Materials and Structures/Matériaux et Constructions, Vol. 32, June 1999, pp 394-399

TECHNICAL REPORTS

In-place compressive strength of concrete: statistical methods to evaluate experimental data

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Paper received: July 29, 1998; Paper accepted: January 8, 1999

ABSTRACT

This paper describes a number of statistical methods to estimate, through an analysis of in-place test results, the in-place compressive strength of concrete expressed as strength of standard specimens. The evaluation of the probable existence of a linear correlation as a function of sample size is discussed. The regression laws and the degree of reliability of the estimates obtained is also discussed. A criterion is proposed for the choice of the number of points to be used in the regression. Finally a method is proposed, knowing the number **m** of replicated in-place tests, to estimate the equivalent number **n** of replicated tests performed on standard specimens.

RÉSUMÉ

Cet article présente diverses méthodes statistiques qui permettent, par le biais de l'analyse des résultats d'essais 'in-situ', l'estimation de la résistance du béton exprimée comme résistance sur échantillons standard. Il est procédé à l'évaluation de l'existence probable d'une corrélation linéaire en fonction de la taille de l'échantillon. Il est procédé à l'analyse des lois de régression, ainsi que du niveau de fiabilité des estimations obtenues. Il est proposé un critère pour l'évaluation du nombre de points à employer dans la régression. Enfin, il est proposé une méthode qui permet, pour un nombre **m** *d'essais répétés selon les méthodes non-destructives, d'estimer le nombre* **n** *équivalent d'essais répétés sur échantillons standard.*

1. INTRODUCTION

In many situations, the compressive strength of the concrete making up a structure cannot be evaluated by performing compressive strength tests on standard – cube or cylinder – specimens manufactured with the same concrete (potential strength); it therefore proves necessary to resort to indirect methods working on in-place concrete. Numerous techniques have been developed to this end, ranging from the classical core tests to various non-destructive methods, such as pull-out, hardness, penetration resistance tests, or any of the other methods described in the literature [1-3].

In general, while the use of indirect methods is rather easy when it comes to the execution of in-place tests, a correct interpretation of the experimental findings is not as straightforward. While keeping in mind that it is not possible to define an "intrinsic" strength of concrete [4] and that even "potential" strength is but a conventional notion adopted for purposes of evaluation, the aim is to fine-tune a method to work out an estimated value of in-place compressive strength from indirect test, which may be expressed as the strength value on standard specimens [5]. This will yield strength values which generally do not coincide with potential strength, even when the tests are conducted on the same type of concrete.

To this end, it proves necessary first of all to know the relationships linking the indirect test measurements with the corresponding standard test values; these relationships can be derived from experimental investigations (conducted on the same concrete) enabling the results of indirect tests to be compared with those of standard tests. Obviously, the testing campaign has to be planned and interpreted by means of statistical methods, by taking into account the fact that, as a rule, only a relatively limited number of test data is available to serve as a statistical sample for the formulation of estimates applicable to all the concrete considered and hence to the entire population.

To study the relationships worked out on the basis of a comparison between indirect and standard tests, it is advisable that both standard and indirect tests be per-

Editorial Note
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formed, as far as possible, on standard specimens. This is so because if the tests are performed on structural members, albeit manufactured with the same concrete, new elements of uncertainty are introduced [5, 6].

It should also be noted that if the tests conducted on standard specimens are compared with core or micro-core tests, we shall be able to express the results in the same units of measure, which facilitates matters. When it comes to interpreting the results, however, it should be kept in mind that the values of strength obtained from this type of indirect tests cannot be directly linked with those obtained from tests on standard specimens.

In most non-destructive methods, instead, the strength values obtained on standard specimens will have to be compared with data measured in units other than strength.

At any rate, regardless of the testing method selected, the procedures for the estimate of direct and indirect relationships are characterised by several common features, as discussed below.

2. DISTRIBUTION OF THE DATA

Let us assume we have tested N types of concrete, for each of which **n** standard specimens (cubes) have been subjected to compressive strength tests, and we have obtained **m** indirect test results. We may consider the N groups of cubes as N samples of size **n**, each of which has been taken from a different population of cubes; similarly, for the indirect tests, we shall have N samples of size **m**, each sample having been taken from a different population.

The first problem to be addressed is the type of distribution of each of the 2N populations of origin of the samples, in particular, by considering whether or not the hypothesis of a normal distribution, as is generally assumed in the drafting of reinforced concrete codes, applies. To this end, we can use different tests, including the χ^2 test, which supplies reliable answers when sample size is large enough, and the Kolmogorof-Smirnov test which can be used on smaller sized samples, as is generally the case in investigations of this sort [7]. Finally it should be noted that in the literature we can also find examples of asymmetrical distributions, other than gaussian distributions [8-10], for which it sometimes proves possible to adopt a log-normal distribution model [2, 9-11].

At all events, it is obvious that the type of distribution to be adopted will necessarily be determined through an examination of the test data with the aid of statistical methods, through a process, that is, which inevitably calls for assumptions of a probabilistic, not of a deterministic nature.

If the outcome of normality tests is satisfactory, the procedures described below will be supported by strong assumptions. If it is not, this means that the proposed techniques may still be used, but the reliability of the estimates will be poorer.

3. CORRELATION

If the N concrete types tested are quite similar to one another, it often proves possible to identify a functional link between cube compressive tests and the results of indirect tests. To this end, it might be useful to process the data concerning the mean values, to identify the function correlating variable Y, *i.e.* the mean strength of the cube sample worked out on the basis of the strengths, y, of the individual specimens, with variable X, *i.e.* the mean value of the results, x, of indirect tests.

By working on mean values, albeit worked out from samples, it proves possible to reduce quickly the uncertainties as to the estimate of the population values, since the standard errors of the mean on cube or indirect test

$$\sigma_Y = \frac{\sigma_y}{\sqrt{\mathbf{n}}} \quad ; \quad \sigma_X = \frac{\sigma_x}{\sqrt{\mathbf{m}}}$$

decrease rapidly with increasing sample size (**n** and **m**), and consequently the two variables to be correlated can be worked out with great accuracy, provided that the experimental basis is large enough.

Going back to the functional link, the simplest and commonest case is a linear correlation between the two variables, X and Y. An estimate of the validity of this correlation can be supplied by the coefficient of linear correlation

$$\rho = \frac{\mathrm{cov}(X,Y)}{\sigma_X \sigma_Y} \tag{1}$$

from which, with the sample data, it is possible to obtain an estimator, r, to be used to evaluate whether or not the linear correlation model is appropriate.

To this end we may perform a significance test by taking as null hypothesis (Ho) the value of the coefficient of correlation corresponding to the absence of correlation, $\rho = 0$. This can be done by making use of the transform of the random variable r

$$\frac{r\sqrt{N-2}}{\sqrt{1-r^2}} \tag{2}$$

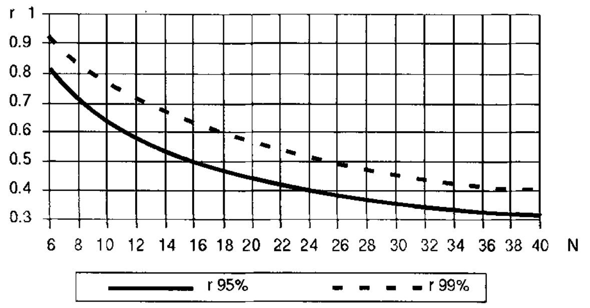
which, as is known [12], follows a Student distribution with N - 2 degrees of freedom in the case of no correlation between the X and Y variables.

Having determined the degree of significance, α, the non correlation hypothesis will be discarded in favour of the alternative hypothesis if the experimental data verify the inequality

$$\left|\frac{r\sqrt{N-2}}{\sqrt{1-r^2}}\right| > t_{1-\frac{\alpha}{2};N-2} \tag{3}$$

where $t_{1-\alpha/2;\,N-2}$ is the quantile of order $1-\alpha/2$ of Students random variable with N - 2 degrees of freedom.

Fig. 1 and Table 1 supply, as a function of the size of the sample N and for α = 0, 0.5 and 0.01, the *minimum* values of $|r|$ for which exp. (3) is verified. In other words, the threshold values of r, in absolute terms, which enable us to reject, with a probability level of 95%



N	r 95%	r 99%
6	0.811	0.917
7	0.755	0.875
8	0.707	0.834
9	0.666	0.798
10	0.632	0.765
11	0.602	0.735
12	0.576	0.708
13	0.553	0.684
14	0.532	0.661
15	0.514	0.641
20	0.444	0.561
30	0.361	0.463
40	0.312	0.403

Table 1 and Fig. 1 – Threshold values for r as a function of sample size and 95% and 99% probability levels.

and 99%, the hypothesis of $\rho = 0$, and consequently, to accept the existence of the correlation. As can be seen, these threshold values decrease fast with increasing N.

Obviously, r being the same, a bigger sample will supply a greater probability of $\rho \neq 0$ and vice versa, sample size being the same, a greater probability level will require higher values of r. An examination of the values of r cannot therefore be limited to the pure observation of the numerical value regardless of the number of points. At all events, a situation in which the tests performed lead to the acceptance of the $\rho = 0$ hypothesis does not rule out the existence of functional links other than linear correlation.

4. REGRESSION

Once the linear correlation hypothesis has been accepted, the least square method makes it possible to identify the regression line:

$$\hat{Y} = a + bX \tag{4}$$

In this connection it should be noted that in order to improve the reliability of the regression several variations to ordinary OLS analysis have been proposed [2, 11, 13] so as to avoid a number of drawbacks associated with the scatter of test data in indirect tests, x. In most cases, however, since the uncertainties as to the estimate of X can be reduced quite easily, it is possible to proceed with the customary methods.

In actual practice, however, using the regression line alone to estimate the mean strength on standard cubes, Y, on the basis of the mean value of the results of indirect tests, X, has little practical validity in view of the fact that, even if the value read on the straight line at a generic X_i represents a punctual estimate of the mean value of Y conditional to the X_i taken into consideration, by doing so we disregard the fact that this result has been worked out from sample data, and therefore it proves possible to identify different lines with varying samples. This is obviated by identifying, at a desired confidence level $(1 - \alpha)$, the confidence interval for the regression line, whose width is:

$$D_{\hat{Y}}(X) = 2t_{\frac{\alpha}{2};N-2}\, s_{Y|X}\sqrt{\frac{1}{N} + \frac{(X-\overline{X})^2}{\sum(X_i-\overline{X})^2}} \tag{5}$$

where $t_{\alpha/2;\,N-2}$ is the quantile of order $\alpha/2$ of Students variable with N - 2 degrees of freedom and $s_{Y|X}$ is the standard error of estimate

$$s_{Y|X}\sqrt{\frac{\sum(Y_i-\hat{Y}_i)^2}{N-2}} \tag{6}$$

Users, however, may be more interested in an estimate of Y, rather than its mean value: this can be obtained, again at a desired confidence level $(1 - \alpha)$, by considering the confidence intervals of this variable, whose width, is given by:

$$D_{Y}(X) = 2t_{\frac{\alpha}{2};N-2}\, s_{Y|X}\sqrt{1 + \frac{1}{N} + \frac{(X-\overline{X})^2}{\sum(X_i-\overline{X})^2}} \tag{7}$$

the confidence level being the same $(1 - \alpha)$, obviously turns out to be greater than the width of confidence interval of the regression line.

However, for the estimate of cube strength on the basis of a different parameter, we should consider that, in general, an overestimate is more dangerous than an underestimate, and therefore it is advisable to select a confidence level ensuring that the risk of the real values of Y being lower than the estimate is limited.

This can be achieved by taking into consideration the lower limit of the confidence interval expressed by:

$$Y = a + bX - t_{\frac{\alpha}{2};N-2}\, s_{Y|X}\sqrt{1 + \frac{1}{N} + \frac{(X-\overline{X})^2}{\sum(X_i-\overline{X})^2}} \tag{8}$$

where $\alpha/2$ is the risk of making an overestimate error which is deemed acceptable.

This is illustrated in Fig. 2 which shows an example of a correlation line and confidence intervals taken from an earlier experimental investigation [6]. The Y variable indicates mean strength values obtained on cubes, whilst the X indicates mean strength values obtained on micro-

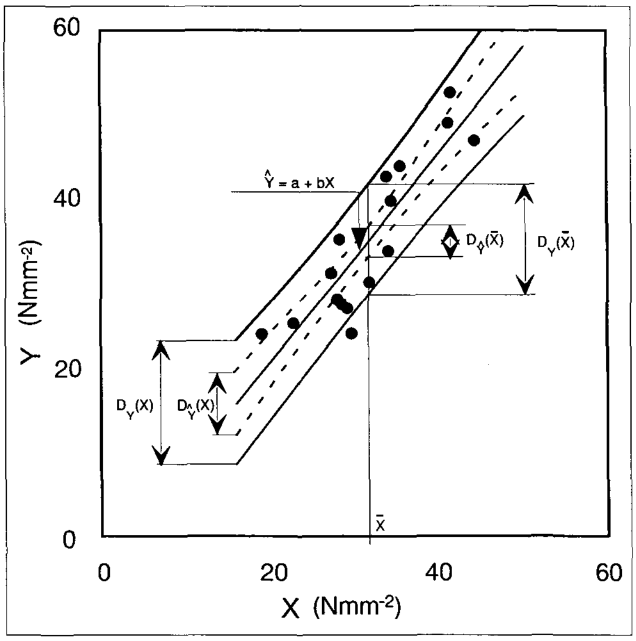


Fig. 2 – Example of two-sided 80% confidence intervals for mean cube strength Y (——), and for the regression straight line $\hat{Y}$ (- - - -) as a function of mean microcore strength, X.

cores drilled from the same concrete types.

The inner band, represented by the broken curves, shows the 80% confidence interval for the Y line, *i.e.*, for mean Y for a given value of X. The outer band shows, for each value of X, the limits between which it is forecast that 80% of the individual observations will lie; it should be noted that at point $X = \overline{X}$, the width of the intervals is minimum.

5. NUMBER OF POINTS FOR THE REGRESSION

At any rate, we are faced with the problem of how many points N are necessary to obtain an estimate of the regression, or to evaluate the lower confidence limit necessary to ensure that the values of $\hat{Y}$ and Y, supplied by equations (4) and (8), are estimated with an acceptable degree of accuracy.

The ACI recommendations [2], which repropose a criterion formulated by Carino [11], indicate an optimal number of points (that is, of X, Y pairs) of between 6 and 9.

According to the proposed technique, from an examination of the $D_{\hat{Y}}(\overline{X})/s_{Y|X}$ ratio between exp. (5), as calculated at point $X = \overline{X}$, and equation (6), as a function of size N, the contribution of additional points after the first 9 would turn out to be negligible. It should be noted, however, that we are more interested in minimising the width of the interval for the estimate of Y, not the mean value of Y, and therefore it seems more advisable to consider the $D_{\hat{Y}}(\overline{X})/s_{Y|X}$ ratio. Working with a confidence level of $(1 - \alpha) = 0.95$, from Table 2 it can be seen that by adopting the method proposed by the ACI, for N = 6 and N = 9, respectively, we get: $D_{\hat{Y}}(\overline{X})/s_{Y|X} = 2.267$ and $D_{\hat{Y}}(\overline{X})/s_{Y|X} = 1.577$. These values show that the width of the interval decreases to a considerable extent if we use 9 as opposed to 6 pairs. On the other hand, by taking into consideration the $D_Y(\overline{X})/s_{Y|X}$ ratios, for the same two sample sizes, we get, respectively: 5.999 and 4.986, with a decrease which is even more appreciable. In either case, these differences in values show that the choice of 6 points only leads to much less accurate results compared to the choice of 9, and that the suggested interval is affected by considerable variability. In any case, the choice of N = 9 seems best, as is obvious, since the estimates will improve with increasing size.

Whether or not N should be increased also depends on other considerations, of a practical nature, and therefore it might be useful to establish parameters to help decide whether the value of N should be increased beyond a limit to be defined.

If we examine the $D_{\hat{Y}}(\overline{X})/s_{Y|X}$ ratios it might be convenient to use, as proposed by Carino [11], the increment:

$$d_{\hat{Y},N} = \frac{D_{\hat{Y},N}(\overline{X}) - D_{\hat{Y},N-1}(\overline{X})}{s_{Y|X}} \qquad (9)$$

If we consider the $D_Y(\overline{X})/s_{Y|X}$ ratios, instead, it might be advisable to take into consideration the increment:

$$d_{Y,N} = \frac{D_{Y,N}(\overline{X}) - D_{Y,N-1}(\overline{X})}{s_{Y|X}} \qquad (10)$$

where $D_{\hat{Y},N}(\overline{X})$ and $D_{\hat{Y},N-1}(\overline{X})$; $D_{Y,N}(\overline{X})$ and $D_{Y,N-1}(\overline{X})$ indicate equations (5) and (7) as a function of a size variation of 1.

Equations (9) and (10), whose values are given in Table 2 and plotted in Fig. 3, measure the variation in the width of the confidence intervals expressed by equations (5) and (7) when the points underlying the regression are increased from N - 1 to N.

As to the values taken on by equation (10), it can be seen that they decrease rapidly over an initial portion, whilst an asymptotic trend $d_{Y,N} \to 0$ is observed for $N \to \infty$.

A limit to the value of N can be selected by establishing a threshold value delimiting a range within which the contribution of a few additional units might seem non negligible. For instance, if we accept for the decrements a threshold value $d_{Y,N} \leq 0.05$, which corresponds to N = 15, we shall find that for $10 \leq N \leq 15$ the decrements take on values of $0.149 \leq d_{Y,N} \leq 0.049$, whilst their sum within the same interval $\sum_{N=10}^{N=15} d_{Y,N} = 0.524$ turns out to be very close to the sum of the decrements $\sum_{N=8}^{N=9} d_{Y,N} = 0.511$. This shows that in the reduction of the

Table 2 – Width of confidence intervals for $\hat{Y}, D_{\hat{Y}}(\bar{X})/s_{Y|X}$ and for $Y, D_Y(\bar{X})/s_{Y|X}$ and values of their increments, $d_{\hat{Y},N}$ and $d_{Y,N}$, as a function of number of points N

N	$D_{\hat{Y}}(\bar{X})/s_{Y\|X}$	$d_{\hat{Y},N}$	$D_Y(\bar{X})/s_{Y\|X}$	$d_{Y,N}$
6	2.267	-0.579	5.997	-0.975
7	1.943	-0.323	5.497	-0.500
8	1.730	-0.213	5.191	-0.306
9	1.577	-0.154	4.986	-0.205
10	1.458	-0.118	4.837	-0.149
11	1.364	-0.094	4.725	-0.112
12	1.286	-0.078	4.638	-0.087
13	1.221	-0.065	4.568	-0.070
14	1.165	-0.056	4.511	-0.057
15	1.115	-0.049	4.462	-0.049
20	0.940	-0.029	4.306	-0.024
30	0.748	-0.014	4.164	-0.010
40	0.639	-0.009	4.092	-0.006
80	0.447	-0.003	4.025	-0.001
120	0.361	-0.002	3.976	-0.001

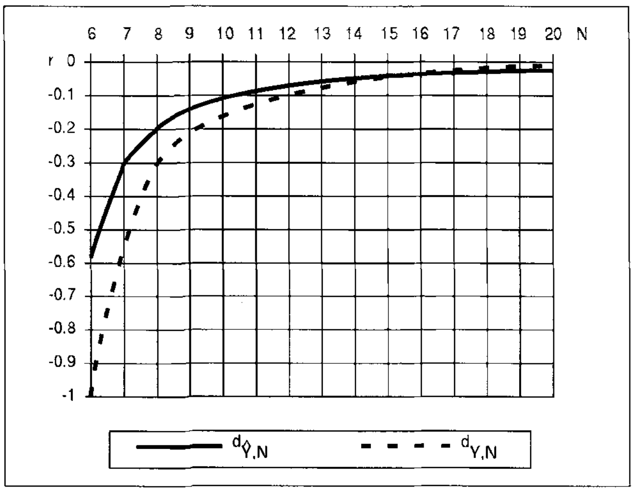


Fig. 3 – $d_{\hat{Y},N}$ and $d_{Y,N}$ increments in the width of confidence intervals for regression line $\hat{Y}$ and variable Y as a function of number of points N.

width of the confidence intervals for the Y's, the addition of 6 points beyond the original 9 corresponds to adding 2 points beyond 7, whilst beyond the threshold of N = 15 it would take the addition of over 100 points to obtain a similar result.

Hence, it would seem justified to raise the number of N from 9 to 15.

6. NUMBER OF EQUIVALENT TESTS

Several problems, including the estimate of mean and characteristic cube strength on the basis of indirect tests, call for an evaluation of the number **m** of repeated indirect tests, which, at a certain strength level, corresponds to a number **n** of tests on cubes.

If the indirect test is a core or microcore test, a solution can be obtained by imposing that the width of the confidence intervals for the means in cube tests be the same as in indirect tests. With the symbols used previously, this entails the equality:

$$t_y \frac{s_y}{\sqrt{\mathbf{n}}} = t_x \frac{s_x}{\sqrt{\mathbf{m}}} \qquad (11)$$

where t_y and t_x represent the quantiles of order $\alpha/2$ of Students variable for **n** - 1 and **m** - 1 degrees of freedom, respectively, whilst s_y and s_x stand for the sample estimates of the standard deviations for cubes and for indirect tests. Knowing the number of indirect tests, **m**, and the s_x/s_y ratio (which must be determined from test data [14]), by means of equation (11) we may obtain the value of $\frac{t_y}{\sqrt{\mathbf{n}}}$.

ISO 2602 provides a table listing the values of this ratio as a function of sample size and in relation to the (1 - α) confidence level. From this table it is possible to determine the value of **n**.

If the indirect test, instead, supplies a value other than the value of strength, as is the case with most non-destructive methods, since we are dealing with a quantity of a different nature we are no longer able to equate the means of the results of the direct and indirect tests nor can we equate their confidence intervals.

At any rate, the latter can be expressed by the following relationships:

$$Y \pm t_y \frac{s_y}{\sqrt{\mathbf{n}}} \qquad (12)$$

for the cube tests, and

$$X \pm t_x \frac{s_x}{\sqrt{\mathbf{m}}} \qquad (13)$$

for the indirect tests.

Then, by dividing equation (12) by Y, *i.e.*, the mean of strength values determined on standard specimens, and exp. (13) by X, *i.e.* the mean of the values obtained from indirect tests, we get, respectively:

$$1 \pm t_y \frac{C_{vy}}{\sqrt{\mathbf{n}}} \qquad (14)$$

and

$$1 \pm t_x \frac{C_{vx}}{\sqrt{\mathbf{m}}} \qquad (15)$$

where C_{vy} and C_{vx} denote the coefficients of variation in direct and indirect tests, which, as is known, are a-dimensional numbers supplying a measure of the variability of the data.

A comparison between equation (14) and equation (15) makes it possible to obtain – from parameters **m** (on which t_x depends) and C_{vx} (*i.e.*, the coefficient of variation of non-destructive tests) – an estimate of $\frac{t_y}{\sqrt{n}}$, and hence the estimated number **n** of tests on standard specimen corresponding to **m**, provided that it proves possible, on the basis of the experimental data, to establish a C_{vx}/C_{vy} ratio.

As a solution to this very problem, ACI Committee 228 proposes [2] the following relationship

$$\mathbf{m} = 2\,(C_{vx}/C_{vy})^2 \qquad (16)$$

which applies in the assumption that the tests on standard specimens are 2. Equation (16) is derived from a more general relationship:

$$\mathbf{m} = \mathbf{n}\,(C_{vx}/C_{vy})^2 \qquad (17)$$

presented in an earlier document [15] by the same Committee. In either case, however, the effects of the terms t^2_y and t^2_x are overlooked, with numerical consequences which may turn out to be non-negligible.

7. CONCLUSIONS

The process leading to the formulation of relationships for the in-place estimate of concrete strength through non-destructive methods, or any kind of indirect methods, calls for the use of statistical methods to process the experimental data to be used as a basis to work out the relationships. It is also necessary to evaluate the type of distribution of the data and, in particular, to determine through tests whether the distributions are normal or not.

When processing the data in order to identify possible correlations between strength as determined on standard specimens, y, and the results of indirect tests, x, it is advisable to work on the mean values, Y and X, because the standard errors of the mean decrease fast with increasing sample sizes, **n** and **m**.

The evaluation of the likely existence of a linear link between Y and X through the correlation coefficient, ρ or r, cannot be limited to the consideration of its numerical value, but has to be performed as a function of the number N of the pairs of X, Y values.

The regression lines, Y vs. X, or, for that matter, any other relationship of a different sort between the two variables, often prove of little practical use if they are not accompanied by the confidence intervals for the regression lines themselves and for the individual observations.

From an analysis of the law of reduction in width of the confidence intervals for the individual observations (which supplies an indicator of the increase in the reliability of the estimates as a function of the number of points used to obtain the regression line), it is possible to propose N = 15 as the recommended number of (X, Y) points to be used.

Finally, a method has been defined which, knowing the number of tests conducted with replicated non-destructive tests, **m**, and the resulting value of quantile t_x, makes its possible to work out, from a comparison of the confidence intervals for the Y and X means, an estimate of the equivalent number of replicated tests, **n**, conducted on standard specimens.

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