Degree of hydration-based description of mechanical properties of early age concrete

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$A B S T R A C T$ **R** ESUME

For the evaluation of the risk of thermal cracking in hardening massive concrete elements, knowledge of the development of strength and deformability of early-age concrete is extremely important. Based on an extensive experimental research program on hardening concrete elements, a degree of hydration-based description for the compressive strength, Young's modulus, the uniaxial **tensile strength, the** splitting tensile strength, the flexural **tensile** strength, Poisson's ratio and the peak strain are all worked out. An extension of the formulation of Sargin for the stress-strain relation for short-term compressive loading leads to a degree of hydration-based stress-strain relation for hardening concrete. Good agreement with experimental results is reported.

1. INTRODUCTION

At the University of Ghent, several research programs have been conducted concerning massive concrete armour units used for breakwaters. More particularly, cracking due to thermal stresses caused by the heat of hydration and impact resistance were studied [1,2]. Due to these thermal stresses, external as well as internal crack formation can occur during hardening. Wave loads can stimulate crack growth. Thus, the service life can be severely reduced by thermal cracks. A thorough knowledge of the thermal stresses and crack formation is necessary to evaluate the influence on the durability. At the University of Ghent, a finite element program for the prediction of thermal stresses due to the heat of hydration in massive concrete elements is being developed. Based on isothermal and adiabatic hydration tests, a new general hydration model was developed, valid for both portland cement and blast furnace slag

Pour bien &aluer le risque de fissuration thermique dans un béton jeune, il est très important de pouvoir en déterminer d'une manière fiable les caractéristiques mécaniques. Sur la base de plusieurs essais mécaniques effectués sur le béton pen*dant sa phase d'hydratation, on a pu établir une expression qui relie au degré d'hydratation la résistance à la compression, le module d'élasticité, la résistance à la traction, la résistance à la flexion, la résistance à la traction par fendage, le coefficient de Poisson et la déformation à la contrainte maximale. Une extension de la relation « contrainte-déformation » établie par Sargin pour un béton durci, mène à une expression basée elle* aussi sur le degré d'hydratation. Pour l'ensemble des caracté*ristiques dtudides, on peut constater une bonne concordance entre les re'sultats et les relations proposdes.*

cement [3]. This hydration model enables the calculation of the heat production rate as a function of the actual temperature and the degree of hydration. By means of improved test methods, a degree of hydrationbased description of the thermal characteristics of the hardening concrete was also elaborated [4]. This contribution focuses on the development of strength and deformability of early-age concrete. As the degree of hydration is considered to be a very fundamental parameter, the evolution of the early age mechanical properties is formulated as a function of the degree of hydration.

2. EXPERIMENTS

The uniaxial tensile strength f_{ct} , the splitting tensile strength $f_{ct sp\ell}$ and the flexural tensile strength $f_{ct ff}$ were determined by means of traditional test set-ups. The

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uniaxial tensile test was performed on cylindrical specimens with diameter 120 mm and height 300 mm. The Brazilian splitting test was performed on cubes with 158-mm sides. For the three-point bending test, specimens 150 mm x 150 mmx 600 mm were used. A span of 500 mm was applied.

The compressive strength f_c , the Young's modulus E_{co} , Poisson's ratio v and the peak strain ε_{c1} were determined simultaneously on a single hardening concrete specimen. At this point, a new test set-up was developed, as shown in Fig. 1. A hardening concrete specimen 150 mm x 150 mm x 400 mm was placed in a testing machine. The longitudinal deformation of the concrete specimen was measured by means of two strain gauges, placed on two opposite faces. By means of an XY-plotter, the stress-strain diagram was recorded continuously, yielding E_{co} , f_c and ε_{c1} quite easily.

The transversal deformation is determined by means of a rigid frame placed around the concrete specimen. This frame is fixed to the concrete specimen by means of two rigid contact points at the rear side and two springy contact points at the front side. When the concrete expands transversally, only a relative deplacement v of the front side of the concrete specimen related to the rigid frame is recorded by means of a very sensitive LVDT (Fig. 2). (The side faces also undergo a relative displacement related to the frame, but this is not relevant.) The measurement of the transversal displacement enables the determination of Poisson's ratio. The rigid frame is supported by means of steel balls, enabling the frame to foilow very small horizontal displacements of the entire concrete section which might occur during testing. Furthermore, the balls are supported elastically in order to enable the frame to follow the small vertical displacements of the concrete section caused by the vertical compaction of the specimen.

The experiments were carried out on concrete with the following composition per m^3 : 300 kg cement, 150 kg water, 670 kg sand 0/2, 1,280 kg gravel 4/14. Three different kinds of cement were used: Portland cement CEM I 52.5 and blast furnace slag cements CEM III/B 32.5 and CEM/III C 32.5. The chemical composition and the fineness of the cement are given in Table 1. Some characteristics of the fresh and hardened concrete are mentioned in Table 2. The concrete specimens were stored at 20°C and 95% relative humidity.

3. COMPRESSIVE STRENGTH fc

Test results are obtained for ages ranging from 12 hours to 28 days. The corresponding degree of hydration r can be calculated as the fraction of heat released, using a newly-developed hydration model (see Appendix). The experimentally-obtained relative strength development $f_c(r)/f_c(r = 1)$ is shown in Figs 3 through 5 for the different cements and for the different test series as a function of the degree of hydration. From these figures, it appears that an almost linear relation exists between compressive strength f_c and degree of hydration r. A

Fig. 1 - Test set-up.

Fig. 2 - Transversal deformation.

linear relationship can be found in the literature [5-7], although several authors also apply parabolic curves [8,9]. A general formulation can be given by:

Fig. 3 - Compressive strength CEM 1 52.5.

Fig. 4 - Compressive strength CEM III/B 32.5.

Fig. 5 - Compressive strength CEM III/C 32.5.

with $f_c(r)$ = compressive strength at degree of hydration r $f_c(r = 1)$ = compressive strength at

> degree of hydration $r = 1$ r_0 , a = parameters

In this formulation, it has been assumed that below a certain value r_0 for the degree of hydration, no strength development occurs. This assumption is supported by the strength results obtained, and by theoretical considerations based on percolation theory [5]. For the parameter r_0 , different values *can* be found in the literature, as indicated in Table 3. The difference in the listed values is mainly due to different concrete compositions. The water/cement ratio, for instance, has a major influence on r_0 . In any event, the existence of a value r_0 , below which no strength development occurs, seems to be widely accepted in the literature.

Regression based on the obtained experimental results yields the parameter values given in Table 4. The strength values $f_c(1)$ are lower than the 28-day cube strength listed in Table 2 (obtained on cubes with sides of 200 mm) because of the size effect. The resulting model (1) is also shown in Figs 3 through 5. It can be shown statistically for instance that for CEM/III B 32.5 the parameter a differs significantly from one.

4. YOUNG'S **MODULUS Eco**

The test results obtained for the different cements and for the different test series are plotted in Figs 6 through 8 as a function of the degree of hydration. A comparison of the stiffness development with the strength development shows that the former evolves faster than the latter. This was already mentioned by Weigler and Karl in 1974 [12], and has now become a well-known phenomenon.

In the literature, the stiffness development is often related to the strength development. Breitenbiicher [13], for instance, uses the following relation :

$$
\frac{E_{\text{co}}(t)}{E_{\text{co}}(28d)} = \sqrt[3]{\frac{f_{\text{c}}(t)}{f_{\text{c}}(28d)}}
$$
(2)

It can be shown mathematically that, given equation (1), the relation (2) can be transformed into:

$$
\frac{E_{\text{co}}(r)}{E_{\text{co}}(r=1)} = \left(\frac{r-r_0}{1-r_0}\right)^{\frac{a}{3}}
$$
(3)

with $E_{co}(r = 1) =$ Young's modulus at degree of hydration $r = 1$.

Equation (3) describes the stiffness development as a function of the degree of hydration. Other stiffnessstrength relations given in the literature can also be transformed into degree of hydration-based descriptions. Very often, the following type of stiffness-strength relation is accepted :

$$
E_{co} = k \cdot (f_c)^n \tag{4}
$$

with **k** and **n** being parameters.

Some given n-values are summarized in Table 5. Given Equation (1), the relation (4) can be transformed into a general degree of hydration-based formulation (with $b = a.n = parameter$).

$$
\frac{E_{\text{co}}(r)}{E_{\text{co}}(r=1)} = \left(\frac{r-r_0}{1-r_0}\right)^b
$$
\n(5)

This formulation was already given by Rostásy et al. [8,9], with $b = 2/3$. From our experimental results, the parametric values can be deduced as given in Table 4. Relation (5) is also plotted in Figs 6 through 8. When the b-values given in Table 4 are transformed into nvalues according to $b = a.n$, a value $n = 0.44$ is found for the blast furnace slag cement. This is in full agreement with the value that can be deduced from the material laws given by Rostásy *et al.* [8,9]. For the Portland cement, a value $n = 0.31$ is found, which is in good agreement with the value $n = 1/3$ given by Breitenbücher, in equation (3) . It can be concluded that the exponent n in equation (4) depends on the cement type used. This might explain the difference in n-values listed in Table 5 according to the different researchers.

5. UNIAXIAL TENSILE STRENGTH f_{ct} , SPLITTING TENSILE STRENGTH $f_{ct,sp,\ell}$, **FLEXURAL TENSILE STRENGTH fct fe**

It has been found experimentally that f_{ct} , f_{ct} sp ℓ and f_{ct} f_{ℓ} all have the same relative development during hardening, as can be seen from Figs 9 through 11. From the absolute values, it was concluded that the well-known conversion factors [18] for the calculation of $f_{\rm ct}$ from $f_{ct\,\,f\ell}$ or $f_{ct\,\,sp\ell}$ also remain valid for early-age concrete. This is why subsequent to this paragraph only f_{ct} is mentioned in the formulas. Similar formulas can be deduced for $f_{ct\ f\ell}$ and $f_{ct\ sp\ell}$. In literature the tensile strength f_{ct} is

Fig. 6 - Young's modulus CEM 1 52.5.

Fig. 7 - Young's modulus CEM III/B 32.5.

Fig. 8 - Young's modulus CEM III/C 32.5.

often related to the compressive strength f_c :

$$
f_{ct} = \ell \cdot (f_c)^m \tag{6}
$$

with ℓ and m being parameters. Some m-values are summarized in Table 6. Given equation (1), the relation (6) can be transformed into a general degree of hydration based-formulation (with $c = a.m = parameter$).

$$
\frac{f_{ct}(r)}{f_{ct}(r=1)} = \left(\frac{r-r_0}{1-r_0}\right)^c \tag{7}
$$

This formulation was already given by Rostásy *et al.* [8,9], with $c = 1$. Regression applied to the obtained experimental results yields the values given in Table 4. Relation (7) is also plotted in Figs 9 through 11. When the c-values given in Table 4 are transformed into m-values according to $c =$ a.m, a value $m = 0.55$ is found for cement CEM I 52.5, while $m = 0.63$ for CEM III/B 32.5 and $m = 0.80$ for CEM III/C 32.5. From the a-, b- and c-values given in Table 4, it is concluded that the tensile strength is developed in a faster way than the compressive strength, though not as fast as the Young's modulus.

6. POISSON'S RATIO v

The experimental results (Fig. 12) clearly show that the Poisson's ratio v is not constant during hardening. In the literature, the evolution of the Poisson coefficient is not always taken into account. Truman *et al.* [22] assume that v remains constant during hydration. Oluokun *et al.* [15] conclude, by means of experiments, that no evolution exists, although they mention some lower values at very early age. Giinzler [23] also found no clear evolution of the Poisson's ratio during hardening. For very early age, Planck [23] found a decrease from $v =$ 0.4 to 0.2. Byfors [6,23] mentions a decrease from $v = 0.48$ to $v = 0.13$ for $f_c \le 1$ to 2 N/mm², and an increase from $v = 0.13$ to $v = 0.28$ for 1 to 2 $N/mm^2 < f_c < 50 N/mm^2$. For fresh concrete, it is accepted that $v = 0.5$, based on the fact that in this case deformation occurs without any volume changes. Based on the experimental results and on the findings

Fig. 13 - Peak strain.

Fig. 14 - Stress-strain relation

mentioned in the literature, a new degree of hydrationbased model for the Poisson's ratio can be deduced:

$$
v(r) = 0.18 \sin \pi r/2 + 0.5.e^{-10r}
$$
 (8)

This model is shown graphically in Fig. 12. Bournazel [24] also gives a maturity-based formulation for v, but the monotonous decrease during hardening is not in agreement with the experimentally-proven existence of a minimum value at very early age.

7. PEAK STRAIN ε_{c1}

In Fig. 13 the experimentally-obtained values for the peak strain ε_{c1} are compared with some results from the literature(Hansen 1956 [23], Kasai 1961 [23], Byfors

[6,23], Wierig 1982 [6], Brameshuber *et al.* [25]). The peak strain ε_{c1} seems to be minimal for compressive strengths of about 10 N/mm^2 . The experimental results can be described mathematically by the following relation :

$$
\varepsilon_{\rm cl} \left(\mathbf{f}_{\rm c} \right) = 0.44 \sqrt[3]{\mathbf{f}_{\rm c}} + \frac{2.1}{\sqrt{\mathbf{f}_{\rm c}}} \tag{9}
$$

with ε_{c1} = peak strain in ‰

 f_c^{th} = compressive strength in N/mm².

Equation (9), resulting from experimental results in the range of 1 N/mm² \leq f_c \leq 55 N/mm², is plotted in Fig. 13. There seems to be strong agreement with results from the literature, especially with Wierig's. It can be shown statistically that for $f_c > 10 \text{ N/mm}^2$, the peak strain ε_{c1} increases significantly.

8. STRESS-STRAIN RELATION FOR SHORT-TERM COMPRESSIVE LOADING

The stress-strain relation for short-term compressive loading changes continuously during hardening, as indicated in Fig. 14 for test series B of blast furnace slag cement CEM III/B 32.5. The evolution of the stressstrain relation during hardening can be summarized by means of the fullness factor v defined as :

$$
\mathbf{v} = \int_0^1 \frac{\sigma_c}{f_c}(\eta) d\eta
$$
 (10)

with

$$
\eta = \varepsilon_c / \varepsilon_{c1} \tag{11}
$$

A value $v = 1$ corresponds with a rectangular stressstrain relation, while $v = 0.5$ corresponds with a linear elastic diagram. For hardening concrete, the fullness factor v takes values between 0.5 and 1.0. This is shown in Fig. 15, where the fullness factor v is plotted as a function of the compressive strength f_c , for the different test series using the different kinds of cement.

For hardened concrete, a widely-accepted formulation for the stress-strain relation for short-term compressive loading has been given by Sargin, and is also given in the Model Code 1990 [18]. An extension of this formulation towards hardening concrete leads to a degree of hydration-based stress-strain relation, valid for $r > r_0$:

$$
\frac{\sigma_c}{f_c} = \frac{A(r)\eta - \eta^2}{1 + [A(r) - 2]\eta}
$$
\n(12)

with

$$
A(r) = \frac{E_{\rm co}(r) \cdot \varepsilon_{\rm cl} (f_{\rm c}(r))}{f_{\rm c}(r)}
$$
(13)

The normalized strain η is given by equation (11), while $f_c(r)$, $E_{co}(r)$ and $\varepsilon_{c1}(f_c(r))$ can be calculated by means of equations (1), (5) and (9). It can be verified that equation (12) provides a good formulation for the stressstrain relation for the short-term compressive loading of hardening concrete. This is illustrated in Fig. 16 for some experimental results obtained with CEM III/C 32.5. The post-peak behaviour $(\eta > 1)$ has not been considered in this study. A verification of equation (12) for this domain has yet to be performed.

9. CONCLUSIONS

- Based on an extensive experimental research program on hardening concrete elements, a degree of hydration-based description for the compressive strength f_c , Young's modulus E_{co} , the uniaxial tensile strength f_{ct} , the splitting tensile strength $f_{ct sp\ell}$, the flexural tensile strength f_{ct f} ℓ , Poisson's ratio v and the peak strain ε_{c1} has been elaborated.

- An extension of the formulation of Sargin for the stress-strain relation for short-term compressive loading leads to a degree of hydration-based stress-strain relation for hardening concrete. Good agreement with experimental results has been reported.

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APPENDIX: CALCULATION OF THE DEGREE OF HYDRATION

The degree of hydration r(t) of a hardening cementitious material is defined as the fraction of cement that has already hydrated:

$$
r(t) = \frac{\text{amount of cement that has reacted at time t}}{\text{total amount of cement at time } t = 0}
$$
 (A.1)

A practical method to estimate the degree of hydration is based on the exothermic character of the hydration process :

$$
r(t) = Q(t)/Q_{\text{max}} \tag{A.2}
$$

where :

 $Q(t) =$ total heat developed at time t, in J/g Q_{max} = total heat development corresponding to complete hydration, inJ/g

The total heat $Q(t)$ can be calculated as:

$$
Q(t) = \int_0^t q(t) dt
$$
 (A.3)

where q(t) represents the heat production, expressed in J/gh.

In [3], a new hydration model is developed, yielding a heat production q depending on the actual degree of hydration $r(t)$ and the actual temperature $\theta(t)$.

$$
q(t) = q[r(t), \theta(t)] \tag{A.4}
$$

From $(A.3)$ and $(A.4)$, it follows that at time t:

$$
dQ(t)/dt = q(t) = q[r(t), \theta(t)]
$$
 (A.5)

Differentiation of (A.2) and introduction of (A.5) yields :

$$
dr(t) = \frac{d Q(t)}{Q_{max}} = \frac{q \left[r(t), \theta(t) \right]}{Q_{max}} dt
$$
 (A.6)

Hence, the value of r can be updated by substituting the previous expression in :

$$
r(t + dt) = r(t) + dr(t)
$$
 (A.7)

In this way, the age of a specimen continuously stored at 20°C can be transformed into the corresponding degree of hydration. For more details concerning the hydration model q[r(t), θ (t)], which is to be determined experimentally, reference is made to [3].

12. REFERENCES

- [1] Van Damme, L., Taerwe, L., Dedeyne, R. and De Rouck, J., 'Quality and durability of concrete armour units', Proc. of the 21st Coastal Engineering Conference, Vol. 3, Chapter 156, ASCE, New York, 1988, pp. 2102-2115.
- [2] Dechaene, R., De Rouck, J., De Schutter, G., Taerwe, L. and Van Der Weeen, F., 'Thermal cracking in hardening concrete armour units', HANG Bulletin, 1994, 82, pp. 61-69.
- [3] De Schutter, G. and Taerwe, L., 'General hydration model for portland cement and blast furnace slag cement', *Cement & Concrete Research,* 1995, 25 (3) 593-604.
- [4] De Schutter, G. and Taerwe, L., 'Specific heat and thermal diffusivity of hardening concrete', *Magazine of Concrete Research,* 1995, 47 (172) 203-208.
- [5] Torrenti, J.M., 'La résistance du béton au très jeune âge', Bull. Liaison Labo P. et Ch., 179, Mai-Juin 1992, pp. 31-41.
- [6] Hamfler, H., 'Berechnung von Temperatur-, Feuchte- und Verschiebungsfeldern in erhärtenden Betonbauteilen nach der Methode der finiten Elemente', Deutscher Ausschus für Stahlbeton, Beuth Verlag GmbH, Berlin, 1988.
- [7] Horden, W.C. and Reinhardt, H.W., 'Temperatuur en spanningen in tetrapods tijdens de verharding', Cement 1986, nr. 12, pp. 49-52.
- [8] Rostasy, F.S. and Lanbe, M., 'Experimental and analytical planning tools to minimize thermal cracking of young concrete', RILEM proceedings 11, Testing during concrete construction, pp. 207-223.
- [9] Rostasy, F.S., Gutsch, A. and Laube, M., 'Creep and relaxation of concrete at early ages - experiments and mathematical modeling', Proceedings of the 5th international RILEM Symposium on Creep and Shrinkage of Concrete, Barcelona, 1993, pp. 453- 458.
- [10] Maatjes, E. and Berlage, A.C.J., 'Beheersing van her verhardingsproces', Cement 1989, hr. 3, pp. 56-62.
- [11] Van Breugel, K., 'Simulation of hydration and formation of structure in hardening cement-based materials', Ph.D. dissertation, Delft, 1991.
- [12] Weigler, H. and Karl, S., 'Junger Beton, Tell 1 : Beanspruchung **-** Festigkeit - Verformung', Betonwerk + Fertigteil-Technik, Heft 6/1974, pp. 392-401.
- [13] Breitenbiicher, R., 'Zwangspannungen und Rissbildung infolge Hydratationswärme', Dissertation, Technischen Universität München, 1989.
- [14] Neville, A.M., 'Properties of Concrete', Pitman Publishing Cie, London, 1963.
- [15] Ohiokun, F.A., Burdette, E.G. and Deatherage, J.H., 'Elastic moduli, Poisson's ratio and compressive strength relationships at early ages', *ACI-Mat.J.,Jan-Feb.* 1991, 3-10.
- [16] Branco, F.A., Mendes, *P.A.* and Mirambell, E., 'Heat of hydration effects in concrete structures', *ACI-Mat.J.,* Mar-Apr. 1992, 139-145.
- [17] Kral, S. and Becker, F., 'Zur Entwicklung mechanischer Betoneigenschaften im Frühstadium der Erhärtung', *Beton 9*/76, 315-320.
- [18] CEB-FIP Model Code 1990, CEB Bulletin d'information n° 213/214, May 1993.
- [19] Derflinger, F., 'Bemessung dicker Fundamentplatten für Eigenund Zwangbeanschpruchung im Hinblick auf die Beschränkung der Stahlspannung und der Rissbreite', Betonwerk+Fertigteil-Technik, Heft 12/1986, pp. 809-814.
- [20] Bastus, E, Carol, I., Mirambell, E. and Vazquez, E., 'Estudio Thermico y tensional de grandes bloques de honnigon fiente a la accion del calor de fraguado', Departamento de Ingenieria de la Construcción de la Universidad Politécnica de Cataluna, Marzo 1990.
- [21] Oluokun, F.A., Burdette, E.G. and Deatherage, H.J.H., 'Splitting tensile strength and compressive strength relationship at early ages', *ACI-Mat.J.,* March-April 1991, 115-121.
- [22] Truman, K.Z., PetruskaA, D.J. and Norman, C.D., 'Creep, shrinkage and thermal effects on mass concrete *structures',Joumal* of Eng. Mech., 117, n° 6, June 1991, 1274-1288.
- [23] RILEM Commission 42-CEA, 'Properties of set concrete at early ages', State-of-the-art-report. Mat. et Constr., 14, nº 84, 399-450.
- [24] Bournazel, J.P., 'Contribution à l'étude du caractère thermomécanique de la maturation du béton', Thèse de Doctorat de l'Université Paris 6, 1992.
- [25] Brameshuber, W. and Hilsdorf, H.K., 'Development of strength and deformability of very young concrete', SEM/KILEM International Conference on Fracture of Concrete and Rock, Houston, Texas, June 1987, pp. 409-421.

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