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CORRECT EVALUATION OF TYPE-B STANDARD UNCERTAINTY

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The evaluation of uncertainty components "by means other than the statistical analysis of series of observations" (type-B evaluation) is considered. The problem comes down to selection of an appropriate a priori probability distribution of a random quantity within specified limits on the basis of all available information. Typical situations are indicated that lead to simple model distributions: rectangular (uniform), triangular, normal, and arcsine. Attention is drawn to the unjustified use of the uniform distribution model in examples of measurement-uncertainty evaluation.

The wide application of the *Guide to the Expression of Uncertainty in Measurement* [1] requires "partial" guides that call for the adaptation of general rules for evaluation and expression of uncertainty to specific measurement problems. Documents [2–4], for example, could serve as those guides. It is obvious that expansion of the sphere of applications will enrich the general concept and be accompanied by examples that are useful in practice. At the same time, broadening will conceal the danger of using simplified or insufficiently substantiated methods in the solution of practical problems. This situation has manifested itself clearly in examples of evaluation of type-B uncertainty.

The possibility of evaluation of uncertainty components by methods that differ from the statistical analysis of a series of observations (type-B evaluation) is a fundamentally important aspect of general metrology as presented in the *Guide*. It makes it possible to go beyond the traditional statistical approach (type-A evaluation) and find the values of uncertainty components for which the required statistical information is difficult or impossible to obtain. Examples of such uncertainty components include those associated with the measurement means (of an established type), deviations of influencing factors from their nominal values (within specified limits), and many others that appear in specific measurements, such as systematic effects.

Evaluation of type-B uncertainty is based on "scientific judgement" employing all available information. This includes data from previous measurements, data based on experience or general information on the behavior and properties of the corresponding materials and instruments, technical conditions of fabrication, etc. It is obvious that the quality of the thus-obtained evaluations is a function of the degree of completeness of the information used and the researcher's ability to interpret it critically.

Let us consider the problem of evaluation of standard uncertainty $\mathbf{u}(x)$, which is related to an influencing factor X whose value is within the limits $[x - \Delta, x + \Delta]$. The evaluation procedure can be represented by the diagram in Fig. 1.

On the basis of the available information, it is required to select an *a priori* probability distribution of possible values of x within specified limits and to find the standard uncertainty $\mathbf{u}(x) = \Delta / k$, where k is a coefficient that depends on the selected distribution function.

The choice of a distribution function depends on the available *a priori* information with respect to x. The following situations are typical.

1) Only the limits of the interval $[a, b]$ of possible x values are known ($b - a = 2\Delta$).

2) The value of $\bar{x}$ is known and the limits of permissible values $\pm\Delta$ are usually symmetric.

3) The interval $[\bar{x} - \Delta_p, \bar{x} + \Delta_p]$, which covers a certain portion p of the probability distribution – for example, 95 or 99% – is known.

4) The quantity x is a periodic function of time – for example, $x = \Delta \sin \omega t$ – such that its variation is negligible in the time necessary to perform the required number of measurements and, therefore, the effect of x is as that of a systematic error. An example is the effect of temperature fluctuations on the measurement result under conditions of thermostatic control.

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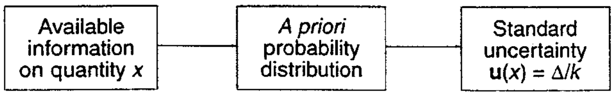

Fig. 1

TABLE 1

No.	Distribution type	Coefficient k in formula $\mathbf{u}(x) = \Delta/k$
1	Rectangular	$\sqrt{3}$
2	Triangular	$\sqrt{6}$
3	Normal	2.6 ($p = 0.99$)
4	Arcsine	$\sqrt{2}$

Each of these situations has its own model probability distribution function and corresponding value of the coefficient k, which is provided in Table 1.

Particularly noteworthy is situation 2), which differs from 1) in that the expected value of $\bar{x}$ is known *a priori*. This knowledge provides a basis for assuming that the probability of values near $\bar{x}$ is greater than near the boundaries $\bar{x} \pm \Delta$. In such cases (see Section F.2.3.3 in [1]), it is reasonable to take a triangular probability distribution as a compromise between rectangular (uniform) and normal distributions.

Despite the detailed recommendations given in the *Guide*, the published examples of uncertainty evaluation do not always take into account this circumstance. The detailed examples in [4], which is devoted to the evaluation of uncertainty in analytic measurements, show that all of the cases described in those examples in which a model of a rectangular distribution was used referred to situation 2) and not situation 1). For example, cases of evaluation of the uncertainty component associated with the use of standard measuring ware in analytic operations. Here, the nominal capacity of a graduated flask, pipette, etc. is an expected value, and the normalized tolerances are the limits of possible volumes of solution. In this situation, therefore, it is more correct to use a coefficient k equal to $\sqrt{6}$ rather than $\sqrt{3}$ to find the standard uncertainty $\mathbf{u}(x) = \Delta/k$.

In essence, evaluation of the components of type-B uncertainty is similar to that employed in domestic practice to find the standard deviations of unexcluded systematic errors (USE) specified in the form of limits. Following the well-known approach [5], this problem is solved, as a rule, exclusively within the framework of a model of a rectangular distribution, since "fairly cautious" estimates of the USE components are obtained. It is obvious, however, that the uncritical use of this model to evaluate standard uncertainty of type B could lead not only to overestimates, as in situation 2), but also (what is more important) to underestimates, as in situation 4).

Correct evaluation of type-B uncertainty requires the thoughtful allowance for all available information. As is noted in Section 3.4.8 of [1], the *Guide* "cannot replace a critical approach, conscientiousness, and professionalism. Uncertainty is neither a routine nor a purely mathematical problem; it depends on detailed knowledge of the nature of the quantity being measured and of the measurement itself."

REFERENCES

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