Dielectric constant and resistivity of epidermal stratum corneum*

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Abstract—*It is important for an understanding of skin impedance that the formative mechanism of the stratum-corneum impedance is analysed. The structural features of the stratum corneum are found in the inhomogeneous and the laminate structure of dead cells. (i) The mathematical expressions for skin impedance have been determined. Both relaxations due to the cellular inhomogeneous structure and the morphological laminate structure are estimated from the complex dielectric constant and integral operation, respectively. (ii) The dielectric constant of the stratum corneum decreases at first and then increases exponentially towards the deeper tissues. Conversely, the resistivity decreases in an exponential fashion. These values depend only on the relaxation due to the inhomogeneous structure. (iii) The model of the skin impedance in which the dielectric component is homogeneous and only the resistive component has the laminate structure, can hardly explain the frequency dispersions of the dielectric constant and the resistivity.*

Keywords—*Dielectric constant and resistivity of skin, Electrical properties of skin, Skin, Skin impedance*

Introduction

THE EFFECT of the epidermal stratum corneum, namely the keratin layers, on the skin impedance is very large, and hence the skin impedance consists mainly of the impedance of the stratum corneum. It is important for the understanding of the skin impedance that the formative mechanism of the stratum corneum impedance is analysed. The structural and morphological features of the stratum corneum are found in the layers of dead cells. As for the relationships obtained between skin impedances and laminations, the decrease of the skin impedance by removing the stratum corneum (LAWLER *et al.*, 1960; TREGEAR, 1966; YAMAMOTO and YAMAMOTO, 1976) and the equivalent circuit of the skin impedance (TREGEAR, 1966) related to the laminations have been previously reported. However, up to the present, the dielectric constants and the resistivities of the keratin layers and the mathematical expression of the skin impedance related to the laminations have not yet been obtained.

The present paper reports the practical values of the dielectric constants and the resistivities of the keratin layers, and then gives the theoretical expressions for the skin impedance depending on relaxation phenomena both from the cellular and the morphological standpoint.

Skin impedance, average dielectric constant and average resistivity

The experimental values for the skin impedance used were those of a previous paper (YAMAMOTO and YAMAMOTO, 1976). Beckman Ag–AgCl electrodes (disc, 9 mm diameter) were placed on the central side of the forearm with a fixed distance of 50 mm between their centres, and Beckman electrode paste was used. Measurements were made with an alternating current bridge. The polarisation impedance of the electrodes and the stray impedance of the lead wires were measured individually, so that their influence on the skin impedance could be eliminated. After applying the electrodes to the skin, a stabilisation period of about 30 min was allowed before measurements were taken. The removal of the stratum corneum was carried out stepwise by cellulose-tape stripping. The total number of strippings was 15. The skin impedance was measured before the initial stripping and at intervals of three strippings, in all, six times. The differences between skin impedances for successive measurements are shown in Fig. 1 and considered as the impedance loci of the stratum corneum removed by three strippings. For reference, perfect circular arcs are shown in Fig. 1 as the dotted lines, from which it is seen that the experimental loci (full lines) are significantly distorted. Next it is assumed that the thickness d of the whole keratin layer is 40 μm, so that the average specific dielectric constant $\bar{\varepsilon}_k$ and the average resistivity $\bar{\rho}_k$ of the stratum corneum are obtained as follows:

$$2R_k = \bar{\rho}_k \frac{2d}{S},$$

$$\frac{C_k}{2} = \bar{\varepsilon}_k \varepsilon_r \frac{S}{2d} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

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where $S = 63{\cdot}6 \times 10^{-6}$ m^2 is the area of an electrode, ε_r is the dielectric constant of vacuum, and $2R_k$, $C_k/2$ represent the parallel impedance components of the whole keratin layers for two electrodes. Fig. 2 shows the experimental values of $\bar{\varepsilon}_k$ and $\bar{\rho}_k$, from which the frequency dependences, namely the frequency dispersions, can be determined.

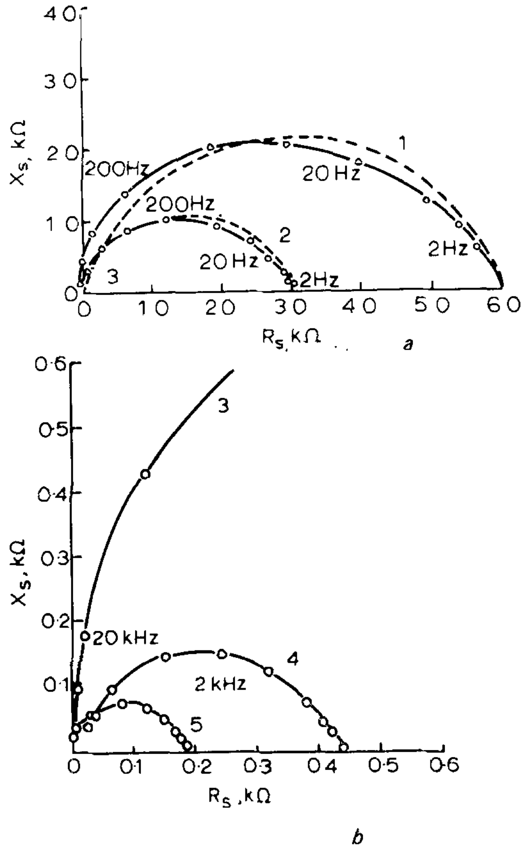

Fig. 1 Series impedance loci of the removed layers calculated from differences between skin impedances of successive measurements
Curve 1 gives the series-impedance locus of layers removed by first three strippings Curve 2 by next three strippings etc.
(b) is a magnified version of (a)

Skin impedance taking into consideration the structure of the stratum corneum

The horny cells are being continuously worn, scraped, or rubbed off. The stratum-corneum cells must then be replaced by a multiplication of the underlying epidermal cells. Epidermal regeneration depends on cellular division in the basal layer. The dividing epidermal cells migrate, or are displaced, towards the skin surface. Because of the morphological variation due to the epidermal flow, horny cells appear in the laminate structure in which flat extinct cells are piled up. To simplify the description, this particular structure is called the laminate structure, from the morphological standpoint, in this paper. The stratum corneum also has a microscopic inhomogeneous structure formed by cell membranes and, for simplification, this structure is called the inhomogeneous structure from the cellular standpoint. The laminate morphological structure gradually varies from the skin surface towards deep tissues, and is an inhomogeneous structure from the cellular standpoint. Since the laminate characteristic is clearly evident in the skin impedance, as shown in Fig. 1, it is considered that the electrical parameters (i.e. the dielectric constant and the resistivity) of the keratin layers vary gradually towards the deeper tissues. Therefore, the Maxwell-Wagner theory of the multilayer configuration of dielectrics can be applied here (OKA and NAKATA, 1962).

To obtain the electrical parameters, the dielectric relaxation obtained from the cellular inhomogeneous structure is considered first. It is well known that the dielectric properties of a biological material having an inhomogeneous structure can be given in terms of the complex dielectric constant ε^* as (SCHWAN, 1957)

$$\varepsilon^* = \varepsilon - j\varepsilon'' = \varepsilon_\infty + (\varepsilon_0 - \varepsilon_\infty)\int \frac{f(T)}{1 + j\omega T}\, dT \quad . \qquad (2)$$

where $\varepsilon_\infty = \lim_{\omega \to \infty} \varepsilon^*$, $\varepsilon_0 = \lim_{\omega \to 0} \varepsilon^*$, T is the relaxation time and the function $f(T)$ characterises the distribution of T. Because of this relaxation the frequency dispersions of ε and ε^*, and then the large dielectric constant appear (SCHWAN, 1957). ε is the effective dielectric constant. If the morphological laminate structure is neglected, the impedance Z_k of the stratum corneum (area S, thickness d) can be calculated as follows:

$$Z_k = \frac{V}{I} = \frac{Ed}{iS} \quad i = \frac{E}{\rho_0} + j\omega D \quad D = \varepsilon^* E$$

$$Z_k = \frac{1}{(1/\rho_0 + j\omega\varepsilon^*)S/d} = \frac{\rho}{(1 + j\omega\varepsilon\rho)S/d}$$

$$= \frac{\rho - j\omega\varepsilon\rho^2}{1 + (\omega\varepsilon\rho)^2}\left(\frac{d}{S}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (3)$$

where

V = voltage
I = current
D = electric flux density
E = intensity of electric field
i = current density

$$\rho = \frac{\rho_0/\omega\varepsilon''}{\rho_0 + (1/\omega\varepsilon'')} = \text{effective resistivity}$$

Here the cellular structure is accounted for by ε^* and the effect on the frequency characteristic of the impedance can be determined. ρ_0 is the resistivity due to the ionic conduction and is independent of the frequency. $(1/\omega\varepsilon'')d/S$ and $(1/\omega\varepsilon)d/S$, respectively, become the parallel resistance and the parallel reactance components of the polarisation impedance due to the relaxation and vary with frequency.

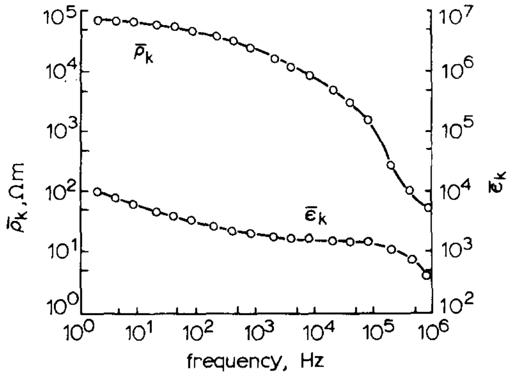

Fig. 2 Average dielectric constant and resistivity of stratum corneum

Next the skin impedance is deduced with consideration for the morphological laminate structure. The distance x is towards the deep tissues from the surface; it is assumed that the parameters ε and ρ vary with x. The skin impedance Z for a pair of electrodes, by manipulation of eqn. 3, becomes

$$Z = Z_c + \int_0^d \frac{2\rho(x)}{1+j\omega\varepsilon(x)\rho(x)} \cdot \frac{dx}{S}$$

$$= Z_c + \int_0^d 2y_R(x)dx - j\int_0^d 2y_X(x)dx \quad . \quad . \quad (4)$$

where

$$y_R(x) = \frac{\rho(x)/S}{1+\{\omega\varepsilon(x)\rho(x)\}^2},$$

$$y_X(x) = \frac{\omega\varepsilon(x)\{\rho(x)\}^2/S}{1+\{\omega\varepsilon(x)\rho(x)\}^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

Z_c is the skin impedance after the complete stripping of the keratin layers, and is the impedance of the deep tissues containing the granular layer. Then, $\lim_{\omega\to\infty} Z_c = Z_{c_\infty} \doteqdot 170\,\Omega$, and $\lim_{\omega\to 0} Z_c = Z_{c0} \doteqdot 400\,\Omega$.

$$Z_0 = \lim_{\omega\to 0} Z = Z_{c0} + \int_0^d \frac{2\rho_0(x)}{S}dx \quad . \quad . \quad (6)$$

where $\lim_{\omega\to 0} \rho(x) = \rho_0(x)$.

Then $Z_\infty = \lim_{\omega\to\infty} Z = Z_{c_\infty}$. The normalised skin impedance $P(\omega)$ is defined as

$$P(\omega) = \frac{Z-Z_\infty}{Z_0-Z_\infty} = \int_0^d \frac{\rho(x)dx}{1+j\omega\varepsilon(x)\rho(x)} \Bigg/ \int_0^d \rho_0(x)dx \quad . \quad . \quad . \quad . \quad . \quad (7)$$

Eqn. 8 is used in the derivation of eqn. 7

$$\left.\begin{aligned} (Z_{c0}-Z_{c_\infty}) &\ll \int_0^d \frac{2\rho_0(x)}{S}dx \\ (Z_c-Z_{c_\infty}) &\ll \int_0^d \frac{2\rho(x)}{\{1+j\omega\varepsilon(x)\rho(x)\}S}dx \end{aligned}\right\} \quad . \quad . \quad (8)$$

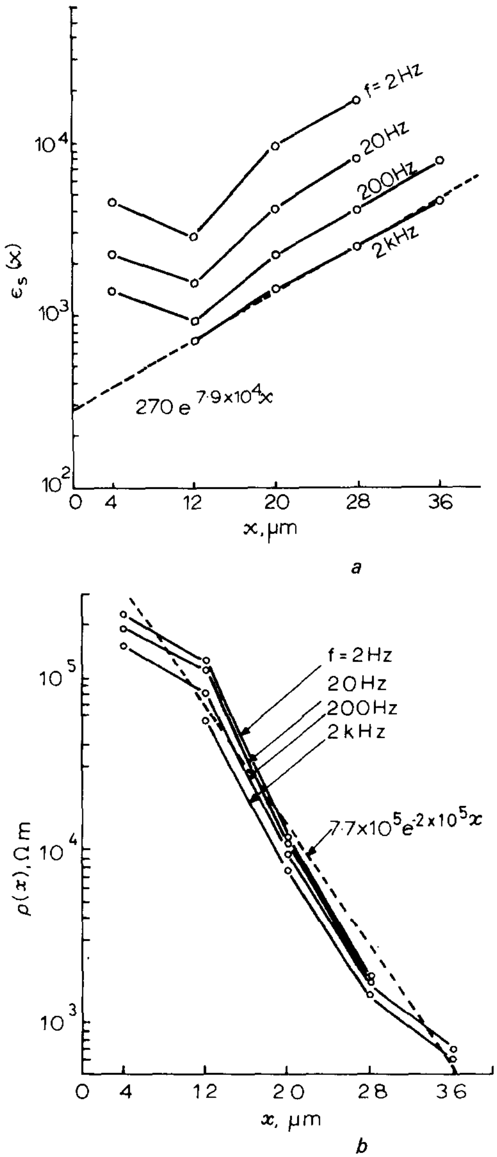

Fig. 3 Effective specific dielectric constant (a) and effective resistivity (b) at every point of stratum corneum

Since Z_c, the impedance of the deep tissues, is very small, eqn. 8 is satisfied. In general, it is well known that $P(\omega)$ can be described by the distribution function $G(\tau)$ of relaxation time τ.

$$P(\omega) = \int_0^\infty \frac{G(\tau)}{1+j\omega\tau} d\tau, \quad \int_0^\infty G(\tau) d\tau = 1 \quad . \quad (9)$$

Eqn. 7 can be transformed as

$$P(\omega) = \int_{\tau_0}^{\tau_d} \frac{\rho(h(\tau))}{1+j\omega\tau}\left(\frac{dx}{d\tau}\right) d\tau \Bigg/ \int_0^d \rho_0(x) dx \quad . \quad (10)$$

(see Appendix I) where

$$\tau = \varepsilon(x)\rho(x) = \phi(x), \quad x = h(\tau) \quad . \quad . \quad . \quad (11)$$

The new variable τ is the relaxation time denoted by eqn. 11, then we obtain $\tau_0 = \phi(0) = \varepsilon(0)\rho(0)$ and $\tau_d = \phi(d) = \varepsilon(d)\rho(d)$. The inverse function of ϕ is h. Eqns. 4, 7 and 10 are formulas for the skin impedance based on the Maxwell–Wagner theory. The relaxation phenomena from the cellular and the morphological standpoint are simultaneously estimated from the complex dielectric constant and integral operation, respectively. From experimental data of the skin impedances, $\varepsilon(x)$ and $\rho(x)$ are obtained using these equations as shown later.

Experimental data for the specific dielectric constant and resistivity of the stratum corneum

The formative mechanism of $\bar{\varepsilon}_k$ and $\bar{\rho}_k$ in Fig. 2 is given by the following equation

$$Z_k = Z - Z_c = \int_0^d \frac{2\rho(x)}{1+j\omega\varepsilon(x)\rho(x)} \cdot \frac{dx}{S}$$

$$= \frac{2\bar{\rho}_k}{1+j\omega\bar{\varepsilon}_k \varepsilon_r \bar{\rho}_k} \cdot \frac{d}{S} \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

As mentioned above, the effects of the inhomogeneous and the laminate structure are estimated from the complex dielectric constant and integral respectively. Then it becomes evident that the frequency dispersions of $\bar{\varepsilon}_k$ and $\bar{\rho}_k$ depend on the inhomogeneous and the laminate structure.

Next, the stratum corneum is divided into equal thicknesses denoted by Δd. The average values of $y_R(x)$, $y_X(x)$ in the ith layer become

$$\bar{y}_{Ri} = \frac{1}{\Delta d} \int_{(i-1)\Delta d}^{i\Delta d} y_R(x) dx,$$

$$\bar{y}_{Xi} = \frac{1}{\Delta d} \int_{(i-1)\Delta d}^{i\Delta d} y_X(x) dx \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (13)$$

Substituting eqn. 13 for eqn. 5, we solve the equations for the dielectric constant and the resistivity, and then obtain the average effective dielectric constant $\bar{\varepsilon}_i \varepsilon_r$ and the average effective resistivity $\bar{\rho}_i$ of the ith layer as

$$\bar{\varepsilon}_i \varepsilon_r = \frac{\bar{y}_{Xi}}{\omega S(\bar{y}_{Ri}^2 + \bar{y}_{Xi}^2)}, \quad \bar{\rho}_i = \frac{(\bar{y}_{Ri}^2 + \bar{y}_{Xi}^2)S}{\bar{y}_{Ri}} \quad . \quad (14)$$

The impedance Z_i per pair of electrodes is shown as

$$Z_i = (\bar{y}_{Ri} - j\bar{y}_{Xi})2\Delta d = \frac{2}{1/R_i + j\omega C_i} \quad . \quad . \quad (15)$$

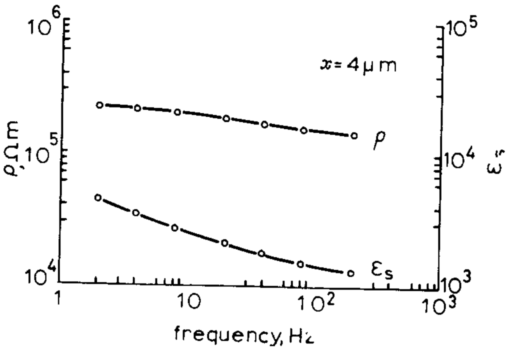

Fig. 4 Frequency characteristics of dielectric constant and resistivity at $x = 4\,\mu m$

where R_i and C_i are the parallel resistance and capacitance of this layer. Finally, from eqns. 14 and 15 we obtain

$$\bar{\varepsilon}_i \varepsilon_r = C_i \frac{\Delta d}{S}, \quad \bar{\rho}_i = R_i \frac{S}{\Delta d} \quad . \quad . \quad . \quad (16)$$

Thus, $\bar{\varepsilon}_i$ and $\bar{\rho}_i$ are simply calculated from eqn. 16 when the experimental values of C_i and R_i are substituted. If the stratum corneum is divided into enough multilayers, it is assumed that the average values $\bar{y}_{Ri}$, $\bar{y}_{Xi}$ approach these values at the centre $x = i\Delta d - (\Delta d/2)$ of the ith interval, i.e.

$$\bar{y}_{Ri} \doteqdot \{y_R(x)\}_{x = i\Delta d - (\Delta d/2)}$$

$$\bar{y}_{Xi} \doteqdot \{y_X(x)\}_{x = i\Delta d - (\Delta d/2)} \quad . \quad . \quad . \quad . \quad . \quad (17)$$

Therefore, $\bar{\varepsilon}_i \varepsilon_r$ and $\bar{\rho}_i$ are also nearly equal to the effective dielectric constant and resistivity at the centre of the interval. The effects of the relaxation due to the laminate structure for $\bar{\varepsilon}_i \varepsilon$ and $\bar{\rho}_i$ become small because the ith layer is sufficiently thin and these effects are neglected. Fig. 3 shows the effective specific dielectric constant $\varepsilon_s(x) = \varepsilon(x)/\varepsilon_r$ and the effective resistivity $\rho(x)$. As the data in Fig. 1 were used, the stratum corneum was divided into five layers, i.e. $\Delta d = 8\,\mu m$. ε_s in Fig. 3*a* decreases at first and then increases exponentially. For reference, the dotted line in Fig. 3a shows the function $270 \exp(7{\cdot}9 \times 10^4 x)$. The effective resistivity in Fig. 3b decreases exponentially and the curve of 2 Hz is approximately expressed by the function $7{\cdot}7 \times 10^5 \exp(-2 \times 10^5 x)$ (dotted line) which was obtained in the previous paper (YAMAMOTO and YAMAMOTO, 1976). The laminate characteristics of ε_s and ρ can be read from Fig. 3. The division into five layers does not provide enough thickness per layer, but it is likely that ε_s and ρ are almost determined by only the cellular inhomogeneous structure at each point $x = (i\Delta d - (\Delta d/2))_{i=1,2,\cdots,5}$. From the comparison of $\bar{\varepsilon}_k$ in Fig. 2 with ε_s in Fig. 3, it seems that the large specific dielectric constant is not obtained from the morphological laminate structure estimated from the integral. Fig. 4, obtained by rearrangement of Fig. 3, shows the frequency characteristics of ε_s and ρ at $x = 4\ \mu$m. A feature of the frequency dispersion of the impedance obtained from ε_s and ρ in Fig. 4 is that the impedance locus is markedly distorted from the circular arc mentioned above.

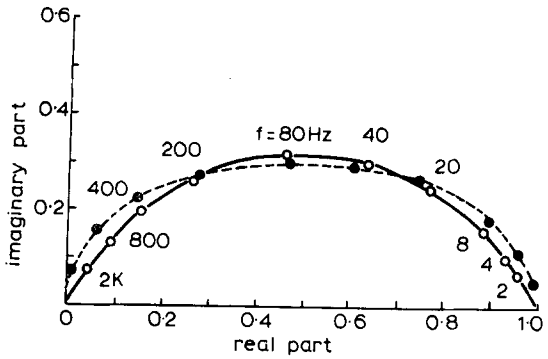

Fig. 5 Normalised impedance loci of P(ω) (—o—) and $P_r(\omega)$ (...o...)

Approximation of skin impedance by rectangular distribution function

As a special case, it is assumed that the dielectric property is homogeneous and only the resistive property has the laminate effect. Therefore,

$$\varepsilon(x) = \varepsilon = \text{constant}, \quad \rho(x) = \frac{A}{a+x} \quad . \quad . \quad (18)$$

where the unknown parameters, ε, A and a, do not contain x and ω and are determined below. From eqn. 18 we obtain

$$\tau = \varepsilon\rho = A\varepsilon/(a+x)$$

therefore

$$\tau_0 = A\varepsilon/a, \quad \tau_d = A\varepsilon/(a+d) \quad . \quad . \quad . \quad . \quad (19)$$

Thus, eqn. 10 becomes

$$P(\omega) = \int_{\tau_0}^{\tau_d} \frac{1}{1+j\omega\tau} \cdot \frac{S}{\tau} \cdot d\tau \Big/ \int_0^d \rho_0(x)dx \quad . \quad (20)$$

In the comparison of eqn. 20 with eqn. 9, we obtain

$$\tau G(\tau) = \frac{A}{\int_0^d \rho_0\, dx} = \text{constant} \quad . \quad . \quad . \quad (21)$$

$$A = \int_0^d \rho_0\, dx / \ln\frac{\tau_d}{\tau_0} = \frac{(Z_0 - Z_{c0})S}{2\ln(\tau_d/\tau_0)} \quad . \quad . \quad (22)$$

The distribution function shown by eqn. 21 is well known as a rectangular distribution. In this case the normalised skin impedance $P_r(\omega)$ becomes

$$P_r(\omega) = \int_{\tau_0}^{\tau_d} \frac{1/\ln(\tau_d/\tau_0)}{1+j\omega\tau} \frac{d\tau}{\tau}$$

$$= \frac{1}{\ln(\tau_d/\tau_0)}\left[\ln\left\{\frac{\omega\tau_d}{\sqrt{1+(\omega\tau_d)^2}}\Big/\frac{\omega\tau_0}{\sqrt{1+(\omega\tau_0)^2}}\right\} - j\tan^{-1}\frac{\omega(\tau_d-\tau_0)}{1+\omega^2\tau_0\tau_d}\right] \quad . \quad . \quad . \quad . \quad (23)$$

Given $\tau_m = \sqrt{\tau_0\tau_d}$ (τ_m = central relaxation time) and the ratio of τ_d/τ_0 we can calculate $P_r(\omega)$ in eqn. 23. In particular, if τ_m and τ_d/τ_0 are suitably chosen, the normalised impedance in the Cole–Cole distribution, i.e. $P(\omega)$ in eqn. 24, can be approximated by $P_r(\omega)$ in eqn. 23.

$$P(\omega) = \frac{1+(\omega\tau_m)^\beta\cos\frac{\beta\pi}{2} - j(\omega\tau_m)^\beta\sin\frac{\beta\pi}{2}}{1+(\omega\tau_m)^{2\beta}+2(\omega\tau_m)^\beta\cos\frac{\beta\pi}{2}} \quad (24)$$

Given τ_m and β, we can calculate eqn. 24. Thus, a condition for the optimum approximation $\{P(\omega) \doteqdot P_r(\omega)\}$ is considered as the relationship between β and τ_d/τ_0 (see Appendix II). For instance, from the skin impedance locus before strippings, values of $\beta = 0{\cdot}711$ and $\tau_m = 2{\cdot}27 \times 10^{-3}$ s were found, so that the optimum rectangular distribution was obtained as $\tau_d/\tau_0 = 105$ and $\tau_m = \sqrt{\tau_0\tau_d} = 2{\cdot}27 \times 10^{-3}$ s. From these values, $P_r(\omega)$ and $P(\omega)$ in Fig. 5 are obtained using eqns. 23 and 24 and $P(\omega)$ could be almost approximated by $P_r(\omega)$. Furthermore, from the values of τ_m and τ_d/τ_0, and eqns. 19 and 22, the parameters were determined as follows:

$$\varepsilon_s = \varepsilon/\varepsilon_r = 1{\cdot}56 \times 10^3, \quad A = 0{\cdot}649\,\Omega\,\text{m}^2$$

$$a = 0{\cdot}385 \times 10^{-6}\ \text{m}$$

where

$S = 63{\cdot}6 \times 10^{-6}\,\mathrm{m}^2$

$d = 40 \times 10^{-6}\,\mathrm{m}, \quad Z_0 - Z_{c0} = 95\,\mathrm{k}\Omega$

ρ becomes $1{\cdot}69 \times 10^6\,\Omega\,\mathrm{m}$ at $x = 0$ and by comparison with the value $7{\cdot}7 \times 10^5\,\Omega\,\mathrm{m}$ obtained in the previous paper (YAMAMOTO and YAMAMOTO, 1976) this value is not unrealistic. On the other hand, $\varepsilon_s = 1{\cdot}56 \times 10^3$ is much larger than the specific dielectric constant of water, and except for the relaxation due to the cellular inhomogeneous structure, this magnitude can hardly be explained. In general, the dispersions in which the dielectric constant and the resistivity exhibit a frequency dependence, arise from the relaxation. The constant value for the dielectric constant, and therefore the present model (i.e. eqn. 18) of the stratum corneum, are unsuitable. From this standpoint, the equivalent circuit of the skin impedance, where it is considered that the dielectric component is homogeneous and only the resistive component has the laminate structure, as previously reported by TREGEAR (1966), is not sufficiently accurate.

Conclusion

The present paper analyses the formative mechanism of the skin impedance due to the structural and morphological particularity of the stratum corneum, and describes a study of the electrical properties of the skin. The essential results of this paper are as follows:

(1) Mathematical expressions for the skin impedance, in which both relaxations due to the inhomogeneous structure from the cellular standpoint and the laminate structure from the morphological standpoint were considered, have been deduced and can be applied to the analysis of the electrical properties of the stratum corneum. The inhomogeneous structure (from the cellular standpoint) and the laminate structure (from the morphological standpoint) were instantaneously estimated from the complex dielectric constant and integral operation respectively.

(2) The experimental values for the effective specific dielectric constant ε_s and the effective resistivity ρ of the stratum corneum were shown. ε_s decreases at first and then increases exponentially towards the deeper tissues away from the surface. Conversely, ρ decreases exponentially. These values are approximately dependent on the relaxation due only to the inhomogeneous structure. The skin impedance locus obtained from the relaxation due to the inhomogeneous structure is significantly distorted from the circular arc. On the other hand, the average dielectric constant and resistivity of whole keratin layers depend on relaxation due to both the inhomogeneous and the laminate structure.

(3) If it is assumed that the dielectric property is homogeneous and only the resistive property has the laminate effect, the Cole–Cole circular arc can be approximated by a simple rectangular distribution. Since the frequency dependences of the dielectric constant and the resistivity, however, can hardly be explained, the model with the rectangular distribution is unsuitable.

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Appendix I: Transformation of the integral variable

If τ is the function of x, i.e. $\tau = \phi(x)$, its inverse function is denoted by $x = h(\tau)$, where h is the inverse function of ϕ. Thus,

$$\int_0^d f(x)dx = \int_{\tau 0}^{\tau d} f(h(\tau))dh(\tau) = \int_{\tau 0}^{\tau d} k(\tau)dh(\tau)$$

where $\tau_0 = \phi(0)$, $\tau_d = \phi(d)$.

The final equation is called the Stieltjes integral on $h(\tau)$ of $k(\tau)$. (The Mathematical Society of Japan, 1968). This integral is transformed in the ordinary Riemann integral as:

$$\int_{\tau 0}^{\tau d} k(\tau)dh(\tau) = \int_{\tau 0}^{\tau d} k(\tau)\left(\frac{dh(\tau)}{d\tau}\right)d\tau$$

Appendix II: Optimum approximation of Cole-Cole circular arc by means of a rectangular distribution

The frequency points are suitably chosen as about 30 points on both the loci of normalised impedances $P(\omega)$ and $P_r(\omega)$. At the same frequency point ω_i the distance $\delta(\omega_i) = |P(\omega_i) - P_r(\omega_i)|$ is calculated. Then the optimum ratio of τ_d/τ_0 is determined so as to obtain the least square value, i.e. the minimum of $S = \sum_{i=1}^{30} \delta^2(\omega_i)$. From this method, only one value of τ_d/τ_0 (i.e. the optimum approximation) is obtained for one value of β.

References

LAWLER, J. C., DAVIS, M. J. and GRIFFITH, E. C. (1960) Electrical characteristics of the skin. *J. Invest. Dermatol.* **34,** 301–308.

OKA, S. and NAKATA, O. (1962) *Theory of dielectric materials in solid state.* Iwanami Shoten, Tokyo, p. 80 (in Japanese).

SCHWAN, H. P. (1957) Electrical properties of tissues and cell suspensions. *Advances in biological and medical physics*, Academic Press, New York, 150, 154, 174, 187.

The Mathematical Society of Japan (1968) *Mathematical dictionary*, Iwanami Shoten, Tokyo, 622 (in Japanese).

TREGEAR, R. T. (1966) *Physical functions of skin.* Academic Press, London, 64, 68.

YAMAMOTO, T. and YAMAMOTO, Y. (1976) Electrical properties of epidermal stratum corneum. *Med. Biol. Eng.* **14,** 151–158.

Constante diélectrique et résistivité de la couche cornée epidermique

Sommaire—Pour comprendre l'impédance de la peau il est important d'analyser le mécanisme formateur de l'impédance de la couche cornée. Les caractéristiques structurales de la couche cornée se trouvent dans la structure hétérogène et lamelleuse des cellules mortes. 1) On a déterminé les expressions mathématiques de l'impédance de la peau. Les deux relâchements dus à la structure hétérogène et à la structure lamelleuse cellulaire sont estimés à partir de la constante diélectrique complexe et de l'opération intégrale respectivement. 2) La constante diélectrique de la couche cornée diminue d'abord; ensuite elle augmente de façon exponentielle vers les tissus plus profonds de la peau. Inversement la résistivité décroît de façon exponentielle. Ces valeurs dépendent uniquement du relâchement dû à la structure hétérogène. 3) Le modèle d'impédance de la peau où le composant diélectrique est homogène et seul le composant résistif a une structure à lamelles expliquent à peine la dispersion des fréquences de la constante diélectrique et de la résistivité.

Dielektrizitätskonstante und spezifischer Widerstand der Hornschicht

Zusammenfassung—Zum Verständnis der Hautimpedanz ist es wichtig, daß der formative Mechanismus der Hornschichtimpedanz analysiert wird. Die strukturellen Merkmale der Hornschicht werden in der inhomogenen und laminaren Struktur abgestorbener Zellen aufgewiesen. (1) Die mathematischen Formeln der Hautimpedanz wurden festgelegt. Sowohl Relaxationen infolge der zellular inhomogenen Struktur als auch infolge der morphologisch laminaren Struktur können anhand der komplexen Dielektrizitätskonstanten bzw. Integraloperation geschätzt werden. (2) Die Dielektrizitätskonstante der Hornschicht nimmt zuerst ab und anschließend in Richtung der tieferleigenden Gewebe von der Hautoberfläche aus exponentiell zu. Der spezifische Widerstand fällt jedoch umgekehrt auf exponentielle Weise ab. Diese Werte werden nur durch die Relaxation infolge der inhomogenen Struktur bedingt. (3) Das Vorbild einer Hautimpedanz, bei welcher die Dielektrizitätskomponente homogener Art ist und nur die ohmsche Komponente eine laminare Struktur besitzt, kann kaum die Frequenzstreuungen der dielektrischen Konstante und den spezifischen Widerstand erklären.