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Scale effect on concrete in tension

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Scale effects in the mechanical behaviour of concrete are important phenomena and are the object of many studies internationally. The Laboratoire Central des Ponts et Chaussées has been working for several years on the scale effect on the distribution function of the tensile strength of concrete. Very many direct tensile tests have been performed on specimens of different volumes, made of concretes having different formulations. From these experimental investigations has been derived a general law that can be used to determine the distribution function (mean and standard deviation) of the tensile strength of the concrete versus the volume of the tensile specimen, for concretes having compressive strengths between 30 MPa and 130 MPa, from knowledge of just the compressive strength obtained on a standardized cylinder (in France) and the ratio of the volume of the tensile specimen to the volume of the coarsest aggregate in the concrete. It should be emphasized that this law is valid only if, on the one hand, the mode of preservation of the direct tensile specimens is identical to that of the compressive specimens and, on the other, the aggregates used in the concretes are common aggregates, which eliminates light aggregates and very hard aggregates with a high Young's modulus.

1. INTRODUCTION

For several years, the Laboratoire Central des Ponts et Chaussées (LCPC) has been developing a numerical model of the cracking of concrete based on a probabilistic approach [1–3]. The main parameter that must be known [1, 2] and incorporated in this model is the distribution function of the tensile strength, which depends on the size of the finite elements of the mesh used. Since, in the model, it is assumed that there is equivalence between the finite element of the mesh and a volume of material, the distribution function of the tensile strength, which depends on the size of the finite elements, can be obtained by performing a large number of direct tensile tests on specimens of different volumes. Although there has been some international research on problems of the scale effect in the cracking of concrete [4, 5], there has not, to our knowledge, been much investigation of the scale effect on the direct tensile strength of concrete. A first study was performed on this subject at the LCPC in 1985 but published only as an internal report. It consisted of the direct tensile testing of cast cylindrical specimens of different diameters and a slenderness ratio of 2. Although highly instructive, this study had two main limitations. (i) The cast specimens, of very small diameter, had wall effect problems (grains too coarse with respect to the diameter of the specimens) that completely perturbed the results, whereas with the specimens of larger diameter there were scale effects that were not sufficiently marked, although significant. (ii) Only one composition of concrete was studied, which made it impossible to draw general conclusions concerning these scale effects. In 1991, it was decided, therefore, to conduct a program of tests that differed in two ways from the previous study: first, the specimens were cored, not cast, and second, three very different formulations of concrete were studied. The results of these two research campaigns are presented here.

2. EXPERIMENTAL INVESTIGATION OF THE DIRECT TENSILE BEHAVIOUR OF CONCRETE

2.1 First study

2.1.1 *Experimental conditions*

In this study, five diameters of cylindrical specimens of slenderness ratio 2 were tested: 20, 36, 89, 113, and 160 mm. As already reported, the specimens were cast. The composition of the concrete studied is given in Table 1. After form removal, the specimens were coated with several layers of resin to prevent drying and so achieve a homogeneous internal moisture content. The tests were performed between 28 and 40 days after the production of the specimens, on a Zwick tensile press having a capacity of 10 tonnes, with a controlled loading

Table 1 Composition of the concrete of the first study, performed in 1985 (per m^3)

Constituents	Weight in kg
Rolled silico-calcareous gravel, 4/12 mm	1100
Rolled silico-calcareous sand, 0/5 mm	700
OPS cement	350
Water	200

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Table 2 Results for the tensile tests of the first study

Specimen diameter (mm)	89	113	160
Number of values	46	54	49
Mean tensile strength (MPa)	3.0	2.8	2.5
Standard deviation (MPa)	0.3	0.3	0.2
Mean compressive strength (MPa)		47.5	

Table 3 Composition of concrete (kg m^{-3})

	Concrete 1	Concrete 2	Concrete 3
Constituent			
Crushed silico-calcareous gravel, 5/20 mm	1114	1236	
Calcareous gravel 4/20 mm			1265
Silico-calcareous sand, 0/4 mm	774		652
Silico-calcareous sand, 0/5 mm		667	
Cement	300	350	421
Water	185	158	112
Silica fume			42.1
Superplasticizer		7	7.6

rate of 0.5 MPa s^{-1}. Steel slabs, of the same diameter as the specimens, were glued to the specimens, then attached to the tensile press. It must be reported that, concurrently with the direct tensile tests, compressive tests were performed on standardized cylindrical specimens (French standard) 160 mm in diameter and 320 mm high. The compressive tests were performed on the same days as the direct tensile tests, and under the same conditions of preservation, i.e., with the specimens coated with several layers of resin.

2.1.2 *Experimental results*

Approximately 50 tensile strength values per specimen diameter were obtained. Tests in which there was a failure in the glue or near the ends of the specimens were eliminated. For the small diameter specimens (diameters 20 mm and 36 mm), it was very quickly discovered, from the results and from the examination of the fracture faces, that the structure of the material was perturbed by wall effects during the casting of the specimens: laitance and bubbling at the surface of the specimens and a large gradient of fines concentration (low at the 'core' of the specimen and large at the perimeter). It was therefore decided to analyse only the results for the specimens 89, 113, and 160 mm in diameter. Table 2 presents the results obtained during this study, and it can be seen that there in fact exists a scale effect, but that it is relatively small: decrease of the mean strength, and of the deviation, with the volume of material loaded.

Statistical tests were performed, Henry's function and the χ^2, to check that the experimental distributions obtained follow known theoretical laws. It was found that these experimental distributions in particular followed two types of law: the truncated normal law (no negative values), and Weibull's law. These are laws that are commonly encountered in investigations of the probabilistic aspects of the failure of heterogeneous and fragile materials.

2.2 Second study

A few years later, for the reasons mentioned above, we performed a second experimental investigation on scale effects in direct tensile testing.

2.2.1 *Experimental conditions*

In this study, three diameters of specimen were used: 30, 60, and 150 mm. Since the first study had shown that the scale effect is small when the specimen diameters range from 90 mm to 160 mm, it was decided to concentrate the tests on smaller diameters. The specimens were cored from large rectangular blocks of concrete having the dimensions 102 cm × 92 cm × 56 cm. Three types of concrete were tested; their compositions are given in Table 3, and we call these concrete 1, concrete 2, and concrete 3. All specimens were protected from desiccation by wrapping, after demoulding, in a sheet of plastic paper followed by two layers of self-adhesive aluminium paper (method developed by the LCPC: Attolou *et al.* [6]). This wrapping was kept during the test. The tests were performed at an imposed loading rate of 0.5 MPa s^{-1} on a 10-tonne Tinus Olsen press. Steel plates were glued to both ends of the specimens, and other steel plates, fitted with ball-jointed tension rods, were bolted to these helmets. The whole set-up was then placed in the press for the test.

In these tests, the displacement measurement, used for determination of Young's modulus, was made using a special extensometer, designed by Boulay [7], that is placed directly on the specimen. This extensometer consists of two aluminium alloy rings, placed at the ends of the specimen (at some distance from the edges to eliminate problems of lateral confinement) using set-screws mounted on elastic blades; these rings constitute the measurement base of three LVDTs placed at 120° around the larger ring.

Concurrently with these direct tensile tests, standardized compressive tests, mentioned above, were performed on the three concretes, at the same age as the tensile tests. The conditions of preservation of the compressive specimens were the same as those of the tensile specimens. The Young's modulus was also determined during these compressive tests, using the same type of extensometer as for the tensile tests. It should be noted that three compressive tests were performed per type of concrete.

Table 4 Results for concretes 1, 2 and 3[a]

	Concrete 1			Concrete 2			Concrete 3		
ϕ_s (m)	30	60	150	30	60	150	30	60	150
N	15	15	16	18	8	9	12	17	7
$m(f_t)$ (MPa)	4.8	3.2	2.4	5.1	4.3	3.3	6.4	6.0	6.0
$\sigma(f_t)$ (MPa)	1.0	0.6	0.2	1.0	0.7	0.2	0.9	0.7	0.2
σ/m (f_t)	0.21	0.18	0.08	0.20	0.16	0.06	0.14	0.12	0.03
$m(E_t)$ (GPa)	35.6	39.0	39.8	42.4	44.2	45.3	52.3	51.8	53.9
$\sigma(E_t)$ (GPa)	3.4	1.8	0.8	3.1	1.8	1.5	2.7	1.6	1.1
$m/\sigma(E_t)$	0.09	0.05	0.02	0.07	0.04	0.03	0.05	0.03	0.02
$m(f_c)$ (MPa)		35.0			55.8			127.5	
$m(E_c)$ (GPa)		39.6			45.7			55.4	

[a] ϕ_s is the diameter of the specimen, N is the number of values, $m(x)$ is the mean value of x, $\sigma(x)$ is the standard deviation of x, $m/\sigma(x)$ is the coefficient of dispersion of x, f_t is the tensile strength, E_t is Young's modulus in tension, f_c is the compressive strength, and E_c is Young's modulus in compression.

2.2.2 *Experimental results*

Since the first tensile study on a given concrete, presented previously, had shown that the distribution function of the tensile strengths followed either a normal law or a Weibull type law, we hypothesized that this is true of all concretes. This hypothesis allowed us, using some statistical laws (based on hypotheses concerning the values of the means and standard deviations), to reduce the number of specimens to be tested while preserving statistical representativeness. Naturally, the number of specimens depends on the volume of each of them (since the mean and standard deviation depend on this volume). It was thus decided that the minimum number of specimens for each volume would be distributed as follows:

ϕ30 mm: 12 specimens

ϕ60 mm: 7 specimens

ϕ150 mm: 7 specimens

Table 4 gives the results for concretes 1, 2, and 3, in the light of which we offer the following observations.

It is found that for the three concretes there exists a scale effect on the direct tensile strength, and that the lower the compressive strength of the concrete the larger this scale effect. If it is assumed that this scale effect is related to the heterogeneity of the material, which is what we think, then this result is readily explained by the fact that the higher the compressive strength the more homogeneous the concrete (the mechanical characteristics of the matrix become closer to those of the grains). This is found again in the coefficients of dispersion of the tensile strength and Young's modulus, which increase as the volume of the specimen becomes smaller and as the compressive strength of the concrete decreases.

It is also observed that there exists a very slight scale effect on the Young's modulus of the concrete, but one that acts in the reverse direction with respect to the effect on tensile strength. It is thus found that the Young's modulus increases as the specimens become larger. This result may seem astonishing, but can be explained, in our opinion, as follows. Because of the different shrinkages existing within the concrete, it contains self-equalizing initial stresses that contribute to 'self-prestressing' the material [8, 9]. Now, when such a medium is cored, a part of these initial stresses is released in the vicinity of the new surfaces generated by the core drilling. To the extent that, for a given concrete, this zone affected by the core drilling is constant (it is related only to the initial stress field, which depends on the concrete), the smaller the specimen, the relatively larger its small mechanical effect (zone of decreased rigidity) on the whole of the specimen. This is what is observed experimentally. In consequence, we think that the mean value of the Young's modulus is independent of the volume of the specimen, but that on the other hand the coefficient of dispersion of this Young's modulus increases as the volume of material decreases and as the strength of the concrete decreases, as indicated later in Tables 6–8. Finally, we may note that for rather similar specimen dimensions ($\phi = 16$ mm, $h = 32$ mm, and $\phi = 15$ mm, $h = 30$ mm) the Young's modulus in compression is equal to the Young's modulus in tension for the three concretes.

3. ANALYTICAL FORMULATION OF A SCALE LAW IN TENSION FOR CONCRETE

The objective of these experimental investigations is, as we stated in the Introduction, to obtain data on the distribution functions of the Young's modulus and direct tensile strength for input to a numerical model based on a probabilistic approach, and we noted that this numerical model poses a problem because of the experimental accessibility of these data. It seems obvious that to perform an experimental investigation as cumbersome as that presented above for each type of concrete is unrealistic, and that this observation calls into question the practical utility of such a model. To overcome this problem, we have attempted to see whether we could not, from the experimental investigation we have just described, arrive at a rather general analytical formulation that would make it possible, for any concrete (except, for the moment, fibre-reinforced concretes and lightweight concretes), using a simple test, to arrive at the distribution functions of interest to us. This is the work we are about to present.

To arrive at these general analytical expressions, we try to judge which are the main parameters (preferably few in number) that govern scale effects in the tensile behaviour of concrete. We have accordingly made two basic assumptions.

(i) The dispersion of the tensile strength is related to the heterogeneity of concrete induced by the different elastic mechanical characteristics of the matrix and grains. More precisely, the closer the scale of the local stress concentrations generated by this heterogeneity to the scale of the specimen, the larger their influence on the macroscopic behaviour of the specimen. This may work out as follows: for given mechanical characteristics of the matrix and grains, the lower the ratio of the volume of the specimen to the volume of the coarsest grains, the larger the dispersion on the strength. This dispersion is also related to the more or less large difference between the tensile strengths of the matrix and grains. In concrete, with the exception of lightweight concretes, compressive strength is governed mainly by the strength of the matrix, the cement paste, which is lower than that of the grains. In consequence, this compressive strength is a good indicator of the quality of the cement paste, together with the difference in strength between the grains and this cement paste. Thus, for the same ratio of volume of specimen to volume of coarsest grain, the dispersion of the tensile strength increases as the compressive strength of the concrete decreases.

Thus, we can say that the dispersion of the tensile strength of a concrete depends, according to our hypotheses, on the ratio 'volume of specimen/volume of coarsest grain', and that the degree of this dependence is a function of the compressive strength of the concrete. It should be emphasized that this analysis is true only in so far as the strength of the cement paste is greater than that of the grains.

(ii) The tensile strength of the concrete is related mainly to that of the cement paste, which is where there are the most weak points (bubbles, microcracks and initial stresses, etc.). This hypothesis is based on Weibull's theory of the weakest link [10], which can be summed up thus: the macroscopic strength of a heterogeneous material is governed by the existence of faults (therefore of weak points) inside it. In consequence, for a given specimen volume and a given (mechanical) quality of matrix and grains, the tensile strength of the concrete depends on the ratio 'volume of specimen/volume of coarsest grain', since depending on this ratio there will be more or less cement paste, and therefore faults. On this hypothesis, it is clear that the better the quality of the cement paste (i.e., the fewer the faults in the matrix), the smaller the scale effect. Now, as we have already stated, the quality of the cement paste of a concrete is manifested in the compressive strength of the concrete. In consequence, the greater or lesser extent of the scale effect in tension in a concrete depends on its compressive strength. We wish to emphasize that we have chosen to reduce everything to compressive strength in part because this characteristic is easy to determine experimentally and in part because it is part of the 'cultural baggage' of any engineer who works with concrete.

In conclusion, we hypothesize that the scale effect on the tensile strength of a concrete is reflected by two relations of the form:

$$m(f_t) = F_\alpha(V_S/V_A) \tag{1}$$

$$\sigma/m(f_t) = F_\beta(V_S/V_A) \tag{2}$$

where the parameters of the functions F_α and F_β depend on the compressive strength of the concrete, and $m(f_t)$ is the mean tensile strength, $\sigma(f_t)$ is the standard deviation of the tensile strength, V_S is the total volume of the specimen, and V_A is the volume of the coarsest grains of the concrete (of a single grain).

We used an optimization program to search for the type of function that best smooths the experimental points taken from Table 4, and found in this way that the functions f_α and f_β are both of the type $y = a(x)^{-b}$. Relations 1 and 2 therefore become

$$m(f_t) = a(V_S/V_A)^{-b} \tag{3}$$

with

$$a = 6.5 \tag{4}$$

$$b = 0.25 - 3.6 \times 10^{-3} f_c + 1.3 \times 10^{-5} f_c^2 \tag{5}$$

for 35 MPa $\leq f_c \leq$ 130 MPa, with f_c the compressive strength obtained on the standardized specimen (the French one), and

$$\sigma/m(f_t) = A(V_S/V_A)^{-B} \tag{6}$$

with

$$A = 0.35 \tag{7}$$

$$B = 4.5 \times 10^{-2} + 4.5 \times 10^{-3} f_c - 1.8 \times 10^{-5} f_c^2 \tag{8}$$

for 35 MPa $\leq f_c \leq$ 130 MPa. It should be stated that, in Relations 3–8, V_A is the volume of the coarsest grain, modelled by a sphere of which the diameter is equal to the mesh of the screen through which this coarse grain passed during grading of the aggregates of the concrete.

We stated earlier that in the light of the experimental results, the Young's modulus of concrete did not seem to be affected by a scale effect, or at least the mean value of this Young's modulus appears to be independent of the volume of the specimen. On the other hand, the coefficient of dispersion, for its part, depends very significantly on the volume of the specimen. This dispersion is related to the fact that the respective Young's moduli of the cement paste and of the grains are different, and that the smaller the specimen dimension, the closer the Young's modulus becomes to that of the cement paste, or to that of the grain, alone. It is therefore the ratio (V_S/V_A) that governs this dispersion, as in the case of

Table 5 Comparisons between the theoretical and the experimental values for the three concretes

	Concrete 1			Concrete 2			Concrete 3		
ϕ_s (mm)	30	60	150	30	60	150	30	60	150
				Theoretical results					
$m(f_t)$ (MPa)	4.6	3.4	2.3	5.2	4.3	3.3	6.4	6.2	6.2
$\sigma/m(f_t)$ (MPa)	0.23	0.16	0.10	0.20	0.12	0.06	0.16	0.08	0.03
σ/m(E) (GPa)	0.08	0.05	0.03	0.07	0.04	0.02	0.05	0.02	0.01
				Experimental results					
$m(f_t)$ (MPa)	4.8	3.2	2.4	5.1	4.3	3.3	6.4	6.0	6.0
$\sigma/m(f_t)$ (MPa)	0.21	0.18	0.08	0.20	0.16	0.06	0.14	0.12	0.03
$\sigma/m(E)$ (GPa)	0.09	0.05	0.02	0.07	0.04	0.03	0.05	0.03	0.02

Table 6 First comparison between experimental results and theoretical results

	Experimental results			Theoretical results		
V_S/V_A	6333	2216	1202	6333	2216	1202
f_c (MPa)			47.5			
$m(f_t)$ (MPa)	2.5	2.8	3.0	2.5	2.8	3.0
$\sigma/m(f_t)$ (MPa)	0.08	0.09	0.10	0.05	0.06	0.07

tensile strength. The closer is the modulus of the matrix to that of the grain, the less this ratio V_S/V_A will influence the dispersion of the Young's modulus. As we have already stated, since the compressive strength of a concrete is an indication of the quality of the cement paste of which it is composed, it controls the dependence between the coefficient of dispersion of the Young's modulus and the ratio V_S/V_A.

In conclusion, we hypothesize that the mean value of the Young's modulus of a concrete depends only on its compressive strength (no scale effect) and is easily determined by performing a compressive test on a standardized specimen. Therefore, it is not necessary to develop an analytical expression to obtain this mean value. On the other hand, the coefficient of dispersion of the Young's modulus depends on the ratio V_S/V_A and on the compressive strength. Using the same optimization software as for tensile strength, and referring to the results given in Table 4, one finds the following equation:

$$\sigma/m(E) = c(V_S/V_A)^{-d} \quad (9)$$

with

$$c = 0.15 \quad (10)$$

$$d = 0.16 + 2.7 \times 10^{-3} f_c - 3.4 \times 10^{-6} f_c^2 \quad (11)$$

for $35 \text{ MPa} \leq f_c \leq 130 \text{ MPa}$.

Table 5 compares the theoretical values obtained with Expressions 3–11 with the experimental ones. It is clear that Equations 3–11 were determined from three experimental values, which is, from a theoretical viewpoint, highly debatable. To justify the pertinence of these equations, we shall compare them with experimental results obtained on types of concrete other than those mentioned in this article.

4. COMPARISON OF SCALE LAWS IN TENSION WITH EXPERIMENTAL RESULTS OBTAINED FOR DIFFERENT CONCRETES

The analytical formulations proposed were compared with experimental results obtained during various studies performed either before or after the one presented above. There are three of these studies. First of all, there is the one mentioned at the beginning of the article (first experimental investigation). The composition of the concrete is given in Table 1, and the experimental results in Table 2. The second study concerns work on the dynamic behaviour of the concrete in tension. In this study, not yet published, static tests were performed, and it is therefore the results of these tests that will be used as the second test to validate our analytical formulations. The third study concerns work on the direct tensile behaviour of fibre-reinforced concretes made with a new metal fibre patented by the LCPC [11].

4.1 First validation test

The comparison between experimental results and theoretical results is given in Table 6.

4.2 Second validation test

In the course of this study, three compositions of concrete were tested. They are given in Table 7. For all three

Table 7 Composition of concretes (kg m^{-3})

Constituent	Concrete 1	Concrete 2	Concrete 3
Microgranite, 6/10 mm	1162	1162	1162
Seine sand, 0/4 mm	681	681	681
OPS cement	365	265	450
Calcareous filler		86	
Silica fume			45
Water	185	185	137
Superplasticizer			7

compositions, the direct tensile tests were performed on cores having the following dimensions:

Concrete 1: $\phi = 74$ mm, $h = 100$ mm

Concretes 2 and 3: $\phi = 68$ mm, $h = 100$ mm

All tests were performed on the same press and under the same experimental conditions as the study that yielded the analytical formulations for scale effects on concrete in tension (in particular as regards determination of the Young's modulus and the preservation of the specimens). For both concretes, 10 specimens were tested. Table 8 gives the comparative results between experiment and theory for both types of concrete.

4.3 Third validation test

The composition of the metal fibre concrete, of which we present here the direct tensile behaviour, is given in Table 9. The tests were performed on cylindrical cores of which the dimensions are $\phi = 150$ mm, $h = 300$ mm. The experimental conditions are identical to those of the second validation test. Six specimens were tested in this study, and Table 10 gives the comparative results between experiment and theory. In the light of these different validation tests, the following observations may be made.

(i) The agreement between the mean tensile strength values obtained experimentally and the theoretical values is very good.

(ii) The agreement between theory and experiment concerning the coefficients of dispersion and their effects on the tensile strength and on the Young's modulus is less good, but still quite satisfactory. This small difference can be analysed on two levels. The first, very generally, consists of recalling that it is always more difficult, in the domain of statistics, to obtain experimental and theoretical access to the coefficients of dispersion of probabilistic parameters, whatever they may be. The second level consists of attempting to explain this disagreement by considering each validation test in a specific manner (at least the validation tests where this disagreement is significant). It may for example be noted that in the first validation test the coefficient of dispersion of the tensile strength is larger experimentally than anticipated by the analytical formula. This can be explained by the fact that, in this first direct tensile study we performed, the experimental conditions were not as good as is possible today (in particular in the other studies used as validation tests). The boundary conditions are not very good (conditions of bonding of the specimens to the press), and spurious bending must surely have occurred, with a larger influence on the standard deviations than on the mean values.

In the study of metal fibre concrete we find, on the other hand, that the theoretical coefficient of dispersion of the tensile strength is greater than the experimental

Table 9 Composition of metal fibre concrete (kg m^{-3})

Silico-calcareous gravel, 5/25	1000
Silico-calcareous sand, 0/5	730
OPS cement	425
Water	190
Superplasticizer	4.25
Metal fibres	78

Table 10 Comparison between the experimental results obtained with the metal fibre concrete and the theoretical results given by the analytical formulations

	Experimental results		Theoretical results
V_S/V_A		648	
f_c (MPa)		50.7	
$m(f_t)$ (MPa)	3.4		3.4
$\sigma/m(f_t)$ (MPa)	0.05		0.08
$m(E_t)$ (GPa)		43.7	
$\sigma/m(E_t)$ (GPa)	0.03		0.02

Table 8 Comparison between the experimental results obtained with concrete 1 and the results obtained with the analytical formulations

	Experimental results			Theoretical results		
V_S/V_A	821	697	697	821	697	697
f_c (MPa)	57	35.3	122.4	57	35.3	122.4
$m(f_t)$ (MPa)	3.6	2.5	6.4	3.6	2.5	6.3
$\sigma/m(f_t)$ (MPa)	0.07	0.10	0.05	0.07	0.11	0.04
$m(E_t)$ (GPa)	42.2	37.4	53.1	42.2	37.4	53.1
$\sigma/m(E_t)$ (GPa)	0.02	0.05	0.02	0.02	0.03	0.01

coefficient of dispersion. This can be explained by the fact that the fibres may act as 'regulators' reinforcing the zones of lowest strength in the matrix. For this, it is obviously necessary for the fibres to be distributed sufficiently uniformly within the matrix, since the contrary would increase the heterogeneity of the material, and therefore the dispersion.

5. CONCLUSIONS

We have presented an experimental investigation concerning the scale effect on the tensile strength of concrete and its Young's modulus. The main findings are the following.

(i) This scale effect is highly significant (increase of the mean value, and of the coefficient of dispersion, as the volume of material increases), and the lower the compressive strength of the concrete, the larger it is.

(ii) The Young's modulus seems not to depend on the volume of concrete but, on the other hand, the coefficient of dispersion of this modulus decreases as the volume and the compressive strength of the concrete increase.

(iii) The conclusions do not concern concretes in which the grains are weaker than the cement paste (lightweight concretes, and some high-strength concretes).

We have also proposed very general analytical formulations that can be used to determine the distribution functions (means and coefficients of dispersion) of the tensile strength and the Young's modulus of any concrete (except those mentioned in (iii)), knowing only the compressive strength of the concrete, determined in accordance with the French standard, and the ratio of the volume of the specimen to the volume of the coarsest grain.

We have compared the outcome of these analytical formulae to experimental results obtained in studies performed both before and after the present study, and have found that the agreement between experiment and theory is very good. It now remains to compare our analytical formulae with a larger body of experimental results, taken in particular from the literature. We suspect, unfortunately, that some difficulties may appear in the course of this validation. (a) There are very few experimental investigations, internationally, of direct tensile tests on concrete (difficulties of implementation of such tests). The results concerning bending tension and splitting tension are not at all equivalent to direct tensile test results, in particular with respect to scale effects. (b) Our analytical formulae contain the compressive strength determined according to the French standard (tests on cylinders), and these formulae would therefore be different if the compressive tests were performed on specimens different from those prescribed by this standard (such as cubic specimens).

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RESUME

Effet d'échelle dans le béton en traction

Les effets d'échelle dans le comportement mécanique du béton sont des phénomènes importants qui font l'objet de nombreuses études au niveau international. Le Laboratoire Central des Ponts et Chaussées travaille depuis plusieurs années sur l'effet d'échelle existant sur la fonction de distribution de la résistance à la traction du béton. De très nombreux essais de traction directe ont ainsi été réalisés sur des éprouvettes de volume différent constituées de bétons de formulation également différente. De ces études expérimentales, il a été tiré une loi assez générale qui permet de déterminer, à partir de la seule connaissance de la résistance à la compression obtenue sur cylindre normalisé (en France), et du rapport du volume de l'éprouvette de traction au volume du plus gros grain de béton, la fonction de distribution (moyenne et écart-type) de la résistance à la traction du béton en fonction du volume de l'éprouvette de traction, cela pour des bétons dont la résistance à la compression est comprise entre 30 et 130 MPa. Il faut souligner que cette loi est valable uniquement si, d'une part le mode de conservation des éprouvettes de traction directe est identique à celui des éprouvettes de compression, et d'autre part les granulats utilisés dans les bétons sont des granulats courants, ce qui élimine les granulats légers et les granulats très durs et à fort module de Young.