Eugene B. Fabes

1937 – 1997

Gene was born in Detroit in 1937. His family later moved to Kansas City, where he grew up. His father and uncles, Jewish immigrants from the Ukraine, built a very successful family business which at its peak, when Gene came of age, owned three luggage factories.

College was conceived to be Gene's last fling of youth before entering the serious business of life. Gene asked his brother what to study at Harvard to prepare for work in the family business. His brother replied that Gene should study whatever pleased him, since he would be able to learn what he needed about the family business once he started working there. Free to do what he liked, Gene majored in mathematics. Harvard did what good colleges sometimes do, encouraging Gene to go in a direction orthogonal to his family's wishes.

Still, Gene tried to live up to what was expected of him. After graduation, he undertook management of one of the family factories, in a small town outside Atlanta. A year later the notions planted by Harvard asserted themselves, and Gene applied to graduate schools, and in January 1961 came to the University of Chicago.

Gene recalled being drawn to Chicago by the presence of Antoni Zygmund. While in college Gene had studied Saks and Zygmund's *Analytic Functions,* which made a deep impression on him. His wife, Esther, recalls another reason why Gene chose Chicago: once Gene decided to go to graduate school he could not wait to do so, and the University of Chicago was one of only two schools which accepted students in the winter term.

Whatever the reason, his decision to come to Chicago was an extremely fortunate one. Gene joined one of the most important mathematics research programs of the century, integrating Real Analysis, Harmonic Analysis, and P.D.E. In the weekly "Calderón-Zygmund Seminar," one learned the new results from Chicago and elsewhere, and then heard a discussion of where they might lead. Bright and ambitious, Gene became Zygmund's student and blossomed in this environment.

One of the advantages of being in a great graduate department is the company of other outstanding students. Nestor Rivière was one such student, who was always eager to integrate friendship with mathematical collaboration. Gene and Nestor teamed up and together transformed the subject of parabolic singular integrals. Gene graduated in 1965 and went to Rice University.

In 1967 Gene and Nestor were reunited when Nestor recruited him to the University of Minnesota. The next few years were glorious. Both Gene and Nestor had a deep commitment to mathematics and worked hard at it, with great joy. It was a pleasure to see their interaction. They had great mutual respect and affection, and a competitive spirit that caused them to bring out the best in each other. Their spirited collaboration ended in 1978 with Nestor's untimely death.

Despite the loss of his closest friend and collaborator, Gene continued to grow, both mathematically and personally. He started an important research program with Luis Caffarelli, Stefano Mortola, and Sandro Salsa. He drew a growing number of graduate students, post-docs, and visitors.

Gene developed contacts with Italian mathematicians, which became an important part of his life. Gene fell in love with Italy and became so fluent in its language that he acquired a heavy Italian accent in his Spanish. S. Salsa wrote: "Since [1976] he often went to Italy to give various kinds of courses and conferences, spreading the new advanced ideas on potential theory, real analysis and PDE. In this way, he directed or started, in various cases, the mathematical research of many young Italian mathematicians."

Intense mathematical activities did not prevent Gene from taking an important role in the leadership of the mathematics department. He served as Director of Graduate Studies, Associate Head, and between 1990 and 1995, Head of the department. The department made great strides during his tenure. Especially remarkable was the recruitment of N.V. Krylov. Gene went to extraordinary lengths to overcome the unwillingness of the Soviet authorities to let talent leave, and the reluctance of U.S. immigration authorities to let talent come. With this and several other excellent appointments, Gene maintained the preeminence of the department in Analysis. Gene also believed that mathematics departments had to contribute to society beyond the flow of new theorems. As Head, Gene established a Master's program in Industrial Mathematics which now serves as a national model. Finally, as a result of Gene's sense of social responsibility, he was deeply interested in mathematics education and from the late 1960s participated in various education projects. Mathematics education in Minnesota flourished during Gene's tenure as Head.

After completing his term, assured by the willingness of an able colleague to take over, Gene decided not to seek a second term as Head, and looked forward to focusing exclusively on his research. But after being convinced that the Geometry Center needed his involvement, he agreed to serve as its Deputy Director.

In the midst of these activities Gene's life was tragically ended by a heart attack.

Gene left a rich professional legacy: some 75 articles, 21 Ph.D. students and numerous post-docs and collaborators all over the world. His mathematical ideas will continue to resonate for years to come. His personal legacy is equally inspiring.

Gene had many, many friends. His relationships were shaped by his generous heart, his keen understanding of people, and his engaging disposition. Even when facing his own personal challenges he continued to embrace life. A colleague wrote: "Meeting Gene, I felt like the day became brighter, the weather better, and worries lighter."

Although Gene spoke simply, eschewing the vocabulary of academe (he would not have used "eschew"), the plain speech expressed clear thinking and, more importantly, a strong resolve. Thus, in 1959, when the Ku Klux Klan demanded that he install segregated drinking fountains in the luggage factory, he refused and, moreover, promoted African-Americans to foremen.

Gene's last year was a fitting celebration of his life. He continued to do excellent mathematics and was honored by his friends and colleagues. His friends in Italy organized a conference marking Gene's 20 years of summer school at Cortona. The University of Chicago invited him to give the Zygmund lectures. The University of Minnesota organized a conference to celebrate his 60th birthday, just weeks before his unexpected death. There was Gene, surrounded by many friends, in the company of his family, each of us trying to say, in our inarticulate way, that we loved him.

His was a life cut short, but one very well lived.

The Mathematical Work of Gene Fabes

Gene Fabes' mathematical career started in the mid 1960s, in the graduate program at University of Chicago. There Gene embraced the Calderón–Zygmund philosophy of using tools of real and harmonic analysis to obtain sharp results in partial differential equations. This was to become Gene's lifelong scientific direction, and he was one of the leading researchers in this area.

In the mid 1960s and 1970s Gene was one of the earliest pioneers in the area of parabolic singular integrals and their applications to parabolic initial boundary value problems (IVBP). Much of this work was done in collaboration with Nestor Rivière. Other collaborators were Gene's first student, M. Jodeit Jr., B.F. Jones and his student J. Lewis. These works are now classics of the field. Gene's thesis also included the beginnings of the theory of singular integrals along curves, which has become an active and important area of harmonic analysis. In this period Gene also initiated a program of finding minimal smoothness conditions, expressed in terms of Dini type conditions on the coefficients of a parabolic operator, which guarantee the solvability of the L^p initial value problem.

This was visionary, and pioneered much future work. The crowning results from this period, in this direction, were obtained in a 1979 joint paper by Gene, his student R. Sroka, and K. Widman. The technique they introduced, using Littlewood–Paley estimates, proved to be prescient and enduring.

The Fabes–Sroka–Widman paper marked the beginning of a period lasting to the mid 1980s in which Gene devoted his work to finding minimal smoothness conditions on coefficients of elliptic and parabolic operators, or on the boundary of domains, for the solvability of IVBPs, or boundary value problems, with L^p data. In a 1978 paper, following A.P. Calderón's proof of the L^2 boundedness of the Cauchy integral on C^1 curves, Gene, together with Jodeit and Riviére, showed that the classical method of layer potentials can be applied to study the Dirichlet and Neumann problems for Laplace's equation with L^p data on C^1 domains. This paper was instrumental in opening the way for the theory of boundary value problems in Lipschitz domains, still a very active area of research. Gene and his students (in particular G. Verchota, R. Brown, and L. Escauriaza) made many important contributions in this field. In a 1981 paper Gene, in collaboration with L. Caffarelli, S. Mortola, and S. Salsa, initiated the study of the boundary behavior of solutions to second-order divergence form elliptic equations with bounded measurable coefficients, on Lipschitz domains. The basic results on the "elliptic-harmonic measure" were established in this, now classic, work. Also in 1981, Caffarelli, Fabes, and Kenig, and independently Modica and Mortola, constructed examples showing that such measures may be singular to Lebesgue measure on the boundary. The important role of the "square Dini" condition on the modulus of continuity of the coefficients is very clear in the Caffarelli–Fabes–Kenig example. This condition was further highlighted in several works of Gene's with Jerison and Kenig, in the mid-1980s. They showed, among other things, that it suffices to guarantee the absolute continuity of the elliptic-harmonic measure. Another noteworthy consequence of these developments was the resolution by Fabes, Jerison, and Kenig, and also, independently, by Coifman, Deng, and Meyer, of the higher dimensional Kato conjecture for square roots of elliptic operators in the "small perturbation of the Laplacian" case. The full conjecture remains open to this day.

From the mid 1980s, Gene's interest turned to Harnack principles, and uniqueness for the Dirichlet problem for operators with non-smooth coefficients. In 1986, in collaboration with his student N. Garofalo, and with S. Salsa, Gene proved a "backward in time" Harnack inequality, for certain solutions to parabolic operators with time independent coefficients. Such inequalities are the first step in understanding the basic properties of the "parabolic measure" associated with the operator. The question left open, that is, the issue of what happens in the case of time dependent coefficients, was settled in 1996 by M. Safonov. This proof, and important subsequent developments, have appeared in joint papers of Fabes, Safonov, and Safonov's student, Y. Yuan.

Also in the 1980s Gene, in collaboration with D. Stroock, revisited Nash's ideas on parabolic divergence form equations, to give a direct proof of Gaussian bounds for the fundamental solution. This approach extended to geometric and degenerate settings. Also in collaboration with D. Stroock, in the mid 1980s, Gene gave an improvement of the classical Alexandrov–Bakelman–Pucci maximum principle, and used this to give a new proof of the Krylov–Safonov–Harnack inequality for non-divergence form elliptic equations. In fact, since the early 1980s, Gene had been interested in studying non-divergence form elliptic equations with bounded measurable coefficients. The boundary behavior, in the case of continuous coefficients, had been studied in the 1982 dissertation of Gene's student P. Bauman. In the case of merely bounded measurable coefficients, one of the hard problems is how to define uniquely a solution of the Dirichlet problem with smooth data. In collaboration with his students Cerutti and Escauriaza, Gene defined a "good solution" by regularizing the coefficients and passing to the limit on a subsequence, using the Krylov–Safonov estimates. In this work they proved that the solution thus obtained is unique (independent of the regularization), if the set of discontinuities of the coefficients is very small. Such results are very delicate in view of Nadirashvili's recent example, which shows that in the case of general discontinuities, uniqueness of the "good solution" may fail.

Gene Fabes made pioneering contributions to the fields of study which he helped create and define. His generous collaborative approach to mathematics research ensures the continuation of his program through the work of his students and friends.

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