that Eqs. (12)-(14) fairly accurately represent the experimental data over a wide range of scales of blasting, with linear dimensions varying by a factor of about 10^4.

Formulas (1) and (14) were checked for nearly optimum conditions giving intensive fragmentation. In critical conditions in which the number of fragments is small or, conversely, when the rock is repeatedly crushed by blasting or machines, the applicability of these expressions is doubtful. It is also very difficult to analyze the crushing of two-component or multicomponent rocks (see Koshelev et al. [2]).

LITERATURE CITED

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