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# Numerical modelling of concrete cracking based on a stochastic approach

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*Concrete is a highly heterogeneous material, because of its composite structure, but also because of the physical phenomena that take place during hardening (initial stresses, drying shrinkage, heat exchanges). This heterogeneity can explain some aspects of the complex mechanical behaviour of concrete, particularly the transition from uniform to localized cracking and the important size effect. A numerical procedure taking the statistical aspects of this heterogeneity into account has been developed and implemented. It permits us to reproduce and explain the principal experimental results for the behaviour of concrete under tension.*

## 1. INTRODUCTION

The influence of the volume of a test specimen on its mechanical behaviour, the so-called size effect, is a well-known phenomenon. It is observed on a macroscopic level, but can be explained through an analysis of local physical and mechanical phenomena.

Concrete, a mix of cement and aggregates, is by nature a heterogeneous material. Its heterogeneity is also attributable to the physical and chemical phenomena that take place during manufacturing and hardening, to microcracking resulting from drying and shrinkage and to porosity due to the presence of water. As regards cracking, this heterogeneity leads to what can be considered as random local mechanical behaviour.

In the work reported herein, this phenomenon is taken into account in a finite-element model of concrete behaviour by introducing statistical distributions of local material characteristics. In a first approach, these are limited to concrete tensile strength.

## 2. PHYSICAL BASES OF THE MODEL

Because of the random spatial distribution of the heterogeneities and as shown by acoustic emission tests, cracking failure of concrete takes place discontinuously.

Local fracture energies are then also random. It follows that the overall energy dissipated during the cracking of a given concrete volume results from these local fracture energies and their spatial distribution.

## 3. NUMERICAL MODELLING: ASSUMPTIONS

Based on the physical reality of concrete cracking, we chose to use a very simple finite-element model. The principal assumptions of this model are as follows:

1. Overall damage in concrete results from accumulated surface energy dissipations leading to failure planes (discontinuities). These planes appear once a limiting critical local fracture energy is reached.
2. Concrete under uniaxial tension is modelled as an elastic-brittle material. With this assumption, the random space distribution of local fracture energies can be modelled by a random space distribution of critical tensile strain values (Fig. 1).
3. Failure planes are modelled by special contact finite elements of infinitely small thickness.

In a volume modelled using triangular elements (Fig. 2), cracks (or microcracks according to the scale being dealt with) can thus propagate in three directions. This allows realistic modelling of the cracked state as can be seen in Fig. 3.

Critical limit tensile strain values drawn from the statistical distribution function will be randomly distributed in space on the contact elements. The contact

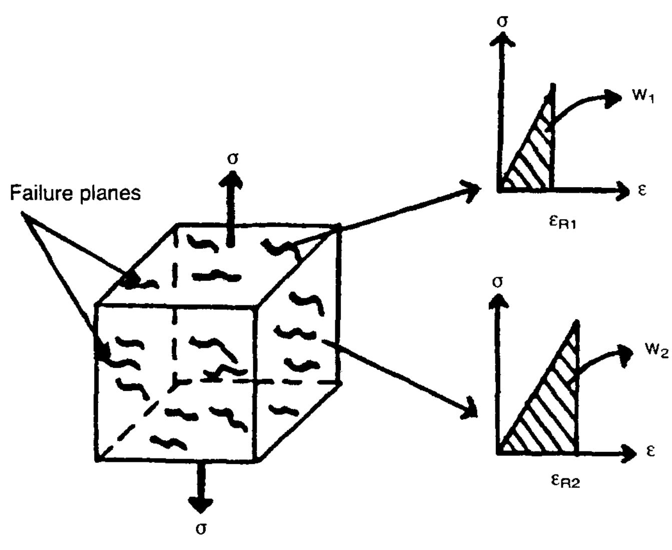


Fig. 1 Concrete modelled as an elastic–brittle material.

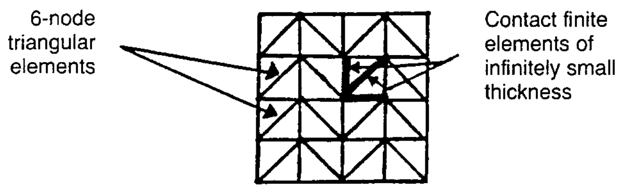


Fig. 2 Finite-element modelling.

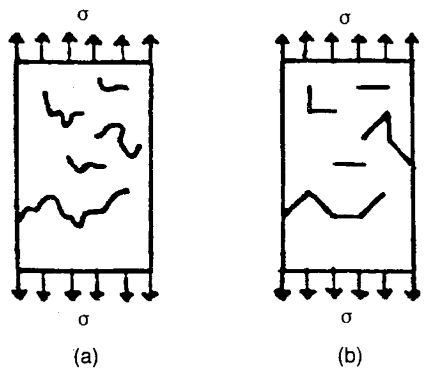


Fig. 3 Comparison between cracking propagation (a) in reality and (b) in modelling.

elements will "open" when the strain normal to their centroid reaches their assigned limit strain values.

## 4. CHARACTERISTICS OF THE CONTACT ELEMENT

This element was developed at the L.C.P.C. in 1985 [1]. It is a 6-node quadrilateral element for plane or axisymmetric contact elements and a 16-node hexahedral or a 12-node pentahedron for 3-D contact problems.

Quadratic and linear shape functions are used to interpolate displacements along the tangent and normal directions to the crack surface, respectively. The element has an infinitely small thickness in this direction. Newton-Côtes numerical integration is used to obtain the stiffness matrix of the element.

The numerical solution of the non-linear contact problem is done by an automatic load incremental iterative procedure.

The stress increments $\{\Delta\sigma\}$ in a given contact element corresponding to strain increments $\{\Delta\varepsilon\}$ are computed with the relationship

$$\{\Delta\sigma\} = [E]\,\{\Delta\varepsilon\} \tag{1}$$

where $[E]$ is the material stiffness matrix of a fictitious contact material. Stresses are computed on the integration points and at the centroid of the element for each load increment.

This element can treat contact as well as coulomb friction type problems. In this study of concrete cracking, only the contact feature of the element is used.

The state of the contact element is given by the following rules, with M, an integration point, $\sigma n$, the stress vector normal to the surface at point M, and $RT$, the tensile strength of the element.

If $\sigma n < RT$ the element remains closed ensuring continuity displacements
if $\sigma n > RT$ the element separates and displacement continuity is no longer ensured thus simulating a failure plane (2)

The state of the element is given by the above-mentioned rules applied at the centroid of the element.

From Equation 1 it is obvious that testing with a tensile-strength limit value is equivalent to testing with limit tensile strain values as was stated before.

## 5. STATISTICAL APPROACH: EXPERIMENTAL DETERMINATION OF THE TENSILE STRENGTH DISTRIBUTION FUNCTION

As was stated, strength or strain limit values can be used in our numerical modelling. Because of experimental procedures, limiting stress values will be used in what follows.

Having determined the bases of the numerical procedure, it is now necessary to determine the statistical distribution function of local tensile strength.

Many direct tension tests were carried out in order to analyse the influence of the test specimen volume on the distribution function of tensile strength. The test specimens were cylinders of different diameters ($d$ = 89, 110 and 160 mm) with a constant length/diameter ratio of 2. 100 tests were conducted for each diameter, under applied stress rate^ to obtain an elastic–brittle behaviour of the material. The concrete composition used is given in Table 1.

Table 1 Concrete composition given for 1 $m^3$

| Constituents | Cement OPC 400 | Sand (0/5) | Gravel (5/12) | Water |
|---|---|---|---|---|
| Weight (kg) | 1110 | 700 | 400 | 190 |

The histograms obtained are shown in Fig. 4 as well as the mean tensile strength and the standard deviation for each test specimen size. From this figure the following can be made:

(i) Mean and standard deviation values are a function of the test specimen volume. The greater the volume, the smaller these values are. This confirms the well-known size effect in heterogeneous materials.

(ii) Both the normal and the Weibull distribution fit the statistical distribution obtained experimentally.

Choosing the normal distribution in a first approach, the functions relating mean tensile strength and standard deviation to concrete volume are easily obtained from the experimental results using an

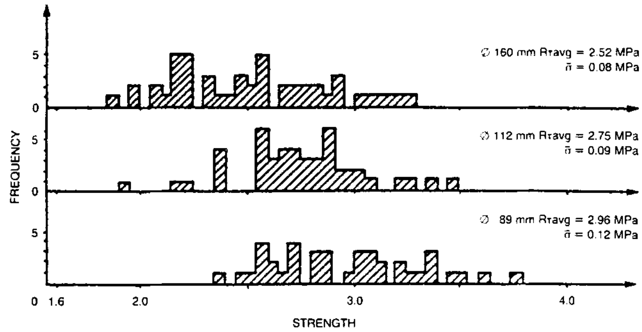


Fig. 4 Histograms of statistical distributions obtained experimentally.

interpolation method. The following relationships are obtained:

Mean average tensile strength (MPa) $RT_{avg}$

$$= 21.95\ V^{-0.142} \quad (3)$$

Standard deviation (MPa) $\bar{\sigma}$

$$= 11.46\ V^{-0.248} \quad (4)$$

where $V$ is the volume of the test specimen in $mm^3$. The tensile strength distribution function can thus be modified according to the volume of each element used.

## 6. NUMERICAL RESULTS

The object of the numerical tests analysis was to reproduce two important phenomena encountered in the direct tension test, namely, the size effect and the localisation of deformations (resulting from the localisation of microcracking) corresponding to a sharp stress decrease.

We chose to model a concrete cube under direct tension. Different cube sizes were modelled to vary the volume stressed. In order to save computer time, each cube was modelled with 2-D elements assuming plane stress conditions. The loading, consisting of applied displacements, and other boundary conditions is shown in Fig. 5.

A uniform mesh was used with 6-node triangular elements, contact elements were used in one direction only in order to model failure planes perpendicular to the loading direction (Fig. 5).

400 6-node elements and 380 contact elements (20 for each failure plane) were used in each mesh. The number of contact elements was considered sufficient to represent the normal distribution used.

The volume of the concrete cube of side $D$ was varied by using different sizes of 6-node elements of side $d$. The following dimensions ($d$, $D$) were used: (2.5 mm, 5 cm); (5, 10); (10, 20); (20, 40); (30, 60). In a previous study [2] the size of the elements $d$ was held constant while varying their number. The size effect was clearly evidenced. In this study $d$ is varied with a view to using this numerical model in a structure analysis.

Parameters $RT_{avg}$ and $\sigma$ of the normal distribution are computed from Equations 3 and 4. The volume considered is that of a square prism of side $d$ and unit thickness (it is a plane stress calculation).

The finite element code CESAR developed at L.C.P.C. was used for the numerical analysis.

Global $\sigma$–$\varepsilon$ behaviour curves for each concrete volume modelled are shown in Fig. 6. Here $\sigma$ is the average stress on the boundary where displacement is applied and $\varepsilon$ is an average strain computed as $\varepsilon = v/D$ ($v$ is the displacement imposed).

Figure 7 shows the $\sigma$–$\varepsilon$ curve as obtained for a cube with $D = 50$ mm and the different states of cracking corresponding to different points on the curve.

## 7. DISCUSSION OF RESULTS OF NUMERICAL ANALYSIS

Based on Figs 6 and 7, the following comments can be made:

1. The finite-element model can reproduce the size effect. The peak stress decreases with larger volumes. Of course, this is expected given the statistical approach adopted. It is, however, important to know that the finite-element model can reproduce the physical reality and that its use on more complex structures can be considered.

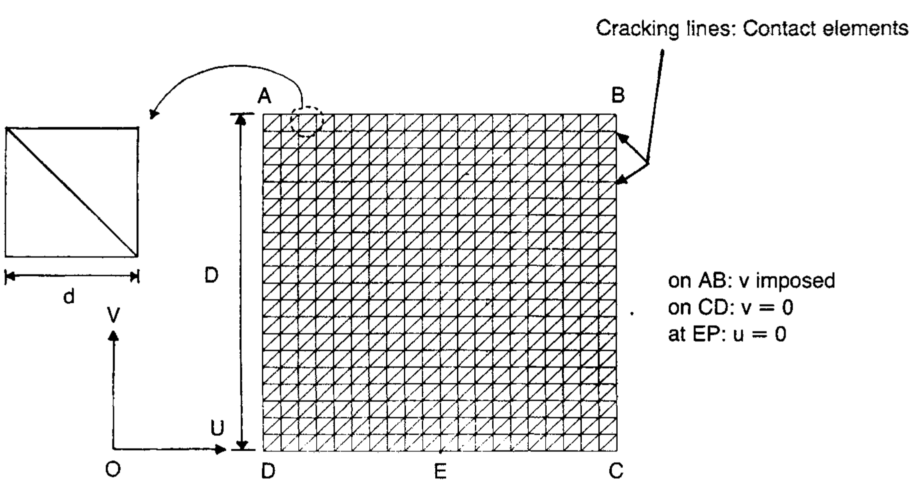


Fig. 5 Boundary conditions of the calculation.

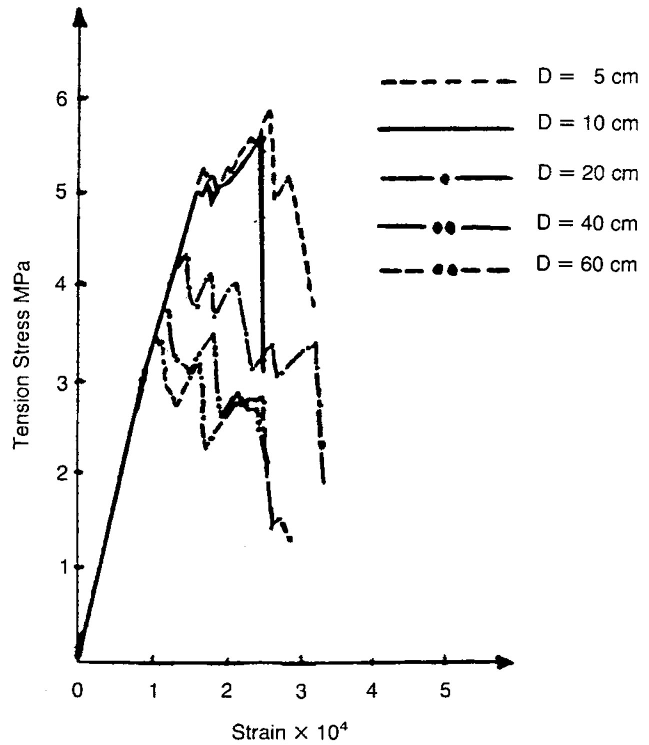


Fig. 6 σ–ε curves for different values of $D$.

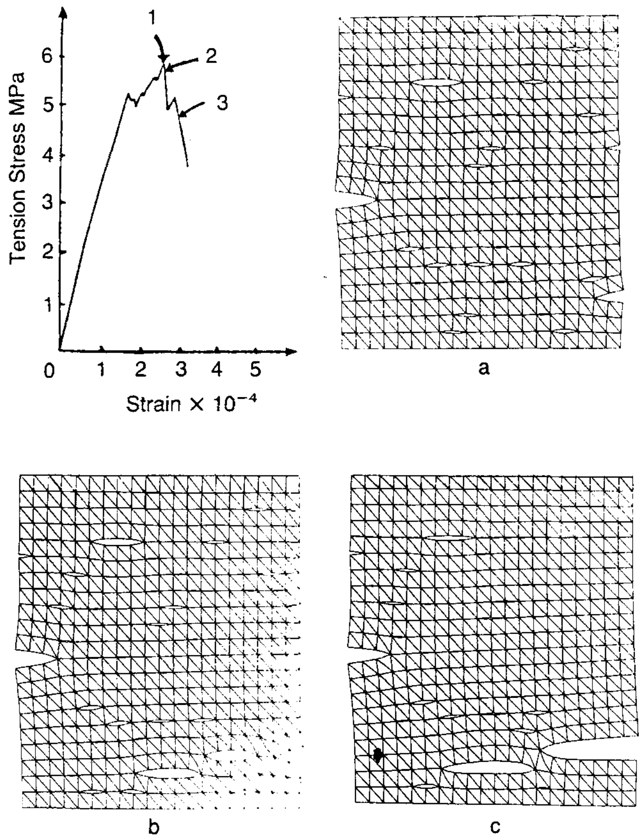


Fig. 7 σ–ε curve for a cube with $D$ = 50 mm and different states of the cracking process: (a) the peak – uniform damage; (b) the post peak – localization of the microcracks; (c) localized cracking propagation.

2. Figure 7 shows that the post-mark softening branch of the σ–ε curve corresponds to localized strains due to concentrated cracking.

Computed peak stresses higher than those normally obtained in experiments can be because:

(i) Boundary conditions in the finite-element normal do not wholly represent those existing in reality. The strain energy stored in the testing machine and its restitution as kinetic energy at the outset of microcracking is not properly accounted for. This kinetic energy accelerates the crack concentration and has thus an effect on the value of the peak stress obtained.

(ii) The incremental load procedure used in the solution introduces an artificial stability in the cracking process of concrete. The effects of kinetic energy on crack propagation are not thus taken into account.

(iii) Modelling failure planes in one direction only also impedes a realistic interaction between cracks propagating in parallel directions. This introduces a certain stability in the propagation of cracks especially on the softening post-peak branch.

## 8. CONCLUSIONS

A model based on a statistical approach to the physical behaviour of concrete has been presented. Its use with finite elements to model the behaviour of concrete under direct tension allowed us to reproduce the principal phenomena encountered in this type of test: the size effect and the relationship between post-peak behaviour and strain concentration.

This model must be improved by taking into account time effects. This could be done, for example, by considering the viscous elastic behaviour of concrete as is necessary when describing the non-stable cracking process of concrete.

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