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Flexural behaviour of concrete corbels containing steel fibers or wrapped with FRP sheets

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ABSTRACT

In the present paper an analytical and experimental investigation referring to the flexural behaviour of reinforced concrete corbels subjected to vertical forces is presented. For fixed shape and dimensions of the corbels the experimental investigation analyses the effects of the following: longitudinal and transverse steel reinforcements; fiber reinforced concrete (FRC) with hooked steel fibers; external wrapping retrofitting technique with a thin layer of carbon fiber sheet (CFRP). The analytical model based on equivalent truss structures, allows one to determine the bearing capacity of corbels, distinguishing the different ultimate states reached. The analytical results are then compared with experimental values, showing good agreement.

RÉSUMÉ

Cet article traite d'une analyse théorique et expérimentale sur le comportement en flexion des poutres en encorbellement de béton armé sous forces verticales. Pour la typologie et les dimensions fixées des poutres en encorbellement, la recherche expérimentale analyse les paramètres suivants : l'armature longitudinale et transversale ; le béton avec fibres d'acier (FRC) ; la consolidation avec des polymères renforcés de fibres de carbone (CFRP). Une analyse théorique a été préconçue pour prévoir la résistance maximale des poutres en encorbellement. Une comparaison des résultats expérimentaux et analytiques indique que les modèles apportent des prévisions satisfaisantes de la résistance.

1. INTRODUCTION

Reinforced concrete corbels are structural members of very common use in reinforced concrete structures and particularly in precast structures. In these members the presence of high values of shear forces reduces their flexural capacity. Moreover, in the presence of cyclic actions or when high strength concrete is utilised, further penalisation occurs in complete flexural capacity and ductility. Several experimental and theoretical studies [1-11] highlight the fact that the principal parameters influencing the structural response of reinforced concrete corbels are: - type (monotonic or cyclic) and direction (vertical or horizontal) of external loads; - shear span to depth ratio; - strength of concrete; - shape and dimensions of corbels; - type, grade and arrangements of longitudinal and transverse steel reinforcements.

Depending on the combination of the above mentioned parameters, the ultimate loads and failure mode can change; in the worst case shear brittle failure (diagonal splitting or constrained shear) occurs, while in the best case ductile flexural failure mode occurs. Recent studies based on extensive experimental investigations [3, 4] highlight the fact that the strength and the ductility of reinforced concrete corbels, both in the presence and in the absence of secondary steel reinforcements, are enhanced by using fiber reinforced concrete instead of ordinary concrete. Moreover, it has been shown that it is possible to completely or partially substitute the secondary shear steel reinforcements (generally constituted by vertical and/or horizontal steel reinforcements) by using fiber reinforced concrete, obtaining analogous performance in terms of both strength and ductility. In this way the difficulty of casting the high percentage of transverse steel required in joint regions is avoided.

In the case of fiber reinforced concrete several studies [1, 3, 5, 8, 14, 15] focus on the choice of the type of fibers and on the optimum volume percentage of fibers to ensure overstrength in shear in reinforced concrete corbels and to avoid brittle failure. The most common fibers utilised are the hooked steel ones and the best percentage is between 1 and 2% by volume of concrete (with the highest values for high strength concrete). Fibers bridge the cracks, increasing the ductility of the member. The bridging action of the fibers is related to the pull-out resistance, resulting in spread cracks in the corbels. Moreover, because of the high tensile strength and high strain

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capacity of the composite, high tensile deformations in longitudinal steel reinforcements are involved and consequently high ductility of the members is achieved.

More recently, in the literature [16-18] the external wrapping of concrete members with fiber reinforced polymer (FRP) is proposed as a very effective flexural and shear reinforcement technique for reinforced concrete beams. Unfortunately, very few data are available for reinforced concrete corbels wrapped with FRP. The choice of the FRP type, the thickness of the reinforcing package, the anchorage length and the tensile strength of reinforced concrete members are some of the most important parameters ensuring the effectiveness of the reinforcing technique. By appropriately choosing the combination of these parameters it is possible to confer high shear and flexural strength on reinforced members with the following advantages: - a very easy casting technique and a very light and removable reinforcing compared to the steel plate-bending technique.

From the theoretical point of view the behavior in flexure and in shear of deep and corbels elements in plain or fibrous concrete was and it is currently object of several researches presented in the literature [7-14]. The focus of these studies was the determination of the bearing capacity with the variation in the shape and the dimension of the elements, in the type and percentages of steel reinforcements, in the strength of concrete, etc.

From the analyses of crack patterns at rupture it emerges that it is possible to simplify the continuum problem by considering an equivalent truss having elements in tension simulating the presence of steel reinforcements and elements in compression simulating the compressed zones of the elements. If fibrous reinforced concrete or external wrapping techniques with FRP sheets are adopted the mentioned models can still be utilized, but they have to be modified considering the further contribution due to the presence of fibers (high values of residual strength in compression and tension or due to FRP in tension.

In the present paper an analytical and experimental research relative to the flexural behavior of reinforced corbels in plain and fibrous concrete with main and transverse steel bars or externally wrapped with carbon fiber reinforced polymers will be presented.

2. EXPERIMENTAL RESEARCH

The focus of the experimental research was the flexural behaviour of reinforced concrete corbels; in the following sections the experimental procedures, the test set-up arrangement, the mechanical characterisation of constituent materials and the tests on corbels will be described; moreover, the experimental results will be presented and discussed.

2.1 Preparation of specimens

Twelve corbels, having the geometry and steel reinforcement details shown in Fig. 1, were prepared to be tested in flexure. Each corbel was constituted by a horizontal beam and a short vertical column. The specimens, two for each different series investigated, with details given in Table 1, were made of: - plain concrete; - fiber reinforced concrete (FRC) with hooked steel fiber at 1% by volume percentage; - concrete reinforced with two longitudinal bars (main bars) having diameter $\phi = 10$ mm and placed at the bottom of the beam; - reinforced concrete with main bars and four horizontal stirrups having diameter $\phi = 6$ mm; - reinforced concrete with main bars and externally wrapped with one ply of flexible carbon fiber reinforced sheet (CFRP) having thickness $t_f = 0.165$ mm; - finally, fibrous concrete and main bars.

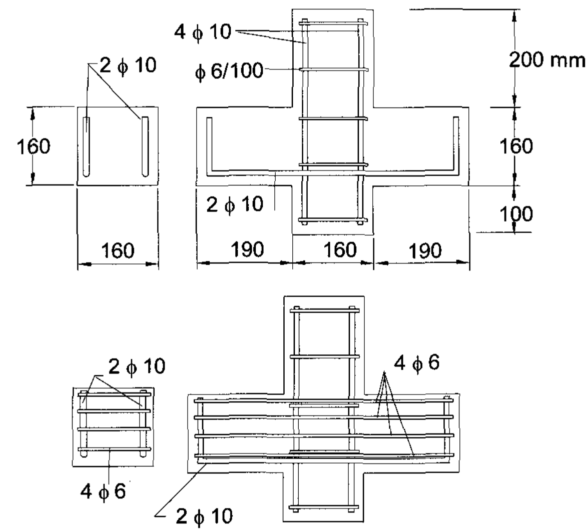

Fig. 1 - Geometry of corbels and details of steel reinforcement.

Table 1 - Details of corbels

Corbel type	Main bars mm	Transverse bars, mm	Fiber content	FRP layer n.
1	/		/	/
2	/		1.0	/
3	2x10	/	/	/
4	2x10	4x6	/	/
5	2x10	/	1.0	/
6	2x10	/	/	1

The steel reinforcements were constituted by deformed bars. To ensure an adequate anchorage length, the main bars were bent at the end portion, while the horizontal steel reinforcement constituted by close stirrups with hooks at the end portions. The column segment was reinforced with four deformed main bars having 10 mm diameter and stirrups having 6 mm diameter placed at a pitch of 100 mm. For all the bars a cover of 10 mm was utilised.

2.2 Characteristics of materials

The concrete utilised for the experimental research was a medium strength concrete having the following proportions in kg/m^3: 1050 of natural gravel at 10 mm maximum size, 850 of sand, 400 of Portland cement type 42.5 MPa and 200 of water. The effect of using 10 mm gravel in 160 mm corbels compared with the more typical 20 mm maximum size and larger corbels was considered acceptable since the scale ratio was not large.

To increase the workability of fiber reinforced concrete and to increase the strength of plain concrete a filler reactive powder (pozzolanic powder) was added to fresh concrete in the percentage of 10% by weight of the cement content. The filler having grain size of 0.1 μm, filled the pores of the cement paste and increased the strength of the material because of porosity reduction.

In the case of FRC, hooked steel fibers having length 30 mm and diameter 0.5 mm were utilised in a percentage of v_f = 1% by volume corresponding to 80 kg/m^3 of fibers. The nominal strength of the fiber was 1115 MPa. The percentage of 1% by volume of fibers commonly utilised in experimental investigations (*e.g.* [3, 4]) in the opinion of the authors [14, 15] is easy to mix in fresh concrete with a traditional technique and it is effective in structural applications, especially as shear reinforcement.

In Table 2 the results of direct tensile tests carried out on steel bars (three for each diameter) are given in terms of mean values of yielding stress f_y, rupture stress f_u and ultimate strain ε_u (measured on a gauge length of five equivalent diameters).

The type of external reinforcing layer was constituted by a single ply of flexible carbon fiber polymer pre-wet with resin and glued externally to the concrete surface by means of an epoxy resin. The mechanical characteristics of sheets as given by the manufacturer were elasticity modulus E_f = 230 GPa, ultimate stress f_u = 3400 MPa, elongation at rupture 1.5%. The corbels were wrapped externally for all the length and for all the height of the beam; an overlap of 100 mm was adopted along the flat portion of the beam. The corbels were cast in the horizontal position and the vibrated concrete was placed and compacted in two layers in the oiled wooden moulds with a thickness of 10 mm. Each batch of concrete was prepared to make two corbels and six cylinders 100 x 200 mm (three for compressive tests and three for split cylindrical tests). All specimens after demolding were placed for 28 days in a room at a temperature of 20°C with 90% relative humidity.

Compressive and indirect tensile tests on cylindrical specimens were carried out by means of a universal testing machine operating in controlled displacement mode.

As shown in Fig. 2 from the trend of the stress-strain curves in compression for plain concrete and fibrous concrete emerges that the addition of fibers did not produce variation in maximum strength, but significantly increased the maximum strain (measured at peak load) and improved the ductility of the material, showing a less marked slope of the softening branch and more residual strength.

Further advantages in using fibers were observed from splitting tensile tests as shown in Fig. 3 in terms of load shortening curves. Fibers improved the maximum tensile strength and corresponding strain and produced very ductile post-peak behaviour characterised by values of residual strength very close to the maximum value.

Table 3 gives mean values of maximum compressive strength f_c', corresponding strain ε_0 and ultimate strain ε_{cu} assumed as the strain reached when the stress value is 0.85f'$_c$ in the softening branch and finally maximum splitting tensile strength f'$_t$.

Table 2 - Mechanical characteristics of steel bars

φ (mm)	f_y (MPa)	f_u (MPa)	ε_u (%)
6	445	627	14.4
10	488	601	18.8

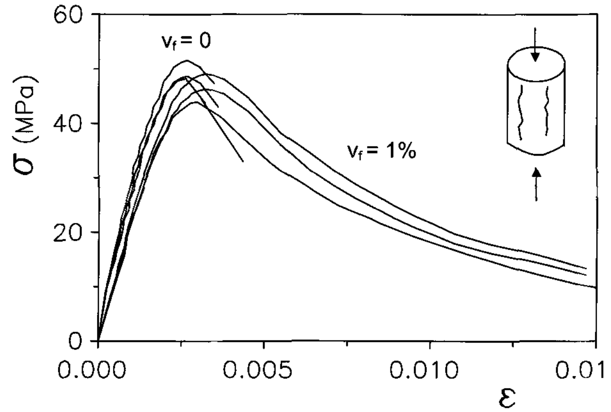

Fig. 2 - Stress-strain curves for concrete in compression.

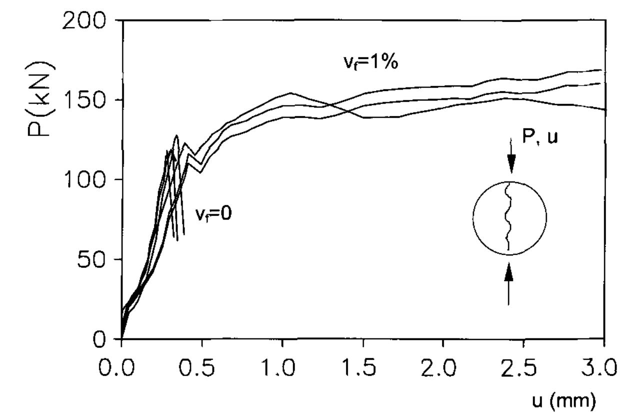

Fig. 3 - Load-displacement curves for concrete in tension.

Table 3 - Mechanical characteristics of plain and fibrous concrete

Type	f'$_c$ (MPa)	ε_0	ε_{cu}	f_t (MPa)
Plain concrete	48.50	0.00263	/	4.09
Fibrous concrete	46.31	0.00308	0.005	4.77

2.3 Test set-up

To perform flexural tests the same universal testing machine as described above was used. The machine was operating in a controlled displacement mode. The load scheme adopted is that shown in the test set-up of Fig. 4.

The corbels were supported symmetrically by two steel rollers placed on the top of the fixed part of the testing machine (basement).

The corbels were loaded through the load jacket of the testing machine (movable part) by means of the column. A slow rate of displacement of 0.5 mm/min was adopted. The load was recorded by a load cell having 60 kN maximum bearing capacity and the displacements were recorded by

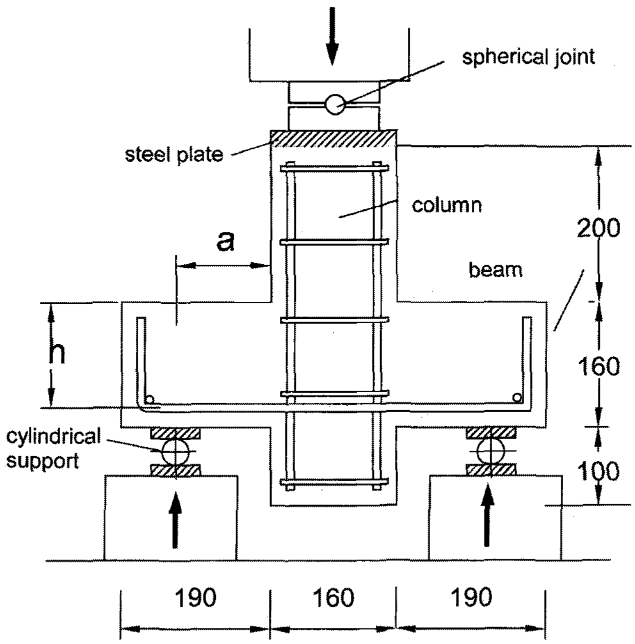

Fig. 4 - Test set-up.

three LVDTs. All data were collected by means of a high speed data acquisition system.

The net deflection δ was obtained as the difference between the displacement of the top of the column (one LVDT) and the average values of the settlements of the support measured at the top of the terminal section of the beams (two LVDTs). The load scheme adopted allows one to subject the corbel to flexure with linear variation in the bending moment and in shear at constant value. The ratio between the shear span (a = 130 mm) and the effective depth (h = 140 mm) of the beam was 0.928.

3. EXPERIMENTAL RESULTS

In the following sections, the results of the flexural tests on reinforced concrete corbels will be given as load-deflection curves and the failure mode will also be discussed.

3.1 Corbels without reinforcements

Fig. 5 shows the load-deflection curves for the corbels in plain concrete and in fibrous concrete without steel reinforcements.

In the case of plain concrete sudden and brittle failure was observed after the first crack appeared and the ultimate load and first cracking load were practically the same. The value of the external load 26.7 kN corresponded to a maximum tensile stress in the plane of the cross-section of the beam of 2.54 N/mm^2 lower than the tensile strength observed experimentally. This circumstance highlights the interaction between shear and flexure characterising the deep elements. In the case of FRC corbels a significant increase in maximum strength was observed and the failure mode was characterised by flexural failure mode; moreover, more ductile behaviour with high deflections and with significant residual strength values (due to the pull-out of fibers) was

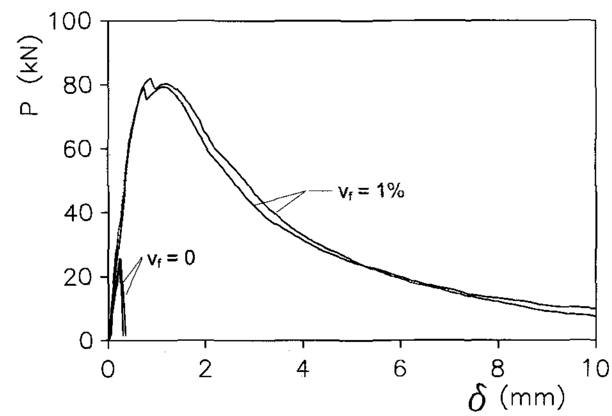

Fig. 5 - Load-deflection curves for corbels without steel reinforcements.

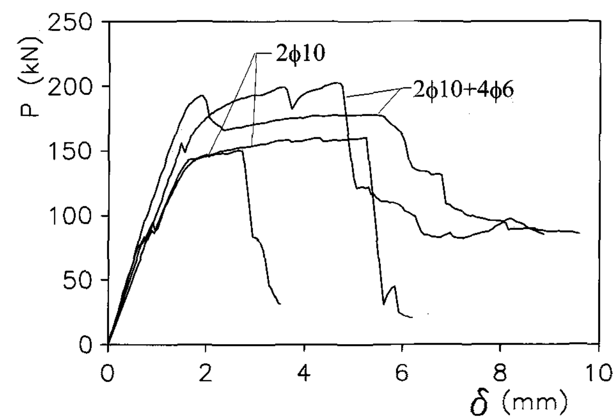

Fig. 6 - Load-deflection curves for steel reinforced concrete corbels.

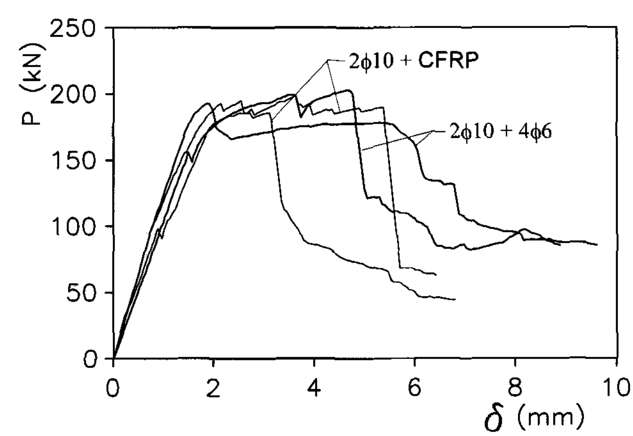

Fig. 7 - Load-deflection curves for corbels with steel reinforcements and CFRP.

observed. Plastic hinges formed at the constrained sections of the beams and the average shear stress in concrete was close to the tensile stress of the material. The crack opening was controlled in width by the presence of fibers.

3.2 Corbels with reinforcements

Fig. 6 shows the load-deflection curves for corbels reinforced with main steel bars with and without transverse stirrups.

Although for both the cases examined a similar ascending linear branch was observed, the load corresponding to the formation of the first crack was higher in the case of corbels with transverse steel and more stiffness was observed until the ultimate load was reached.

In the presence of stirrups, more cracks formed and they were finer and the ultimate strength measured was higher than that of corbels with main steel only. After the peak load was reached, ductile behaviour characterised by a progressive fracturing process (controlled in the width by the elongation of transverse steel) was observed. In both cases shear failure was observed.

Fig. 7 shows the load-deflection curves for corbels with longitudinal steel bars and wrapped with one layer of CFRP; for comparison the curves relating to corbels with longitudinal and transverse steel are also given. From the trend of the curves it can be observed that the overall behaviour of the corbels was quite similar in terms of both initial stiffness and maximum load, and the failure mode was in shear.

The resistant mechanism involved in the wrapped corbels was also characterised by the interaction phenomena at the interface between the concrete surface and the CFRP sheet. After the principal cracks in shear opened, debonding of sheet occurred at the interface with the concrete. Moreover, if the sheet did not break in tension (because of the stress-concentration at the sharp corners) debonding occurred in the overlap length. The failure mechanism observed experimentally in CFRP was in tension and in debonding.

Fig. 8 shows the load-deflection curves for corbels in FRC with longitudinal steel bars and for comparison the curves relating to corbels with longitudinal and transverse steel bars are given.

In the case of FRC, the complete flexural capacity of beam was reached and ductile behaviour was observed. This circumstance highlights the effectiveness of fibers as shear reinforcement and in improving the confinement of the compression zone of the beam.

Figs. 9 and 10 show the evolution in the development of the cracks recorded and the condition of the specimens at the end of the tests.

Table 4 shows the results of the flexural tests in terms of first cracking P_f, maximum load P_{max}, ultimate load P_u and

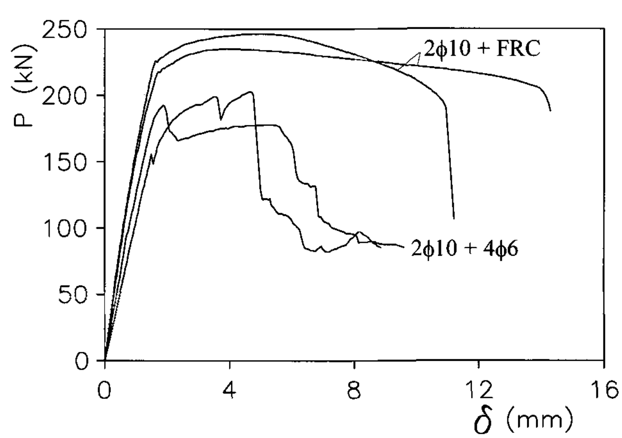

Fig. 8 - Load-deflection curves for corbels with steel reinforcement and FRC.

Table 4 - Results of flexural tests

	P_f	P_{max}	P_u	δ_{max}	δ_u
2 ϕ 10	64.50	155.20	25.95	3.93	4.27
2 ϕ 10 +4 ϕ 6	60.50	197.65	85.45	3.26	5.40
2 ϕ 10 +FRP	/	192.20	93.12	3.35	4.52
2 ϕ 10+ FRC	104.00	240.75	147.35	4.60	12.75

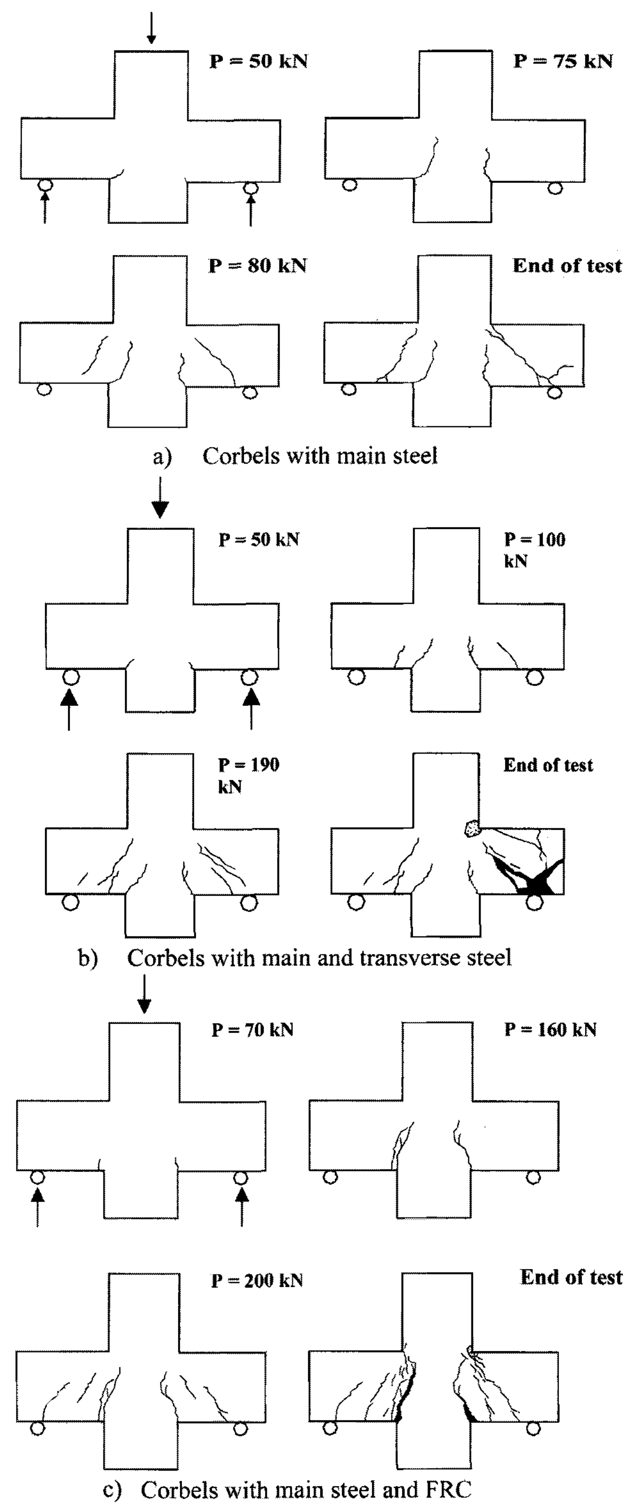

Fig. 9 - Evolution in the crack pattern for reinforced concrete corbels.

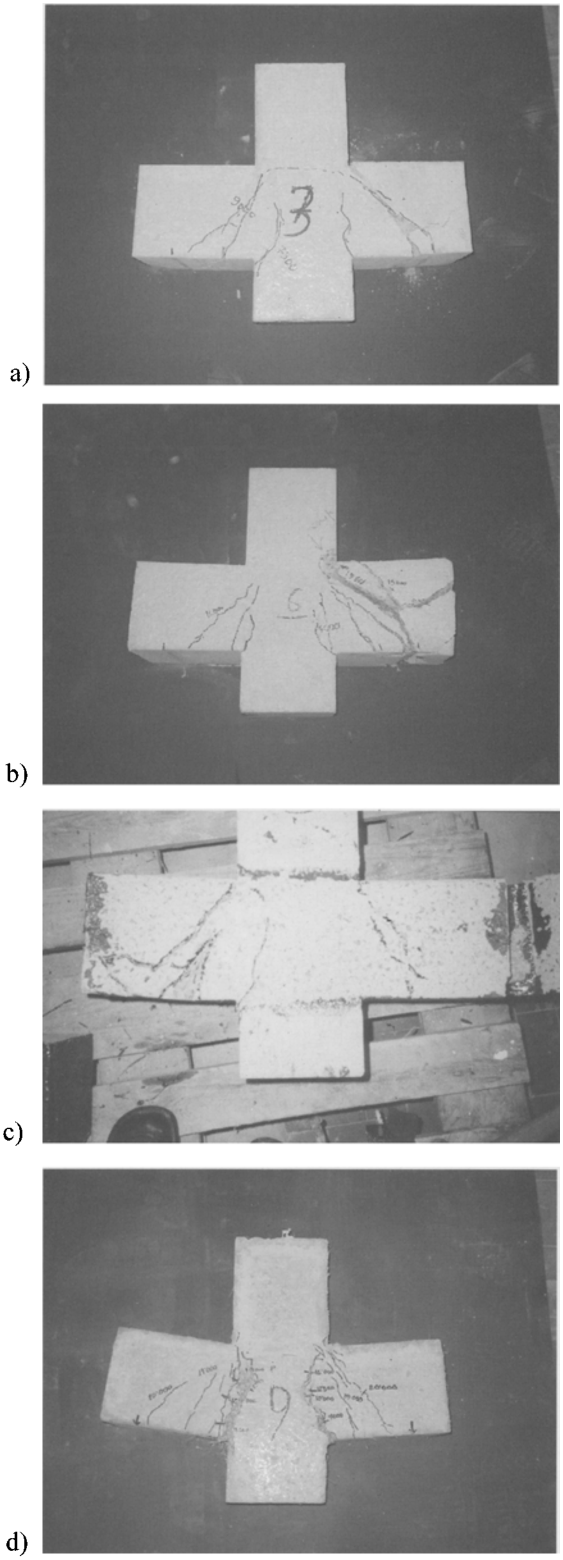

Fig. 10 - Cracking patterns of steel reinforced corbels with:
a) main steel bars; b) main steel bars and transverse tirrups;
c) main steel and wrapped with CFRP; d) main steel bars and FRC.

the corresponding deflections δ_{max}, δ_u.

In the case of corbels with main steel the rupture is related always to the brittle failure of compressed zone arising after the yielding of steel bars occurs. In the case of

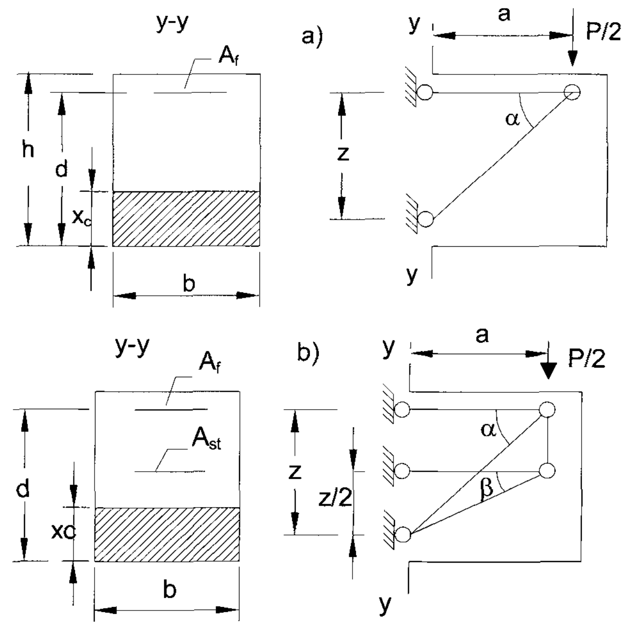

Fig. 11 - Equivalent truss model: a) single truss; b) multiple truss.

external wrapping with CFRP similar mode of failure was observed but the compressive rupture is consequent to the failure of CFRP wraps in tension.

In the presence of transverse steel reinforcements more ductile behaviour is observed (because of the bridging action of transverse steel against the principal cracks), but the flexural capacity is not completely achieved. In the case of FRC ductile balanced flexural failure is observed.

4. ANALYTICAL MODEL

On the basis of the experimental results obtained and the crack patterns observed at rupture, it appears reasonable to substitute the cracked continuum with an equivalent single or multiple truss, as shown in Figs. 11 a) and b).

The single truss refers to corbels with the main steel in plain or fibrous concrete, while the multiple truss refers to corbels with the main and transverse steel in plain concrete or with the main steel in plain concrete and externally wrapped with FRP.

The single truss shown in Fig. 11 a) is constituted by a diagonal member of compressed concrete (inclined α with respect to the horizontal direction) and by a horizontal member in tension representative of the main steel.

For fibrous concrete the presence of fibers is taken into account by also considering the vertical projection of the strength contribution of fibers (bridging the cracks formed perpendicular to the diagonal direction and translated into high values of residual tensile strength).

The multiple truss shown in Fig. 11 b) is constituted instead by two single trusses, one, mentioned below as the principal truss, with the compressed member (inclined α) connected to the main steel; and the other, mentioned below as the secondary truss, having the compressed member inclined β and connected to the transverse steel (or the FRP).

In the principal truss z is the arm of the internal forces of the fixed cross-section, while in the secondary truss the transverse reinforcements are supposed to be applied to z/2.

In the next section more details will be given for the determination of the value of z and for the definition of the effective depth to assume for the compressed diagonals (strut members).

With reference to the compressed members it is necessary to distinguish the case of plain concrete from that of fibrous concrete. For plain concrete, because of the biaxial stress state, a softening behaviour of concrete due to the presence of transverse cracks reducing the uniaxial compressive strength f_c' has to be taken into account, *e.g.* by adopting the reductive coefficient proposed in [13] and defined as:

$$\varsigma = \frac{5.8}{\sqrt{f'_c}}\frac{1}{\sqrt{1+400\varepsilon_r}} \le \frac{0.9}{\sqrt{1+400\varepsilon_r}} \qquad (1)$$

ε_r being the average strain in the direction perpendicular to the principal cracks and f_c' given in MPa. In a simplified way, as again suggested in [13], if failure of compressed members follows the yielding of steel (this is the case examined here) the value of ε_r can be assumed equal to the yielding value of steel.

In the case of fibrous concrete in a high percentage, as in the case examined here, it appears reasonable to assume that the compressive uniaxial strength does not need to be reduced because of the presence of the biaxial stress state in relation to the high residual strength in tension of the composite.

In the case of external wrapping of corbels with FRP, it has to be observed that, although the FRP sheets can reach the maximum strain in tension, when the FRP is fixed externally to the concrete surface, the maximum strain allowed is that related to the delamination load or due to failure in the overlap connection [16-18].

As shown in [18] the maximum force that can be transferred by adhesion increases with the anchorage length up to a fixed maximum value F_b for the length L_b.

As suggested in [18], the following expressions of F_b and L_b, respectively can be assumed:

$$F_b = \psi_F \cdot h_w\sqrt{E_f t_f f_t'} \qquad (2)$$

$$L_b = \psi_L\sqrt{\frac{E_f t_f}{f_t'}} \qquad (3)$$

f_t' being the tensile strength of the concrete, and ψ_F and ψ_L numerical coefficients assumed equal to 0.65 and 0.85 respectively. The units in Equations (2) and (3) are expressed in S.I.

The overlap length L_{anc}, indicated in Fig. 12, was chosen in such a way to avoid the detachment, as in [19].

5. BEARING CAPACITY OF CORBELS

As observed experimentally, the external load is transferred from the point of application to the fixed cross-section of the corbels because it forms a compressed cracked zone (extending from the point at which the external load is applied to the centre of the compressed zone of the fixed cross-section) connected to the main steel reacting in tension. The area of the cross-section of the equivalent strut inclined α can be defined as $A_c = h_{eff}$ b, b being the depth of the corbel and h_{eff} the effective width equal to $x_c\cdot \cos\alpha$, with x_c the depth of the neutral axis of the fixed section.

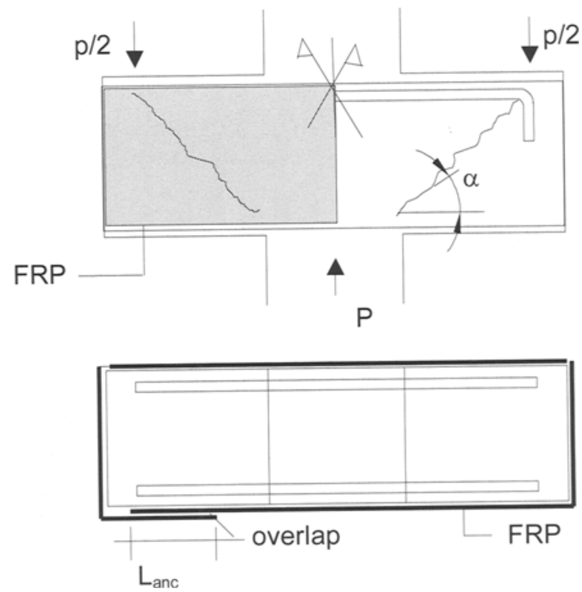

Fig. 12 - Corbels wrapped with FRP.

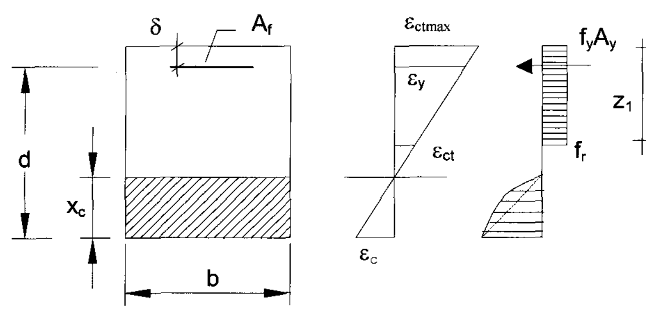

Fig. 13 - End fixed cross-section of the corbel.

Below, the condition considered to determine x_c is that in which the main steel reinforcements yields, while the strain values in the compressed zone are small and for this reason the effective nonlinear behaviour can be replaced for simplicity by a linear trend.

In the case of FRC, as shown in Fig. 13, the equilibrium condition also takes into account the contribution of fibers.

This contribution considered by means of the residual strength in tension of the fibrous concrete, assumed to be equal to f_r and constant for the length z_1, the latter being the distance between the most stressed fiber in tension of the concrete and the fiber in which the peak stress in tension is reached in the fibrous concrete. Simple geometrical considerations give:

$$z_1 = (d - x_c) \cdot \left(1 - \frac{f_{ctf}}{f_y} \cdot \frac{E_s}{E_{ct}}\right) + \delta \tag{4}$$

f_{ctf} and f_y being the tensile strength of the fibrous concrete and the yielding stress of the main steel respectively, E_{ct} and E_s being the elasticity modulus and δ the cover thickness respectively.

In the case of FRC it is possible to relate the maximum and the residual strength to the characteristics of plain concrete and to the geometrical and mechanical characteristics of the fibers. Specifically, the ultimate tensile strength, as shown in [20], is equal to:

$$f_{ctf} = f_t' \cdot (1 - v_f) + \alpha_1 \cdot \alpha_2 \cdot \tau \cdot v_f \frac{L_f}{D} \tag{5}$$

in which τ is the average bond strength at the fiber-matrix interface, α_1 the coefficient representing the fraction of bond mobilised at first matrix cracking and α_2 the efficiency factor of fiber orientation in the uncracked state.

Regarding post-cracking strength, Naaman [20] suggests the following suitable expression:

$$f_{ctu} = \lambda_1 \cdot \lambda_2 \cdot \lambda_3 \cdot \tau \cdot v_f \frac{L_f}{D} \tag{6}$$

in which λ_1 is the expected pull-out length ratio, λ_2 the efficiency factor of orientation in the cracked state, λ_3 the group reduction factor associated with the number of fibers pulling out per unit area. According to [20] possible values for steel fibers with aspect ratio equal to 100 are $\alpha 1 = 0.1$, $\alpha 2 = 0.5$, $\tau/f_{ct} = 2$, $\lambda_1 = 0.25$, $\lambda_2 = 1.2$, $\lambda_3 = 1$. In the present paper these values were assumed.

5.1 Corbels with main steel

With reference to the single truss shown in Fig. 11 a), it is possible to obtain the ultimate load by imposing the condition of failure of the main steel reinforcement or of the compressed concrete in the equilibrium equation.

In the case of yielding of steel we have:

$$P_{u1} = 2 \cdot f_y \cdot A_f \cdot \tan\alpha \tag{7}$$

and in the case of failure of the compressed zone we have:

$$P_{u2} = 2\xi \cdot f'_c \cdot b \cdot x_c \cdot \cos\alpha \cdot \text{sen}\alpha \tag{8}$$

With reference to fibrous concrete, considering also the vertical projection of the bridging action exerted by the fibers across the diagonal cracks, we have:

$$P_f = 2 \cdot \frac{f_r \cdot z_1 \cdot b}{\text{sen}\alpha \cdot \cos\alpha} \tag{9}$$

The last term is to be added to those obtained using Equation (7) or Equation (8).

5.2 Corbels with main and secondary steel reinforcement or with FRP wraps

For corbels with principal and secondary steel or with principal steel and wrapped with FRP we refer for the calculus of the maximum load to the multiple truss shown in Fig. 11 b).

As already observed, the principal truss has the main steel as its tension reinforcement, while the secondary truss has as its tension reinforcement the resultant of the secondary steel or of the FRP.

In the multiple truss it was supposed that the same vertical displacements occur in the loaded joints. The distribution of the external load P/2 between the two trusses occurs in relation to the stiffness of the main and secondary trusses, denoted in the following as R_1 and R_2, respectively. These stiffnesses can be expressed as:

$$R_1 = \frac{1}{\dfrac{a}{E_s A_{f1} \tan^2\alpha} + \dfrac{\sqrt{a^2 + z^2}}{E_c A_{c1} \sin^2\alpha}} \tag{10}$$

$$R_2 = \frac{1}{\dfrac{a}{E_s A_{f2} \tan^2\beta} + \dfrac{\sqrt{a^2 + (z/2)^2}}{E_c A_{c2} \sin^2\beta}} \tag{11}$$

$A_{c1} = bx_c\cos\alpha$ and $A_{c2} = bx_c\cos\beta$ being the areas of the equivalent compressed struts, A_{f1} and A_{f2} being the area of the main steel and secondary reinforcements (constituted by transverse steel or FRP) respectively. Moreover, it has to be observed that if FRP is utilised in Equation (11) E_s has to be replaced by the E_f modulus of FRP.

The load supported by the main truss is $\eta P/2$, with $\eta < 1$, while the load supported by the secondary truss is $(1-\eta)P/2$, where:

$$\eta = \frac{R_1}{R_1 + R_2} \tag{12}$$

It has to be mentioned that the elasticity modulus of the concrete (and therefore the stiffness of the trusses) varies with the load levels, and with reference to the failure conditions it can be assumed equal to 2/3 of the initial value. Excluding cases in which the secondary truss yields before the principal truss, the load corresponding to the failure in steel and concrete proves to be:

- in the case of steel failure

$$P_{u1} = \frac{2}{\eta} \cdot f_y \cdot A_f \cdot \tan\alpha \tag{13}$$

- in the case of compressive failure

$$P_{u2} = \frac{2 \cdot \xi \cdot f'_c \, bx_c \text{sen}\alpha \cdot \cos\alpha}{1 - \eta} \tag{14}$$

In the presence of FRP, if failure occurs because of debonding, the maximum allowable load proves to be:

$$P_{u3} = \frac{2 \cdot F_b \tan\beta}{(1-\eta)} \tag{15}$$

Finally, Table 5 gives the experimental and analytical values of the maximum load, showing good agreement between them.

Table 5 - Comparison between analytical and experimental results

Type	Load P (kN)			
		Analytical		
	Exper.	η	P_{u1}	P_{u2}
/	155.20	1	149	184
4 ϕ 6	197.65	0.77	168	225
FRP	192.20	0.90	146	195
CFR	240.75	1	232	319

6. CONCLUSIONS

In the present paper an analytical and experimental research regarding the flexural behaviour of reinforced concrete corbels is presented. The effects of traditional steel reinforcements constituted by longitudinal steel bars and transverse stirrups are compared with the effects produced by the use of fiber reinforced concrete or due to external wrapping with fiber reinforced polymers. The results obtained show the effectiveness of the two non-conventional reinforcing techniques in improving the maximum strength. It is also shown that in the case of fiber reinforced concrete very ductile behaviour is observed, characterised by flexural failure.

The analytical model proposed is able to determine the bearing capacity of corbels for all cases examined. It is based on the analysis at rupture of corbels by means of an equivalent multiple truss including all the possible modes of failure observed experimentally, such as steel yielding, concrete crushing, external wrap debonding and fiber pull-out. The model shows good agreement with the experimental results and also gives a physical interpretation of the behaviour of corbels at rupture.

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