THERMAL ROCK BREAKING

L. S. Dudoladov

UDC 622.83

When a solid body is subjected to intense thermal actions nonuniformly distributed over its surface, it is observed to break up by mechanical ejection of pieces of material from the surface—so-called spalling [1, 2]. As a rule, spalling usually occurs in solids with low thermal conductivity k, such as rocks [1]. This is evidently because the temperature stresses which arise as a result of nonuniform heating become concentrated owing to the low thermal conductivity of the rock near the surface.

To study the features of the state of stress arising in massive solid bodies as a result of nonuniform surface heating, let us consider the quasistatic axisymmetric thermoelastic problem for a half-space $z \geq 0$, the boundary of which is free from stress:

$$\sigma_z = \tau_{rz} = 0 \quad \text{for} \quad z = 0.$$

The temperature field satisfies the Fourier heat conduction equation,

$$k \nabla^2 \theta = \frac{\partial \theta}{\partial t} \tag{1}$$

with the following initial and boundary conditions:

$$\begin{aligned} \theta &= 0 && \text{for} \quad t = 0; \\ \theta &= \frac{T_0 \lambda^3}{(r^2 + \lambda^2)^{3/2}} && \text{for} \quad z = 0. \end{aligned} \tag{2}$$

This assignment of the temperature on the boundary corresponds to a dome-shaped distribution with a value T_0 on the axis of symmetry; as the distribution parameter λ increases, the dome gets shallower and more spread out.

Using the results in [3], we can write the solution of the temperature problem (1), (2) in the form:

$$\theta = \frac{z T_0 \lambda^2}{2\sqrt{k\pi}} \int_0^t e^{-\frac{z^2}{4k\tau}} \frac{d\tau}{\tau^{3/2}} \int_0^\infty \xi e^{-\lambda\xi - k\tau\xi^2} J_0(\xi r)\, d\xi, \tag{3}$$

where $J_0(\xi, r)$ is a Bessel function of the first kind.

The solution of the thermoelastic problem [3] can be written in the form:

$$\sigma_r = -\frac{E\beta\theta}{1-\nu} + \frac{2\mu}{1-2\nu}\psi - \frac{2\mu(1-\nu)}{1-2\nu}\frac{1}{r^2}\int_0^r r\psi\, dr - \sigma_z + \frac{1}{r^2}\int_0^r r\sigma_z\, dr;$$

$$\sigma_\varphi = -\frac{E\beta\theta}{1-\nu} + \frac{2\mu\nu}{1-2\nu}\psi + \frac{2\mu(1-\nu)}{1-2\nu}\frac{1}{r^2}\int_0^r r\psi\, dr - \frac{1}{r^2}\int_0^r r\sigma_z\, dr;$$

$$\tau_{rz} = -\frac{1}{r}\int_0^r r\frac{\partial\sigma_z}{\partial z}\, dr, \tag{4}$$

Institute of Mining, Siberian Branch, Academy of Sciences of the USSR, Novosibirsk. Translated from Fiziko-Tekhnicheskie Problemy Razrabotki Poleznykh Iskopaemykh, No. 2, pp. 102-106, March-April, 1969. Original article submitted August 20, 1968.

where in this case the axial stress σ_z and the harmonic function ψ are

$$\sigma_z = \frac{E\beta k T_0 \lambda^2}{1-\nu}\left\{\frac{z}{2\sqrt{k\pi}}\int_0^t (t-\tau)\, e^{-\frac{z^2}{4k\tau}} \frac{d\tau}{\tau^{3/2}} \int_0^\infty \xi^3 e^{-\lambda\xi - k\tau\xi^2} J_0(\xi r)\, d\xi\right.$$

$$+ z\int_0^\infty \xi^3 e^{-(\lambda+z)\xi}\left[\frac{1}{2k\xi}\operatorname{erf}(\xi\sqrt{kt}) + \frac{\sqrt{t}}{\sqrt{k\pi}} e^{-kt\xi^2} - t\xi \operatorname{erfc}(\xi\sqrt{kt})\right]$$

$$\left.\times J_0(\xi r)\, d\xi - t\int_0^\infty \xi^3 e^{-(\lambda+z)\xi} J_0(\xi r)\, d\xi\right\}; \tag{5}$$

$$\psi = \frac{1-2\nu}{1-\nu}\frac{E\beta T_0\lambda^2}{\mu}\int_0^\infty \xi^2 e^{-(\lambda+z)\xi}\left[\frac{1}{2\xi}\operatorname{erf}(\xi\sqrt{kt}) + \frac{\sqrt{kt}}{\sqrt{\pi}}\right.$$

$$\left.\times e^{-kt\xi^2} - kt\xi\operatorname{erfc}(\xi\sqrt{kt})\right] J_0(\xi r)\, d\xi,$$

where E is Young's modulus, μ is the shear modulus, ν is Poisson's ratio, and β is the coefficient of linear expansion.

In (3) and (5) let us transform to a new variable of integration $x = \lambda\xi$ and introduce the dimensionless variables $\zeta = z/\lambda$, $\rho = r/\lambda$, $t^* = \frac{2\sqrt{kt}}{\lambda}$. Assuming that t^* is small (which corresponds to finite values of λ and small times t), we can expand the functions containing this parameter in the well-known series

$$e^{-\frac{t^{*2}x^2}{4}} = 1 - \frac{t^{*2}x^2}{4} + \frac{1}{2!}\left(\frac{t^{*2}x^2}{4}\right)^2 - \ldots$$

$$\operatorname{erf}\left(\frac{xt^*}{2}\right) = \frac{2}{\sqrt{\pi}}\left[\frac{xt^*}{2} - \frac{(xt^*)^3}{1!3\cdot 2^3} + \ldots\right].$$

Integrating term by term we get the solution in the form of an expansion in a small parameter. In particular, restricting ourselves to the first term, we get

$$\frac{\theta}{T_0} = \frac{1}{(1+\rho^2)^{3/2}}\operatorname{erfc}\left(\frac{\zeta}{t^*}\right);$$

$$\frac{(1-\nu)\sigma_z}{E\beta T_0} = \frac{3t^*}{\sqrt{\pi}}\frac{\zeta(1+\zeta)[2(1+\zeta)^2 - 3\rho^2]}{[(1+\zeta)^2+\rho^2]^{7/2}}; \tag{6}$$

$$\psi = \frac{1-2\nu}{1-\nu}\frac{E\beta T_0}{\mu}\frac{t^*}{\sqrt{\pi}}\frac{2(1+\zeta)^2-\rho^2}{[(1+\zeta)^2+\rho^2]^{5/2}}.$$

Using (4) we easily get an expression for the residual components of the stress tensor.

$$\frac{(1-\nu)\tau_{rz}}{E\beta T_0} = -3\rho\left\{\frac{1}{(1+\rho^2)^{5/2}}\left[\zeta\operatorname{erfc}\left(\frac{\zeta}{t^*}\right) - \frac{t^*}{\sqrt{\pi}} e^{-\frac{\zeta^2}{t^{*2}}}\right]\right.$$

$$\left. + \frac{t^*}{\sqrt{\pi}}\frac{(1+\zeta)}{[(1+\zeta)^2+\rho^2]^{5/2}} - \frac{t^*}{\sqrt{\pi}}\frac{\zeta[4(1+\zeta)^2-\rho^2]}{[(1+\zeta)^2+\rho^2]^{7/2}}\right\};$$

$$\frac{(1-\nu)\sigma_r}{E\beta T_0} = -\frac{1}{(1+\rho^2)^{3/2}}\operatorname{erfc}\left(\frac{\zeta}{t^*}\right) + \frac{2t^*}{\sqrt{\pi}}\frac{2(1+\zeta)^2-\rho^2}{[(1+\zeta)^2+\rho^2]^{5/2}}$$

$$-\frac{2(1-\nu)t^*}{\sqrt{\pi}}\frac{1}{[(1+\zeta)^2+\rho^2]^{3/2}} - \frac{3t^*}{\sqrt{\pi}}\frac{\zeta(1+\zeta)[(1+\zeta)^2-4\rho^2]}{[(1+\zeta)^2+\rho^2]^{7/2}};$$

$$\frac{(1-\nu)\sigma_\varphi}{E\beta T_0} = -\frac{1}{(1+\rho^2)^{3/2}}\operatorname{erfc}\left(\frac{\zeta}{t^*}\right) + \frac{2\nu t^*}{\sqrt{\pi}}\frac{2(1+\zeta)^2-\rho^2}{[(1+\zeta)^2+\rho^2]^{5/2}}$$

$$+\frac{2(1-\nu)t^*}{\sqrt{\pi}}\frac{1}{[(1+\zeta)^2+\rho^2]^{3/2}} - \frac{3t^*}{\sqrt{\pi}}\frac{\zeta(1+\zeta)}{[(1+\zeta)^2+\rho^2]^{5/2}}. \qquad (7)$$

Using (6) and (7) we calculated the stress fields for a Poisson ratio of $\nu = 0.3$. The graphs given below were constructed for the reduced stresses,

$$S_r = \frac{(1-\nu)\sigma_r}{E\beta T_0}; \quad S_\varphi = \frac{(1-\nu)\sigma_\varphi}{E\beta T_0}; \quad S_z = \frac{(1-\nu)\sigma_z}{E\beta T_0};$$

$$S_\tau = \frac{(1-\nu)\tau_{rz}}{E\beta T_0}; \quad T = \frac{(1-\nu)\tau_{max}}{E\beta T_0}.$$

When $t^* = 0.1$, the state of stress is as follows. At the surface of the half-space $\zeta = 0$, the radial stress σ_r and the peripheral stress σ_φ are compressive for all values of ρ (Fig. 1). As ζ increases, σ_r and σ_φ also increase, and starting from a certain cross section become tensile (Figs. 2, 3). Simultaneously, as ζ increases, axial stresses σ_z and tangential stresses τ_{rz} develop, which, beginning from a certain cross-section, become comparable with the stresses σ_r and σ_φ (Figs. 4, 5).

Figure 6 gives the values of the stresses at the point $\zeta = 0.1$, $\rho = 0.6$, versus t^*. At this point σ_z and τ_{rz} are monotonic functions of t^*, and for all the values of t^* studied, σ_z remains tensile. As t^* increases, the radial and peripheral stresses reach their maximum tensile values, and then rapidly fall and become compressive. Figure 7 gives the values of the principal stresses,

$$S_1 = \frac{1}{2}\left[S_r + S_z + \sqrt{(S_r - S_z)^2 + 4S_\tau^2}\right],$$

$$S_2 = \frac{1}{2}\left[S_r + S_z - \sqrt{(S_r - S_z)^2 + 4S_\tau^2}\right]$$

at the point $\zeta = 0.1$, $\rho = 0.6$, plotted versus t^*.

Going on to analyze the possible fractures, we begin with the following model of the rock. We shall consider it to be a homogeneous elastic body in which are variously oriented cracks. These are of such size that they do not appreciably affect the stress distribution.

According to the theory of quasibrittle fracture, owing to the tensile stresses σ_z (see Fig. 4), fractures may develop perpendicular to the z axis in the neighborhood of the symmetry axis. Following [4], we write the condition of propagation of a fracture in the form

$$\int_0^R \frac{r\sigma_z(r)\,dr}{\sqrt{R^2 - r^2}} = K\sqrt{\frac{R}{2}}, \qquad (8)$$

where K is the modulus of cohesion and R the radius of the fracture (crack).

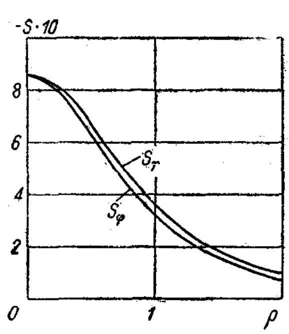

Fig. 1

Expanding σ_z (6) in the neighborhood of the symmetry axis as a Taylor series, we get, to the first order of small quantities,

$$\frac{(1-\nu)\sigma_z}{E\beta T_0} = \frac{12\sqrt{kt}}{\lambda\sqrt{\pi}}\frac{\zeta}{(1+\zeta)^4}. \qquad (9)$$

Putting (9) into (8) and integrating, we get

$$\frac{12\sqrt{kt}}{\lambda\sqrt{\pi}}\frac{\zeta}{(1+\zeta)^4}\frac{E\beta T_0}{1-\nu}R = K\sqrt{\frac{R}{2}}. \qquad (10)$$

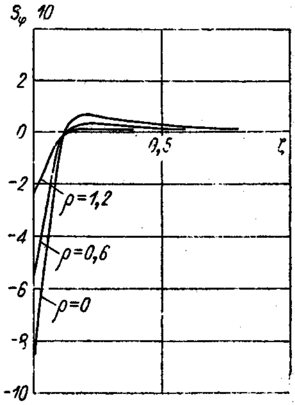

Fig. 2

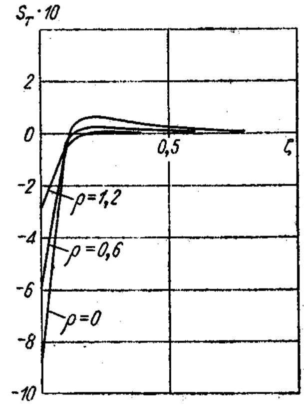

Fig. 3

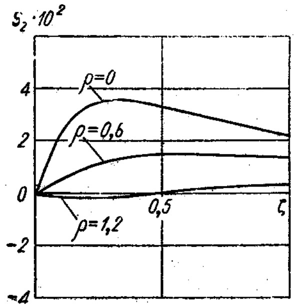

Fig. 4

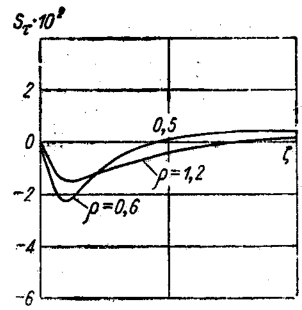

Fig. 5

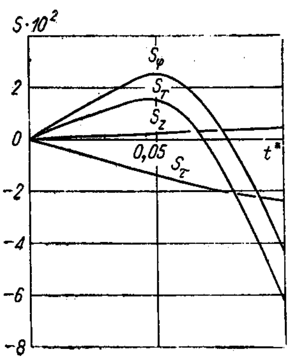

Fig. 6

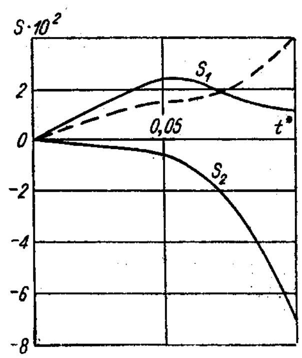

Fig. 7

We shall take $R/t = v_{cr}$ to be the velocity of propagation of the crack. Since the function $f(\zeta) = \zeta/(1+\zeta)^4$ has a maximum at $\zeta = 1/3$, from (10) we get the condition of crack propagation in the form

$$\lambda \leqslant 1.01 \frac{E\beta T_0 \sqrt{kR}}{(1-\nu) K \sqrt{v_{cr}}}.$$

Further fracture may result from loss of stability of individual plates.

Fissures may open at some point not lying on the symmetry axis owing to the tensile force S_1 (see Fig. 7). The fissure will obviously be perpendicular to the direction of $\vec{S}_1$. Further fracture may occur owing to the maximum tangential stress, which increases monotonically with t_* (see Fig. 7, dashed line). The orientation of the areas on which the maximum tangential stress acts at the point $\zeta = 0.1$, $\rho = 0.6$ is given by the following relations between the principal stresses:

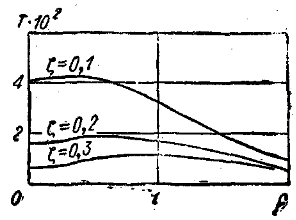

Fig. 8

$$S_\varphi > S_1 > S_2 \text{ for } 0 \leqslant t^* < 0.06;$$
$$S_1 > S_\varphi > S_2 \text{ for } 0.06 \leqslant t^* \leqslant 0.1.$$

Figure 8 gives the maximum tangential stresses versus ρ for $t^* = 0.1$ at the cross sections $\zeta = 0.1$, $\zeta = 0.2$, $\zeta = 0.3$. The relation between the principal stresses is

$$S_1 > S_\varphi > S_2 \text{ for } \zeta = 0.1;$$
$$S_\varphi > S_1 > S_2 \text{ for } \zeta = 0.2,\ 0.3.$$

In the cross sections $\zeta < 0.1$, the maximum tangential stresses for $t_* = 0.1$ are monotonically decreasing functions of the radial coordinate ρ.

For a given thermal action (λ being fixed) on the rock surface, the time dependence of the formation of the state of stress at any fixed point inside the body depends essentially on the thermal diffusivity of the rock. This becomes evident if we draw the time graphs in Figs. 6 and 7. From the formula $t = \frac{t^{*2}\lambda^2}{4k}$ it follows that the state of stress corresponding to some fixed value of t_* will be attained for low thermal conductivity in a fairly short time. The number of open fissures thus rises, which favors further fracture.

From the graphs we also infer that the process of thermal fracture is also directly influenced by the value of Poisson's ratio ν and by the combination of parameters $E\beta T_0$. As these increase, the dimensional values of the stresses also increase.

LITERATURE CITED

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