ACTA MECHANICA SINICA (English Series), Vol.13, No.2, May 1997 ISSN 0567-7718
The Chinese Society of Theoretical and Applied Mechanics
Chinese Journal of Mechanics Press, Beijing, China
Allerton Press, INC., New York, U.S.A.

THE MOTION OF A SMALL BUBBLE IN WAVES*

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ABSTRACT: The motion of a single spherical small bubble due to buoyancy in the ideal fluid with waves is investigated theoretically and experimentally in this article. Assuming that the bubble has no effect on the wave field, equations of a bubble motion are obtained and solved. It is found that the nonlinear effect increases with the increase of the bubble radius and the rising time. The rising time and the motion orbit are given by calculations and experiments. When the radius of a bubble is smaller than 0.5mm and the distance from the free surface is greater than the wave height, the results of the present theory are in close agreement with measurements.

KEY WORDS: bubble, bubble motion, air-sea exchange

1 INTRODUCTION

The background of our researches of bubbles is air-sea interaction. During the rise of a bubble, the exchange of mass, momentum and heat between the bubble and the water around will take place. Investigating the law of the bubble motion is important to the study of air-sea exchange.

For a long time , most researches about bubbles focused on the eigenfrequency of a bubble and the nonlinear effect on it. Some studies considered the transfer and the diffusion of the mass. After 1970's, there were some researches on the dynamics of a nonspherical bubble. They concerned about the nonspherical deformation and the interaction between a bubble and the wall. These early investigations were reviewed by Plesset and Prosperetti(1977)[1]. There are less work about the translation of a bubble. Moore(1965)[2] studied the rising of a bubble with small deformation by asymptotical expansion; Ryskin and Leal(1984)[3] studied the rising of a bubble with large deformation by numerical simulation. During the rising , the bubble radius will vary due to the change of the pressure outside. All the research above didn't consider the effect of the change of the bubble radius on the rising. B.B. Chakraborty and G.S. Tuteba(1993)[4] studied the motion coupled the rising to the change of the radius. They established the equations of the motion, and obtained the effect of the change of the radius on the rising by numerical calculation. T. Watanabe and Y. Kukita (1993)[5] studied the effect of acoustic waves on the motion of a bubble, but didn't consider the rising of a bubble due to gravity.

All the research above didn't consider the effect of waves on the bubble motion. In ocean environment waves is inevitable, so the motion of a bubble in waves with the gravity

Received 8 October 1996, revised 20 March 1997

* The project supported by the National Natural Science Foundation of China

should be understood. In this article, we studied the motion of a bubble with three freedoms of rising, horizontal motion and the change of the radius. Under some assumption, the simplified equations can be derived and solved. Then, we do some experiments, and compare the experimental results with the theory. The nonlinear effect of the bubble motion, the mean rise velocity, the orbit and the rising time of a bubble, and the change of the bubble radius in waves are discussed.

2 EQUATION AND NUMERICAL CALCULATION

Since bubbles in the ocean are very small, and their radius are rarely more than the order of a millimeter. When the wind speed is 11m/s, the radii of most bubbles are 60 μm~ 200 μm. We assume that the nonspherical deformation of a bubble can be ignored, and the bubble can be considered as a sphere. We also assume that the fluid around the bubble is incompressible and the movement is potential. There is no effect of a bubble on waves because the wave length in the ocean is much larger than the diameter of a bubble. And the bubble can be looked as a particle in the wave field. The total velocity potential ϕ can be divided into two parts, one is the velocity potential of waves ϕ_w, the other is the velocity potential ϕ_b produced by the motion of a bubble , then

$$\phi = \phi_w + \phi_b \tag{1}$$

ϕ_b satisfies the equations

$$\left.\begin{aligned} &\nabla^2\phi_b = 0 \\ &\nabla\phi_b = 0 && \text{at the infinity} \\ &\frac{\partial\phi_b}{\partial n} = \boldsymbol{U}_b\cdot\boldsymbol{n} + \frac{\mathrm{d}R}{\mathrm{d}t} && \text{at the bubble boundary} \end{aligned}\right\} \tag{2}$$

where $\boldsymbol{U}_b = \boldsymbol{U} - \nabla\phi_w$, $\boldsymbol{U}$ is the total velocity of the bubble. Subscript b, w means the bubble and the wave respectively. If the bubble is not very near the free surface and the fluid is incompressible, the propagation velocity of a perturbation is infinity,the velocity potential ϕ_b can be written approximately as

$$\phi_b = -\frac{U_b R^3\cos\theta}{2r^2} - \frac{R^2\dot{R}}{r} \tag{3}$$

in the coordinate with its origin located at the center of the spherical bubble. Since the coordinate is moving, we use Cauchy-Lagranian integration in the moving coordinate[7] to get the pressure outside the bubble. Considering the drag, the equations of bubble motion can be obtained as

$$\left.\begin{aligned} &R\frac{\mathrm{d}^2R}{\mathrm{d}t^2} + \frac{3}{2}\left(\frac{\mathrm{d}R}{\mathrm{d}t}\right)^2 - \frac{U_b{}^2}{4} = \frac{1}{\rho_l}\left(P_g - P_w - \frac{2\sigma}{R}\right) \\ &\frac{\mathrm{d}}{\mathrm{d}t}\left(R^3\boldsymbol{U}_b\right) = -2R^3\nabla P_w - \frac{3}{4}|U_b|\boldsymbol{U}_b R^2 C_d \end{aligned}\right\} \tag{4}$$

where, $P_w = P_a - \rho_l\left[\frac{\partial\phi_w}{\partial t} + \frac{1}{2}(\nabla\phi_w)^2 + gz\right]_b$, P_a is the value of barometric pressure on the free surface, ρ_l is the density of the fluid. Let P_g express the pressure inside the bubble.

Under the assumption of the adiabatic process , we have $P_g = P_0\left(\frac{R_0}{R}\right)^{3\gamma}$. where, P_0 is the value of pressure when $R = R_0$, γ is the adiabatic ratio. The drag acting on the bubble is expressed in the form of drag coefficient. Here, we use the experimental result by Crum[8]

$$C_d = 27.0Re^{-0.78}$$

where

$$Re = \frac{2U_bR}{\nu}$$

The Crum's result is applicable for $Re = 1 \sim 200$. This is also suitable for our research.

We take the initial bubble radius $R(0)$, the steady rising velocity of a bubble in still water U_0, the density of water ρ_l as the characteristic length, speed, density, respectively. The equations can be changed into the first-order equations, and written in dimensionless form as

$$\left.\begin{aligned}
\frac{\mathrm{d}R}{\mathrm{d}t} &= Q \\
\frac{\mathrm{d}Q}{\mathrm{d}t} &= \frac{1}{R}\left(P_g - P_w - \frac{2}{RW} - \frac{3}{2}Q^2 + \frac{U_b^2}{4}\right) \\
\frac{\mathrm{d}U_{bx}}{\mathrm{d}t} &= -2\frac{\partial P_w}{\partial x} - \frac{3}{4}\sqrt{U_{bx}{}^2 + U_{bz}{}^2}U_{bx}C_d/R - 3QU_{bx}/R \\
\frac{\mathrm{d}U_{bz}}{\mathrm{d}t} &= -2\frac{\partial P_w}{\partial z} - \frac{3}{4}\sqrt{U_{bx}{}^2 + U_{bz}{}^2}U_{bz}C_d/R - 3QU_{bz}/R \\
\frac{\mathrm{d}X}{\mathrm{d}t} &= U_x = U_{bx} + \frac{\partial \phi_w}{\partial X} \\
\frac{\mathrm{d}Z}{\mathrm{d}t} &= U_z = U_{bz} + \frac{\partial \phi_w}{\partial Z}
\end{aligned}\right\} \qquad (5)$$

where, $W = \rho_l U_0{}^2 R(0)/\sigma$ is Weber number, Eqs. (5) are solved by the fourth-order Runge-Kutta method. The initial values are

$$R(0) = 1 \quad Q(0) = Q_0 \quad U_{bx}(0) = U_{bz}(0) = 0 \quad X(0) = 0 \quad Z(0) = Z_0$$

For simplicity, the velocity potential of deep water is used

$$\phi_w = \phi_0 e^{kz}\cos(kx - \omega t + \theta_0)$$

3 EXPERIMENTAL APPARATUS AND METHODS

The experiments were carried out in wind-wave water tunnel. The side walls of the tunnel are made of glass so that we can observe the flow. On the bottom of the tunnel, there is a glass window. The light can penetrate this window. At one end of the tunnel the wave generator is located. At the other end there is a dissipation system which can minimize the wave reflection. The bubble generator is at the middle of the tunnel. Below the window on the bottom , there is a lamp for the camera and video-camera to record the radius of the bubble and the orbit of the bubble respectively. Figure 1 is the diagram of the experimental apparatus.

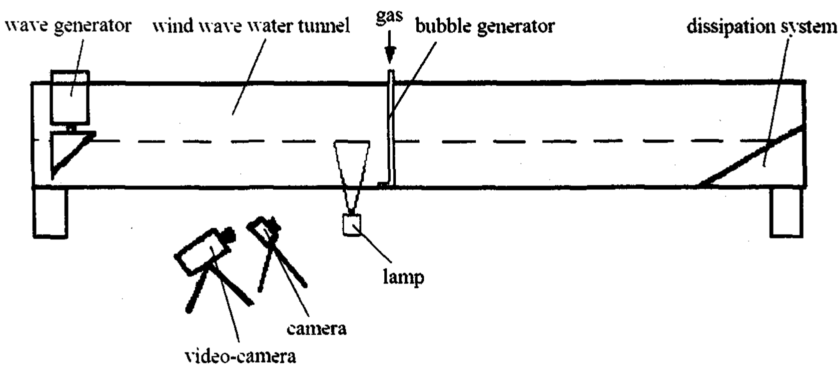

Fig.1 The diagram of experimental apparatus

Bubble Generation Bubble can be generated by bubble generator. Bubble generator is made up of a metal tube plunged into the water and a rubber ball. Injecting gas into the tube by pressing the rubber ball, bubbles can be formed at the other tip of the tube in water. By controlling the volume of the gas injected , bubbles of different radius can be produced and analyzed. The bubble generator, camera and video-camera operated synchronistically to ensure that the same bubble was recorded by them.

Wave Characteristics Measurements Wave characteristics can be measured by wave gauge. The error on the wave amplitude is ± 0.1 mm. Wave signal was changed into electrical signal by wave gauge. The noise of background was eliminated by wave filter. The signal was amplified and changed into digital signal by A/D transformation, then collected by personal computer. The wave characteristics was given by the software which dealt with the wave process.

Bubble Diameter Measurements The diameter of the bubble can be obtained according to the bubble image recorded by the camera in experiments. The bubble image on the film was enlarged by slide projector. The diameter of the enlarged bubble image can be measured directly. The diameter of the real bubble was given by the scale of the enlarged image to the real field.

Orbit of the Bubble and Rising time Orbits of bubbles were recorded on the magnetic tape by video-camera. The scale of the image to the real field was determined before the record. The picture from a videocorder was changed into digital signal and handled. The videocorder was shown frame by frame with the interval 0.02s. Bubble orbits recorded on the tap was input into a personal computer frame by frame. According to the scale of the image to the real field, the position of a bubble in the real field can be obtained from the position of the bubble in every picture which can be given by the image process software. According to the number of the frames during the rising, we got the rising time.

4 RESULTS AND DISCUSSION

4.1 The Nonlinear Effect on the Motion of Bubble

When a bubble is very small, the rising velocity is small too , so we can take the motion of a bubble in the wave field as the superimposition of the motion induced by waves and the steady rise of a bubble. When the bubble is larger, the rising velocity become large, and the nonlinear effect will appear. We introduce $\epsilon = \dfrac{|U_b - U_0|}{|U_0|}$ where U_0 is the steady rising velocity of a bubble, it can be determined by the balance of drag and buoyance $\frac{4}{3}(\rho_l - \rho_g)\pi R^3 g = \frac{1}{2}AU_0{}^2C_d$. The variation of ϵ with time t is given in Fig.2. A periodicity in the variation of ϵ can be seen. We can also see that ϵ decreases from 1 very fast at the beginning and soon increases slowly with the time. When the time is not too long, ϵ is small, and can be ignored. But when time becomes long, ϵ becomes larger and larger, the nonlinear coupled effect should be considered for large bubbles. From Fig.2, we can get the following points. (1) The initial acceleration time of a bubble is much shorter than the rising time; (2) The mean rising velocity of a bubble in waves is approximately equal to the steady rising velocity of a bubble in still water; (3) The nonlinear effect increases with the time and the bubble radius (at long time). The relationship between the rising velocity and the diameter of bubbles is given in Fig.3. In the range of our research, when the bubble diameter is smaller than 1 mm, both experiments and theory showed that the mean rising velocity of the bubble in waves is near the steady rising velocity of the bubble in still water. When the diameter of the bubble is larger than 1 mm, the theory is not suitable. From experiments, we can see that the mean rising velocity of the bubble in waves doesn't increase with the increase of the bubble diameter, it will come to a constant.

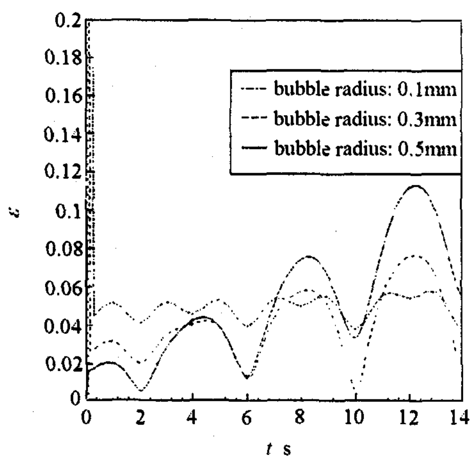

Fig.2 The nonlinear effect on the bubble motion
wave length: 25 m,
wave height: 0.5 m

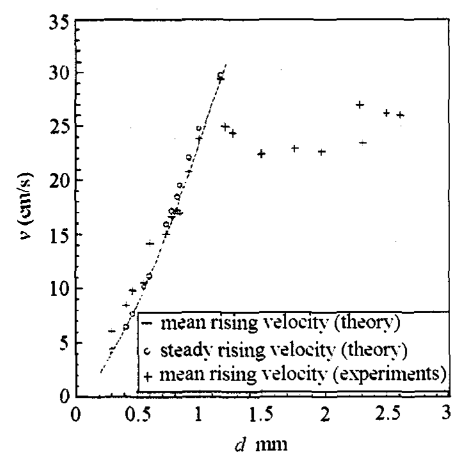

Fig.3 The relationship of rising velocity and bubble radius
wave height: 4.5 cm,
wave length: 1.59 m,
wave frequency: 0.95 Hz,
initial depth: 40 cm

4.2 Variation of Bubble Radius

During the rising of a bubble in waves, the pressure outside decreases, the bubble radius will increase. In Fig.4 we can see that the variation of the dimensionless bubble radius is lower than 0.1 in our calculation. Because we assume the bubble is at rest initially, and the frequency of waves($\sim$ 1 Hz) is much lower than the eigenfrequency(10^3 Hz $\sim$ 10^4 Hz) of the bubble, there is no energy exchange between them. Their motion will not be coupled, so the oscillation of the bubble can not appear, the bubble radius increases smoothly. When the bubble is not at rest initially, the bubble will oscillate at first, but the oscillation decays very fast, later the bubble radius increases monotonically. In this case, in initial stage Δt used in the calculation is very small because of the oscillation, and it costs much time in calculation. For time saving, in the calculation we used larger Δt when the bubble is at rest initially instead of small Δt. When the bubble oscillate initially, the results for large Δt is the mean results for small Δt. The results for larger Δt are the same whether the bubble oscillated initially or not. So the high frequency oscillation of the bubble radius can be ignored when we study the motion of the bubble. This view can be explained by the equations of the motion. When the bubble oscillates with high frequency, we take average over a period for Eqs.(5). If the oscillation is small , the second-order quantities can be ignored, and the average quantities satisfy the same equations as Eqs.(5).

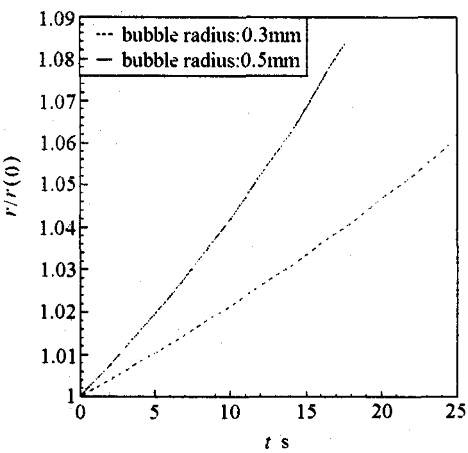

Fig.4 The variation of bubble radius
wave length: 25 m,
wave height: 0.5 m

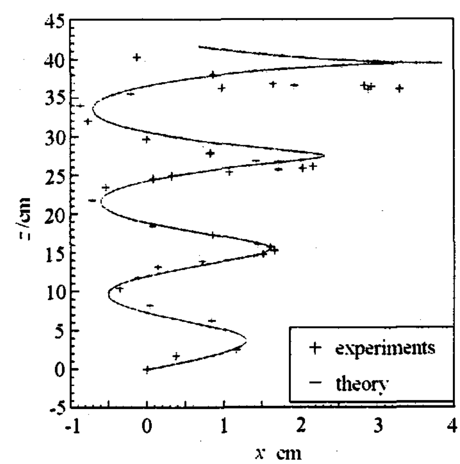

Fig.5 The orbit of the bubble motion
wave height: 4.5 cm,
wave frequency: 0.95 Hz,
wave length: 1.59 m,
bubble radius: 0.3 mm,
initial depth: 40 cm

4.3 The Orbit of the Bubble

The theoretical orbits of the bubble motion were compared with experiments (Fig.5). We can see that when the distance between the bubble and the free surface is larger than the wave height, the results of the present theory are in close agreement with the measurements. When a bubble is near the free surface, there are some differences between experiments and theory. The reason is that we didn't consider the effect of the free surface in our theory.

Acknowledgments We would like to thank Professor Huhe Aude for the helpful comments and advices. The author appreciate sincerely Huang Zhenhua and Tang Chi for performing the experiments.

REFERENCES

1 Plesset MS, Prosperetti A. Bubble dynamics and cavitation. *Ann Rev Fluid Mech*, 1977, 9: 145

2 Moore DW. The velocity of rise of distorted gas bubbles in a liquid of small viscosity. *J Fluid Mech*, 1965, 23: 749

3 Ryskin G, Leal LG. Bubble deformation in an axisymmetric straining flow. *J Fluid Mech*, 1984, 148: 37

4 Chakraborty BB, Tuteba GS. Motion of an expanding, spherical gas bubble in a viscous liquid under gravity. *Phys Fluids(A)*, 1993, 5(8): 1879

5 Watanabe T, Kukita Y. Translational and radial motions of a bubble in an acoustic standing wave field. *Phys. Fluids(A)*, 1993, 5(11): 2 682

6 Vagle S, Farmer DM. The measurement of bubble-size distribution by acoustical backscatter. *J of Atmospheric and Oceanic Technology*, 1991, 9: 630

7 Dept of Engineering Mech, Univ of Tsinghua. Foundation of Fluid Mechanics. Beijing: Mechanical Industry Press,1980. 169 (in Chinese)

8 Crum LA. Bjerknes forces on bubbles in stationary sound field. *J Acoust Soc Am* 1975, 57: 1363