Vertical Eddy Diffusion Coefficient in Lake Zürich

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ABSTRACT

The vertical eddy diffusion coefficient in Lake Zürich is a strong function of both depth and season. In the deep lake, the annual oxygen consumptions by the oxidation of organic carbon is balanced out by the equal amount of oxygen supply from above by diffusion.

SCHMIDT [2], MORTIMER [1] and others have estimated the vertical eddy diffusion coefficient $D(z, t)$ of many lakes by the equation:

$$F(z, t) = \int_{z}^{z_1} (\partial T(z, t)/\partial t)_z \, dz$$

$$= - D(z, t) \cdot (\partial T(z, t)/\partial z)_t ,$$

where $F(z, t)$ = the total heat diffused through a unit horizontal area per unit time at depth z and time t.

$\partial T/\partial z$ and $\partial T/\partial t$ = the partial differentials or the observed slopes of temperature plotted against depth and time respectively. z_1 = the depth where $\partial T/\partial t = 0$, or bottom depth.

Equation (1) is applicable only under the conditions that: a) there is no vertical advection of lake water, b) the net horizontal heat transport is negligible i.e., the horizontal advection and/or horizontal temperature gradient are negligibly small and c) the heat exchange between water and bottom sediments is small. Fortunately Lake Zürich satisfies the above conditions (more or less).

ZIMMERMANN [3] summarized the average monthly temperatures of Lake Zürich as a function of depth down to the bottom at one station over a period of 10 years (1948–57, Fig. 1). The advantage of using the average temperature of several years is that any random factors (either climatic or observational) have been smoothed out, so the reading of $\partial T/\partial t$ and $\partial T/\partial z$ for each observational point from the smoothed curves (in Fig. 1 and monthly T-z plots) becomes easier. $D(z, t)$ calculated from April to December at a depth above 60 m by using Zimmermann's data are given in Table 1 and plotted in Figure 2. At a depth below 60 m, the $\partial T/\partial t$ and $\partial T/\partial z$ terms become so small and the uncertainty in $D(z, t)$ so big, that no attempt is made to calculate D

Table 1. The vertical eddy diffusion coefficients of Lake Zürich $D(\mathrm{cm}^2/_{\mathrm{sec}})$.

Depth (m)	April	May	June	July	August	Sept.	Oct.	Nov. and Dec.	Depth (m)	Also for Feb. (Jan. and March)
5	0.71	0.35	0.17	0.110	0.18	–	–	–	60	9.78
7.5	0.88	0.21	0.065	0.054	0.052	–	–	–	70	4.20
10	1.06	0.14	0.064	0.041	0.026	0.020	–	–	80	3.34
12.5	1.35	0.18	0.073	0.039	0.035	0.020	–	–	90	2.31
15	1.51	0.28	0.090	0.082	0.050	0.023	–	–	100	1.21
17.5	1.66	0.37	0.22	0.110	0.065	0.046	–	–	110	0.95
20	1.86	0.61	0.29	0.14	0.11	0.072	0.074	–	120	0.68
25	2.23	0.81	0.37	0.18	0.18	0.108	0.11	0.51		
30	2.70	0.90	0.44	0.27	0.20	0.17	0.12	0.64		
40	4.02	1.00	0.48	0.31	0.25	0.18	0.16	0.68		
50	4.30	1.07	0.45	0.33	0.28	0.17	0.18	0.59		
60	3.75	1.35	0.43	0.34	0.28	–	–	0.62		

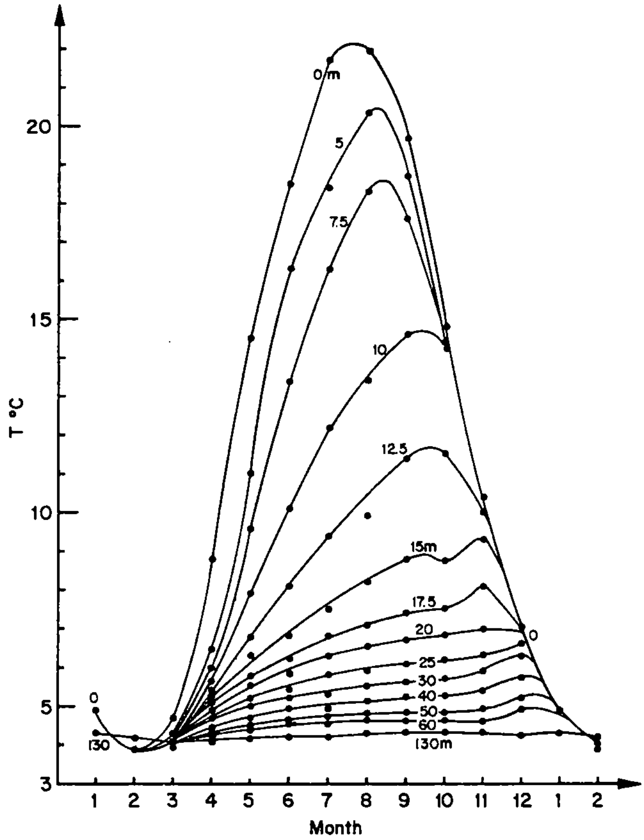

Fig. 1. The monthly variation of temperature in Lake Zürich for the different depths.

below 60 m from the temperature data. Also at shallow depths where the disturbance of water body is strong, D was not calculated. The uncertainty of D above 40 m is about 10%, and below 40 m, about 20% between April and June and 30% or more between July and October. The uncertainty of D can be largely attributed to the uncertainty of $\partial T/\partial t$ at a depth below 60 m, therefore the accurate measurement of temperatures at greater depths is very critical[1]). As shown in Figure 2, the D minimum (May to September) corresponds nicely to the thermocline. D also changes with

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[1]) Unfortunately, most of recent continuous-temperature-profile measurements by thermocouple devices were usually calibrated at the surface of a lake so that the monthly temperature measurements at depth fluctuate even by as much as 0.4 °C, which can hardly be explained by a sudden climatic change. So my personal plea is that the continuous temperature profile should be calibrated against the temperature at depth by using reversible thermometers as 'old time' limnologists used to do.

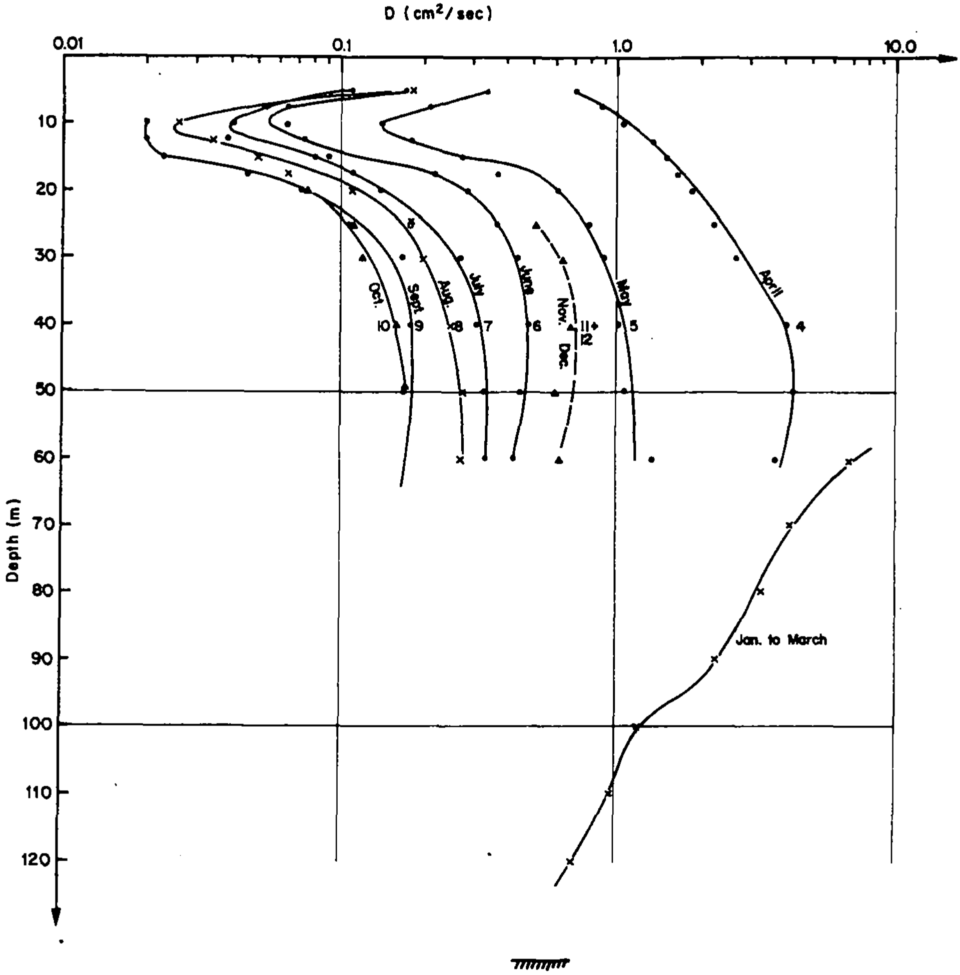

Fig. 2. The vertical eddy diffusion coefficients as a function of both depth and month.

Table 2. Oxygen fluxes at depths 10, 20 and 60 m and the net gain and loss of oxygen in the water columns.

	(mg $O_2/cm^2 \cdot$ month)				
	10 m	20 m	60 m	(10–20)	(20–60)
April	3.05	2.76	3.02	0.29	–0.26
May	1.40	0.82	1.06	0.57	–0.23
June	0.84	0.13	0.51	0.71	–0.38
July	0.70	–0.20	0.23	0.90	–0.43
August	0.52	–0.23	0.23	0.75	–0.45
September	0.55	–0.17	0.20	0.72	–0.37
October	0.56	–0.23	0.18	0.79	–0.41
November			0.45		
December			0.35		
January			11.91		
February			13.44		
March			7.61		
Total			39.19		

depth by almost one order of magnitude and decreases from April to October more than one order of magnitude i.e., the lake as a whole becomes increasingly stagnant till October. Certainly in any calculation dealing with material transport by diffusion in the lake, one can not neglect the fact that D is a strong function of both depth and season.

ZIMMERMANN [3] also summarized the average monthly oxygen concentration $[O_2]$ of Lake Zürich over the same 10 year-period as plotted in Figure 3. The average rate of decrease of oxygen concentration during the summer stratification period, J_z, and the rate of increase of oxygen concentration in February, $(\partial[O_2]/\partial t)_z$, are also plotted in Figure 4 (J_z and $(\partial[O_2]/\partial t)_z$ were both directly read from Fig. 3). Both J_z and $(\partial[O_2]/\partial t)_z$ represent a net effect of a) the rate of oxidation of organic carbon by biological processes and b) the supply rate of oxygen by diffusion from above. Fortunately, the latter is much smaller than the former at a depth below 60 m during the summer stratification period as will be shown later. Therefore, J_z at a depth below 60 m is very close to the true oxidation rate of organic carbon. Since the temperature at any depth below 60 m changes annually by no more than 0.5 °C which exerts a negligible influence (for our purpose) on the rate of biological processes. Thus, one may assume the same value for J_z holds in winter as well as in summer. As a consequence, $J_z + (\partial[O_2]/\partial t)_z$ represents approximately a true increasing rate of oxygen supply in winter. By re-formulating equation (1) for oxygen, one can calculate D in February at a depth below 60 m by the equation:

$$\text{Vertical Flux of } O_2 = \int_{z}^{z_1} (J + \partial[O_2]/\partial t)_z \, dz = -D\,(\partial[O_2]/\partial z)_t. \qquad (2)$$

All pertinent data are given in Figure 4 and the results in Figure 2 and Table 1. D at a depth below 60 m decreases exponentially with depth and shows a change of

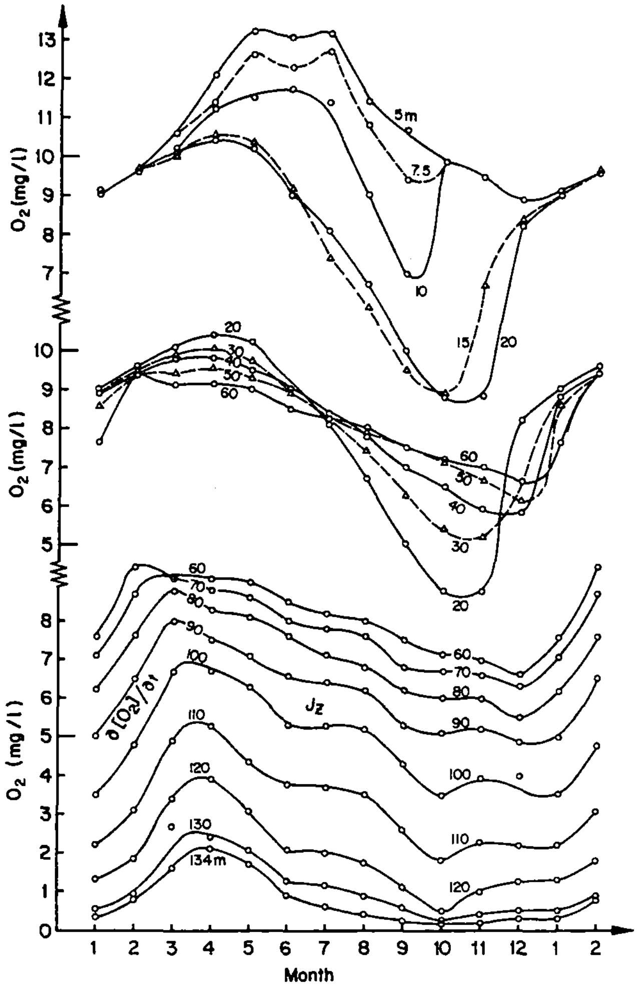

Fig. 3. The monthly variation of oxygen concentration in Lake Zürich for the different depths.

slope at a depth of 100 m. Above 60 m, the water is well mixed in regard to oxygen concentration (Fig. 4).

In order to study the oxygen budget of Lake Zürich, the monthly oxygen flux by diffusion at 10 m and 20 m depths during the summer stratification period and at 60 m depth for a whole year were calculated by using the obtained $D(z,t)$ (Table 1) and the observed $(\partial[O_2]/\partial z)_t$. The results are given in Table 2 along with the monthly net gain or loss of oxygen from water columns (i.e., 10 to 20 m and 20 to 60 m intervals). Adding up the values of the 4th column of Table 2, the total annual flux of oxygen from above at 60 m depth is about 39.19 $mg/cm^2 \cdot$ year. On the other hand, the total annual consumption of oxygen by oxidation of organic carbon in the water column below 60 m is 39.24 $mg/cm^2 \cdot$ year, obtained by integrating J_z in the water column below 60 m and multiplying by 12, in good agreement with the above diffu-

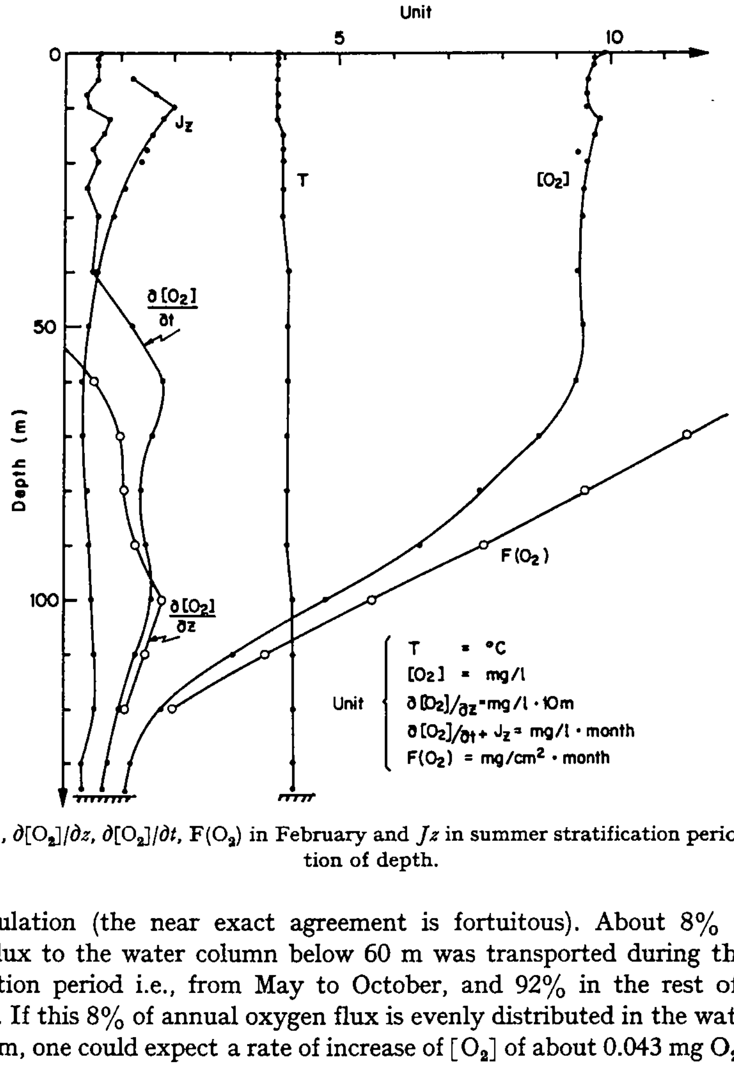

Fig. 4. $[O_2]$, $\partial[O_2]/\partial z$, $\partial[O_2]/\partial t$, $F(O_2)$ in February and Jz in summer stratification period as a function of depth.

sion calculation (the near exact agreement is fortuitous). About 8% of annual oxygen flux to the water column below 60 m was transported during the summer stratification period i.e., from May to October, and 92% in the rest of the year (Table 2). If this 8% of annual oxygen flux is evenly distributed in the water column below 60 m, one could expect a rate of increase of $[O_2]$ of about 0.043 mg O_2/l · month which is only about 10% of the apparent oxidation rate of organic carbon, J_z, in the same water column. (J_z ranges from 0.32 to 0.56 mg/l · month and in average 0.44 mg/l · month.) In a water column between 20 m and 60 m oxygen is transported out at both ends by diffusion throughout the whole year, thereby the water column loses on the average about 0.072 mg O_2/l · month, which is again relatively small as compared with J_z (average 0.7 mg O_2/l · month) in the same water column. Meanwhile, in the water column between 10 m and 20 m, the average gain of oxygen by diffusion during the summer stratification period is about 0.74 mg O_2/l · month which is no longer a negligible amount as compared with J_z (average 1.6 mg/l · month, ranging from 1.4 to 2.0 mg/l · month) in the same water column. The true average oxidation rate of organic carbon in the water column between 10 m and 20 m then should be $1.6 + 0.74 = 2.34$ mg O_2/l · month.

Summary

1. The vertical eddy diffusion coefficient of Lake Zürich is a strong function of both depth and season. 2. The oxygen budget in the deep lake can be explained by the oxygen supply from above by diffusion and the oxidation of organic carbon by biological processes. 3. During the summer stratification period, the oxygen transport by diffusion has only a minor effect on the oxygen concentration in the water column below 20 m, but in the zone of the oxygen minimum (10 to 20 m), the oxygen transport by diffusion becomes important as compared to the magnitude of oxygen consumption by biological processes.

ZUSAMMENFASSUNG

1. Der vertikale Austauschkoeffizient im Zürichsee hängt streng von der Tiefe und der Jahreszeit ab. 2. Der Sauerstoffhaushalt in der Seetiefe lässt sich durch austauschbedingte Zufuhr von oben und durch die Oxidation organischen Kohlenstoffs durch biologische Prozesse erklären. 3. Während der sommerlichen Stagnationsperiode wirkt sich der Sauerstofftransport durch Austausch nur sehr schwach auf die Sauerstoffkonzentration in der Wassertiefe unterhalb 20 m Tiefe aus. In der Zone des metalimnischen Sauerstoffminimums (10 bis 20 m) hingegen erlangt der Sauerstofftransport durch Austausch gegenüber dem biogenen Sauerstoffverbrauch eine grosse Bedeutung.

RÉSUMÉ

1. Le facteur de diffusion verticale dans le lac de Zurich dépend étroitement de la profondeur et la saison. 2. La concentration actuelle en oxygène de la couche profonde est déterminée par l'apport de la surface lors de la diffusion et par l'oxydation du carbone organique lors des processus biologiques décomposants. 3. Pendant la période de stratification estivale l'oxygène apporté lors de la diffusion n'a qu'une très faible influence sur la concentration d'oxygène dans les couches en-dessous de 20 m de profondeur. Dans la zone du metalimnion où l'oxygène est réduit à son minimum (10 à 20 m) l'apport d'oxygène par diffusion – comparé avec la consommation biologique d'oxygène – reprend une grande importance.

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