BULLETIN OF
MATHEMATICAL BIOPHYSICS
VOLUME 2, 1940

FURTHER CONTRIBUTIONS TO THE THEORY OF CELL POLARITY AND SELF-REGULATION

N. RASHEVSKY
THE UNIVERSITY OF CHICAGO

In continuation of a previous study a case of self-regulating polarity is investigated, in which a metabolite neither enters nor leaves the cell, being produced in one part of it and consumed in another. Such a cell possesses a larger degree of autonomy and independence of external conditions than the previously discussed one. Application to cellular locomotion is indicated.

This study is a continuation of a previous paper (Rashevsky, 1940; hereinafter referred to as *loc. cit.*).

Consider a spherical cell of radius r_0, inside of which a substance is produced at a rate of q gm cm^{-3} sec^{-1}. Let the same substance break down and therefore disappear at a rate proportional to its concentration c, and given by $-bc$ gm cm^{-3} sec^{-1}.

Let the membrane of the cell be completely impermeable to that substance, so that the latter neither enters nor leaves the cell. Under these conditions a stationary state is approached, in which the rate of production q of the substance is equal to its rate of consumption bc. This gives $c = q/b$. The concentration is uniform throughout the cell.

Consider now, as in the previous paper (*loc. cit.*) that the cell contains colloidal particles, which act as an inhibitory catalyst upon the rate of formation q of the substance. Denoting by n the concentration of the colloidal catalyst, we shall again use equation (24) of *loc. cit.* For a constant c, n is also constant throughout the cell.

Let now, due to any disturbance, a small increase in n arise in some region of the cell, producing thus an asymmetry. In this region q will decrease and therefore c will decrease. This will result in a negative gradient of c in the direction of that region and therefore in an accumulation of colloidal particles in that region with a resulting further decrease of c. As in *loc. cit.* we thus will find an asymmetry or polarity automatically established and maintained. The conditions for stability are the same as before.

The calculations proceed very much as in *loc. cit.* Denoting by the subscript 1 the quantities referring to one hemisphere and by the subscript 2 those referring to the other hemisphere and remembering that in the stationary state the total amounts produced in one hemisphere equal the total amount consumed, plus the total amount diffusing into the other hemisphere, we have

$$\begin{aligned} &\tfrac{2}{3}\pi r_0^3 q_1 - \tfrac{2}{3}\pi r_0^3 b c_1 - \pi r_0 D(c_1 - c_2) = 0\,, \\ &\tfrac{2}{3}\pi r_0^3 q_2 - \tfrac{2}{3}\pi r_0^3 b c_2 + \pi r_0 D(c_1 - c_2) = 0\,. \end{aligned} \tag{1}$$

Solving for c_1 and c_2 and calculating the quantity $c_1 - c_2$, we find:

$$c_1 - c_2 = \frac{r_0^2}{r_0^2 b + 3D}(q_1 - q_2)\,. \tag{2}$$

For the relation between the quantity $x = c_1 - c_2$ and $n_2 - n_1$ we have equation (23) of *loc. cit.* In its dependence on $q_1 - q_2$ equation (2) is similar to equation (18) of *loc. cit.* Hence, putting

$$\frac{r_0^2}{r_0^2 b + 3D} = A \tag{3}$$

we shall have for $x = c_1 - c_2$ formally the same expressions as those obtained previously.

In view of the assumed impermeability of the cell membrane to the substance in question, such a cell possesses an autonomy and independence of external conditions to a much greater degree than before.

An interesting consequence may be considered here. Let the membrane, while impermeable to the metabolite, be freely permeable to water. The osmotic pressure due to the concentration of the metabolite and directed outward, will be greater at the pole, at which the concentration is greater. Therefore a net resultant force in the direction of that pole will be present, and the cell in that case will move forward with the pole of higher concentration. At first sight this conclusion sounds like a paradox, for we seem to have a movement due to internal forces. This however is not so, since inside the cell the substance flowing from one pole to another exerts a diffusion drag (Young, 1939) upon the water, which even inside the cell remains stationary, due to the assumed complete permeability of the cell membrane to water.

Somewhat similar cases of spontaneous locomotion have been studied by Gale Young (unpublished) for the case of an asymmetric flow of a substance from the cell. In that case the mechanism of locomotion is much like that of a rocket. In the present case the mechanism may be to some extent compared with a sort of caterpillar drive. Such mechanisms may well be responsible in some cases for spontaneous locomotion of unicellular organisms.

The author is indebted to Mr. H. D. Landahl for checking the algebra.

This work has been aided in part by a grant from Dr. Wallace C. and Clara A. Abbot Memorial Fund of the University of Chicago.

LITERATURE

Rashevsky, N. 1940. "An Approach to the Mathematical Biophysics of Self-Regulation and of Cell Polarity." *Bull. Math. Biophysics*, 2, 15-25.

Young, Gale. 1938. "Theory of Diffusion Forces in Metabolizing Systems." *Growth*, 2, 165-180.