Applied Mathematics and Mechanics
(English Edition, Vol. 20, No. 4, Apr. 1999)

Published by SU,
Shanghai, China

HAMILTONIAN FORMULATION OF NONLINEAR WATER WAVES IN A TWO-FLUID SYSTEM *

Lu Dongqiang (卢东强), Dai Shiqiang (戴世强), Zhang Baoshan (张宝善)

Shanghai Institute of Applied Mathematics and Mechanics,

Shanghai University, Shanghai 200072, P R China

(Received Dec. 24, 1997; Revised Aug. 27, 1998)

Abstract: *In this paper, it is dealt with that the Hamiltonian formulation of nonlinear water waves in a two-fluid system, which consists of two layers of constant-density incompressible inviscid fluid with a horizontal bottom, an interface and a free surface. The velocity potentials are expanded in power series of the vertical coordinate. By taking the kinetic thickness of lower fluid-layer and the reduced kinetic thickness of upper fluid-layer as the generalized displacements, choosing the velocity potentials at the interface and free surface as the generalized momenta and using Hamilton's principle, the Hamiltonian canonical equations for the system are derived with the Legendre transformation under the shallow water assumption. Hence the results for single-layer fluid are extended to the case of stratified fluid.*

Key words: two-fluid system; Hamilton's principle; nonlinear water waves; shallow water assumption; Hamiltonian canonical equations

Introduction

The geometrization of mechanics is a tendency of the development of continuum mechanics and draws extensive attention of researchers. Through the efforts in one and half a century, the geometrical theory of dynamics of particles and rigid bodies has laid a solid foundation and during the past decades, people turned to explore the corresponding theory for continuum mechanics. In recent years, the study of Hamiltonian structure, symmetries and conservation laws for water waves has become one of important subjects in dynamics of water waves. And thus a new system of treating water waves was brought about, in which the aim is to turn the investigation of water waves into that of infinite-dimensional Hamiltonian structure and to study the water waves with the concepts and techniques in classical Hamiltonian mechanics and modern theory of mathematics, such as the differential manifold, the symplectic geometry, the Lie group and the Lie algebra.

J. C. Luke first established the variational principle for water waves[1], and the G. B. Whitham continued to investigate the variational principles for several problems of water waves and their applications[2]. V. E. Zakharov[3] found that the governing equations for water waves, under the assumptions of inviscidity, irrotationality, incompressibility and uniformity of density, form the Hamiltonian dynamical system, in which the positive-definite Hamiltonian functional is the total energy of fluid and the elevation η of free surface and the velocity potential Φ at free surface represent the canonical variables. In fact, η is the generalized displacement of infinite di-

* Project supported by the National Natural Science Foundation of China (19672035)

mension, and Φ is the corresponding generalized momentum. V. E. Zakharov's pioneering work opened a new way to explore the Hamiltonian canonical structure of the problems on water waves, a novel approach to study water waves[3]. J. W. Miles and D. M. Milder investigated rather completely Hamilton's principle and the Hamiltonian canonical equations for water waves[4, 5]. T. B. Benjamin and P. J. Olver (1982) analyzed the relation between the Hamiltonian structure for the original and approximate of water waves problems[6]. The existing fruitful results on the Hamiltonian formulation of water waves have been briefly reviewed in Ref. [7].

The shallow water approximation and the Boussinesq assumption have been used for many years. The related idea and techniques are based on the estimation of magnitude order, that is, preserving the principal nonlinear terms and dispersion terms in the equations of motion. The advantage of doing so is the appropriate simplification and the accompanying drawback is the destruction of Hamiltonian structure of the problems. In the Hamiltonian formulation, the Hamiltonian canonical variables have not been changed and the corresponding approximation are automatically canonical, i. e., Hamiltonian-structure-preserving. For the Hamiltonian formulation of single-layered fluid under the shallow water approximation, W. Craig and M. D. Groves (1994) derived the approximate Hamiltonian equations of various orders for 2-D shallow water waves by using a convergent Tayler expansion of the Dirichlet-Neumann operators[8]. Starting form Hamilton's principle and using the Legendre transformation, the authors presented the Hamiltonian canonical equations for single-layered fluid[9] and the obtained results agreed with those in Ref. [8], while the approach was simpler and the physical meaning clearer. In this paper, we shall further extend the results in Ref. [9] and under the shallow water assumption, consider a two-fluid system studied in Ref. [10]. We shall expand the velocity potential for the lower fluid as a series in terms of that at the bottom and the velocity potential for the upper fluid as a series in terms of that at the interface. And then the kinetic thickness ζ_1 of lower fluid and the reduced kinetic thickness ζ_2 of the upper fluid are taken as the generalized displacements; the velocity potential Φ_1 of lower fluid at the interface and the velocity potential Φ_2 of upper fluid at the free surface as the generalized momenta. That is to say, (ζ_1, Φ_1) and (ζ_2, Φ_2) constitute the pair of dual variables. Then with the Legendre transformation, the corresponding Hamiltonian canonical equations are derived for the system. Hence, the results in Refs. [3, 4, 9] are extended to a two-fluid system and other forms of equations governing nonlinear shallow water waves in the system are given, slightly different from those presented in Ref. [10], but more concise, which are in agreement with each other in the linearized approximation.

1 Variational Principle and Hamiltonian Canonical Equations for the Two-Fluid System

We consider the irrotational motion of two layers of immiscible incompressible constant-density inviscid fluids. Suppose that the ratio of density of upper and lower fluids is $\sigma(=\rho_2/\rho_1)$; the (static) thicknesses of upper and lower fluids are h_2 and h_1; the horizontal coordinates are $x = (x_1, x_2)$, and the vertical coordinate is y, while the horizontal bottom is at $y = 0$; the elevation of interface is $\eta_1(x, t)$, and the elevation of free surface is $\eta_2(x, t)$. Denote $\Omega: S_0 \times R_{(T)}$ as the considered domain of time and space. Assume that the stratification of fluids is statically stable (i. e., $\sigma < 1$) and the ratios of h_1 and h_2 to the characteristic wave-length obey the shallow water assumption, i. e., $h_1/\lambda \ll 1$ and $h_2/\lambda \ll 1$.

From Hamilton's principle

$$\tilde{L}_1 = \int_0^{\eta_1}\left(\frac{1}{2}\mid \nabla \phi_1 \mid^2 - gy\right)\mathrm{d}y,\ \tilde{L}_2 = \sigma\int_{\eta_1}^{\eta_2}\left(\frac{1}{2}\mid \nabla \phi_2 \mid^2 - gy\right)\mathrm{d}y, \quad (1a,b)$$

$$\delta\iint_\Omega \tilde{L}\mathrm{d}\boldsymbol{x}\mathrm{d}t = \delta\iint_\Omega (\tilde{L}_1 + \tilde{L}_2)\mathrm{d}\boldsymbol{x}\mathrm{d}t = 0, \quad (1c)$$

we can obtain the governing equations for the two-fluid system but cannot get the complete boundary conditions. From the variational principle

$$L_1^* = \int_0^{\eta_1}\left(\phi_{1,t} + \frac{1}{2}\mid \nabla \phi_1 \mid^2 + gy\right)\mathrm{d}y, \quad (2a)$$

$$L_2^* = \sigma\int_{\eta_1}^{\eta_2}\left(\phi_{2,t} + \frac{1}{2}\mid \nabla \phi_2 \mid^2 + gy\right)\mathrm{d}y, \quad (2b)$$

$$\delta\iint_\Omega L^*\mathrm{d}\boldsymbol{x}\mathrm{d}t = \delta\iint_\Omega (L_1^* + L_2^*)\mathrm{d}\boldsymbol{x}\mathrm{d}t = 0, \quad (2c)$$

it follows that

$$\nabla \phi_1 = 0, \qquad 0 < y < \eta_1, \quad (3a)$$

$$\nabla \phi_2 = 0, \qquad \eta_1 < y < \eta_2, \quad (3b)$$

$$\eta_{2,t} + \nabla \phi_2 \nabla \eta_2 - \phi_{2,y} = 0, \qquad y = \eta_2, \quad (3c)$$

$$\phi_{2,t} + \frac{1}{2}(\nabla \phi_2)^2 + gy = 0, \qquad y = \eta_2, \quad (3d)$$

$$\eta_{1,t} + \nabla \phi_1 \nabla \eta_1 - \phi_{1,y} = 0, \qquad y = \eta_1, \quad (3e)$$

$$\eta_{1,t} + \nabla \phi_2 \nabla \eta_1 - \phi_{2,y} = 0, \qquad y = \eta_1, \quad (3f)$$

$$\left(\phi_{1,t} + \frac{1}{2}(\nabla \phi_1)^2 + gy\right) - \sigma\left(\phi_{2,t} + \frac{1}{2}(\nabla \phi_2)^2 + gy\right) = 0, \qquad y = \eta_1, \quad (3g)$$

$$\phi_{1,y} = 0, \qquad y = 0, \quad (3h)$$

where ϕ_1 and ϕ_2 are the velocity potentials of upper and lower fluids respectively; $\phi_{i,t} = \partial\phi_i/\partial t$, $\phi_{i,x} = \partial\phi_i/\partial x$, $(i = 1, 2)$; g is the gravitational acceleration; ∇ is the Hamilton's operator. Eqs. (3a ~ 3h) are just the governing equations and boundary conditions for the two-fluid system, so the variational principle (2) (i. e., the principle of stationary pressure) can be regarded as the generalization of Luke's variational principle for single-layered fluid[1]. Comparing the Lagrangian function L^* in Eqs. (2) and the Lagrangian function $\tilde{L}$ in Hamilton's principle (1), we have

$$\begin{aligned}\tilde{L} = &- L^* + \int_0^{\eta_1}(\mid \nabla \phi_1 \mid^2 + \phi_{1,t})\mathrm{d}y + \int_{\eta_1}^{\eta_2}(\mid \nabla \phi_2 \mid^2 + \phi_{2,t})\mathrm{d}y = \\ &- L^* - [\phi_1(-\eta_{1,t} - \nabla \phi_1 \nabla \eta_1 + \phi_{1,y})]_{y=\eta_1} - [\phi_1\phi_{1,y}]_{y=0} - \\ &\int_0^{\eta}\phi_1 \nabla^2\phi_1\mathrm{d}y - [\phi_2(-\eta_{2,t} - \nabla \phi_2 \nabla \eta_2 + \phi_{2,y})]_{y=\eta_2} - \\ &[\phi_2(\phi_{2,y} + \nabla \phi_2 \nabla \eta_1)]_{y=\eta_1} - \int_{-h_0}^{\eta} \phi_2 \nabla^2\phi_2\mathrm{d}y + \\ &\frac{\partial}{\partial x_i}\int_0^{\eta_1}\phi_1 \frac{\partial}{\partial x_i}\phi_1\mathrm{d}y + \frac{\partial}{\partial t}\int_0^{\eta_1}\phi_1\mathrm{d}y - \frac{\partial}{\partial x_i}\int_{\eta_1}^{\eta_2}\phi_2 \frac{\partial}{\partial x_i}\phi_2\mathrm{d}y + \frac{\partial}{\partial t}\int_{\eta_1}^{\eta_2}\phi_2\mathrm{d}y,\end{aligned} \quad (4)$$

where the dummy indices follow the summation convention and the additional terms other than the term of divergence in Eq. (4) embody the conservation of mass. If ϕ_i and η_i satisfy Eqs. (3), then those additional terms will disappear, while the last four terms in Eq. (4) will disappear as the boundary terms in the variational approach. Therefore L^* and $\tilde{L}$ can be considered to be dynamically equivalent.

Since ϕ_1 is the harmonic function and satisfies the boundary condition at bottom $\phi_{1,y}|_{y=0} = 0$, therefore ϕ_1 can be expanded as

$$\phi_1 = \varphi_1(x,t) - \frac{1}{2}y^2\nabla^2\varphi_1(x,t) + O(h_1^4/\lambda^4), \tag{5}$$

where $\varphi_1(x,\ t) = \phi_1(x,\ 0,\ t)$.

ϕ_2 is also the harmonic function and can be expanded as

$$\phi_2 = \varphi_2(x,t) + (y-\eta_1)\psi - \frac{1}{2}(y-\eta_1)^2\nabla^2\varphi_2(x,t) + O(h_2^3/\lambda^3), \tag{6}$$

where $\varphi_2(x,\ t) = \phi_2(x,\eta_1,\ t)$ and $\psi = [\partial\phi_2/\partial y]_{y=\eta_1}$. Under the shallow water assumption, we have

$$\Phi_1 = \varphi_1 - \frac{1}{2}\eta_1^2\nabla^2\varphi_1 + O(h_1^4/\lambda^4), \tag{7a}$$

$$\Phi_2 = \varphi_2 + \xi\psi - \frac{1}{2}\xi^2\nabla^2\varphi_2 + O(h_2^3/\lambda^3), \tag{7b}$$

in which $\xi = \eta_2 - \eta_1$ is the "kinetic thickness" of lower fluid; Φ_1 is the velocity potential of lower fluid at the interface; Φ_2 is the velocity potential of upper fluid at the free surface. Substituting Eq. (5) into Eq. (2a) and neglecting the smaller quantities lead to

$$L_1^* = \eta_1\left(\varphi_{1,t} + \frac{1}{2}\varphi_{1,x_i}\varphi_{1,x_i}\right) + \frac{1}{2}g\eta_1^2 - \frac{1}{6}\eta_1^3[\nabla^2\varphi_{1,t} + \varphi_{1,x_i}\nabla^2\varphi_{1,x_i} - (\nabla^2\varphi_1)^2]. \tag{8}$$

Inserting Eq. (7a) into Eq. (8) and taking

$$\iint_\Omega \eta_1\Phi_{1,t}\mathrm{d}x\mathrm{d}t = \iint_\Omega \frac{\partial}{\partial t}(\eta_1\Phi_1)\mathrm{d}x\mathrm{d}t - \iint_\Omega \eta_{1,t}\Phi_1\mathrm{d}x\mathrm{d}t, \tag{9}$$

into account, the first term of right hand side in Eq. (9) turns into the boundary term in variational approach, so Eq. (8) is reduced to

$$L_1^* = -\eta_{1,t}\Phi_1 + \left[\frac{1}{2}\eta_1\Phi_{1,x_i}\Phi_{1,x_i} + \frac{1}{2}g\eta_1^2 - \frac{1}{6}\eta_1^3(\nabla^2\Phi_1)^2\right]. \tag{10}$$

Substituting Eq. (6) into Eq. (2b) and using Eq. (7b), we obtain

$$\int_{\eta_1}^{\eta_2}\phi_t\mathrm{d}y = \xi\varphi_{2,t} + \frac{1}{2}\xi^2\psi_t - \frac{1}{6}\xi^3\nabla^2\varphi_{2,t} = \xi\Phi_t - \frac{1}{2}\frac{\partial}{\partial t}(\xi^2\psi) + \frac{1}{3}\frac{\partial}{\partial t}(\xi^3\nabla^2\varphi_2), \tag{11}$$

$$\frac{1}{2}\int_{\eta_1}^{\eta_2}\nabla\phi_2\nabla\phi_2\mathrm{d}y = \frac{1}{2}\Big[\xi(\nabla\varphi_2)^2 + \frac{1}{3}\xi^3(\nabla\psi)^2 - \frac{1}{3}\xi^3\nabla\varphi_2\nabla\nabla^2\varphi_2 + \xi^2\nabla\varphi_2\nabla\psi + \xi\psi^2 - \xi^2\psi\nabla^2\varphi_2 + \frac{1}{3}\xi^3(\nabla\varphi_2)^2\Big] =$$

$$\frac{1}{2}\xi(\nabla\Phi_2)^2 - \frac{1}{6}\xi^3(\nabla^2\Phi_2)^2 + \frac{1}{2}\xi\psi^2 - \frac{1}{3}\xi^3(\nabla\psi)^2 - \frac{1}{2}\xi(\nabla\xi)^2\psi^2 -$$

$$\xi^2 \nabla \xi \nabla \psi\psi - \frac{1}{2} \nabla (\xi^2 \psi \nabla \varphi_2) + \frac{1}{3} \nabla (\xi^3 \nabla \varphi_2 \nabla^2 \varphi_2). \tag{12}$$

At the interface of two fluids, the normal components of velocities are equal and by using Eq. (3e) we approximately have

$$\left.\frac{\partial \phi_1}{\partial y}\right|_{y=\eta_1} = \left.\frac{\partial \phi_2}{\partial y}\right|_{y=\eta_1} = \psi = \eta_{1,t}, \tag{13}$$

because under the shallow water assumption $\nabla \eta_1$ and $\nabla \eta_2$ are small.

Discarding the boundary terms which have no effect on the results in variational approach, we can obtain the Hamiltonian in the corresponding Hamilton's principle

$$\tilde{L} = -L^* = \eta_{1,t}\Phi_1 + \sigma\xi_t\Phi_2 - \left[\frac{1}{2}\eta_1(\nabla \Phi_1)^2 + \frac{1}{2}g\eta_1^2 - \frac{1}{6}\eta_1^3(\nabla^2\Phi_1)^2\right] - \sigma\left[\frac{1}{2}\xi(\nabla \Phi_2)^2 - \frac{1}{6}\xi^3(\nabla^2\Phi_2)^2 + \frac{1}{2}\xi\eta_{1,t}^2 + \frac{1}{2}g\xi(\xi + 2\eta_1)\right]. \tag{14}$$

Set $\zeta_1 = \eta_1$ and $\zeta_2 = \sigma\xi$. Then Eq. (14) reduces to

$$\tilde{L} = \zeta_{1,t}\Phi_1 + \zeta_{2,t}\Phi_2 - \left[\frac{1}{2}\zeta_1(\nabla \Phi_1)^2 + \frac{1}{2}g\zeta_1^2 - \frac{1}{6}\zeta_1^3(\nabla^2\Phi_1)^2\right] - \left[\frac{1}{2}\zeta_2(\nabla \Phi_2)^2 - \frac{1}{6\sigma^2}\zeta_2^3(\nabla^2\Phi_2)^2 + \frac{1}{2}\zeta_2\zeta_{1,t}^2 + \frac{1}{2\sigma}g\zeta_2(\zeta_2 + 2\sigma\zeta_1)\right]. \tag{15}$$

We may as well choose (ζ_1, ζ_2) as the generalized displacements of the two-fluid system. By definition we have

$$p_1 = \frac{\partial \tilde{L}}{\partial \zeta_{1,t}} = \Phi_1, \quad p_2 = \frac{\partial \tilde{L}}{\partial \zeta_{2,t}} = \Phi_2, \tag{16}$$

which indicates that the generalized momenta of the system are (Φ_1, Φ_2). For single-layer fluid, the generalized displacement and momentum are respectively the elevation η of free surface and the velocity potential Φ at the free surface and thus (η, Φ) constitute the canonical variables of the system, Eqs. (15) and (16) indicate that for the two-fluid system, two pairs of dual variables, (ζ_1, Φ_1) and (ζ_2, Φ_2) form the canonical variables, in which (ζ_1, ζ_2) are the "kinetic thickness" of lower fluid and the "reduced kinetic thickness" of upper fluid respectively. Making the Legendre transformation for Eq. (15) gives the Hamiltonian of the two-fluid system

$$\tilde{H} = \left[\frac{1}{2}\zeta_1(\nabla \Phi_1)^2 + \frac{1}{2}g\zeta_1^2 - \frac{1}{6}\zeta_1^3(\nabla^2\Phi_1)^2\right] + \left[\frac{1}{2}\zeta_2(\nabla \Phi_2)^2 + \frac{1}{2}g\zeta_2\left(\frac{\zeta_2}{\sigma} + 2\zeta_1\right) - \frac{1}{6\sigma^2}\zeta_2^3(\nabla^2\Phi_2)^2 + \frac{1}{2}\zeta_2\zeta_{1,t}^2\right]. \tag{17}$$

Thus far we have obtained the Hamiltonian functional for the considered system

$$H = \iint_{S_0} \tilde{H}\,\mathrm{d}\boldsymbol{x}. \tag{18}$$

It is easy to verify that the following relation holds:

$$\tilde{H} = \int_0^{\eta_1}\left(\frac{1}{2} | \nabla \phi_1 |^2 + gy\right)\mathrm{d}y + \sigma\int_{\eta_1}^{\eta_2}\left(\frac{1}{2} | \nabla \phi_2 |^2 + gy\right)\mathrm{d}y, \tag{19}$$

which means H in Eq. (18) is the total energy of the system. It can be shown that from the variational principle

$$\delta\iint_\Omega (\zeta_1\Phi_1 + \zeta_2\Phi_2 - \tilde{H})\,\mathrm{d}\boldsymbol{x}\,\mathrm{d}t = 0, \tag{20}$$

we can derive the governing equations and the complete set of boundary conditions. And Eq.

(20) is an alternative formulation of Eq. (1), which has dynamical equivalence with Eq. (2) (see Ref. [9]). Following the procedure in Ref. [9], we can give out the Hamiltonian canonical equation for the system

$$U_t = J\frac{\delta H}{\delta U}, \tag{21}$$

where $U = (\zeta_1, \zeta_2, \Phi_1, \Phi_2)^{\mathrm{T}}$, $J = \begin{pmatrix} 0 & I_2 \\ -I_2 & 0 \end{pmatrix}$, I_2 is the 2×2 unit matrix and J is the measure matrix of simplectic geometry. Substituting Eqs. (17) and (18) into the Hamiltonian canonical equation (21) yields the governing equations for the two-fluid system

$$\zeta_{1,t} = \frac{\delta H}{\delta \Phi_1} = -\nabla(\zeta_1 \nabla \Phi_1) - \frac{1}{3}\nabla^2(\zeta_1^3 \nabla^2 \Phi_1), \tag{22a}$$

$$\zeta_{2,t} = \frac{\delta H}{\delta \Phi_2} = -\nabla(\zeta_2 \nabla \Phi_2) - \frac{1}{3\sigma^2}\nabla^2(\zeta_2^3 \nabla^2 \Phi_2), \tag{22b}$$

$$\Phi_{1,t} = -\frac{\delta H}{\delta \zeta_1} = -\frac{1}{2}(\nabla \Phi_1)^2 - g(\zeta_1 + \zeta_2) + \frac{1}{2}\zeta_1^2(\nabla^2 \Phi_1)^2 + (\zeta_{1,t}\zeta_2)_t, \tag{22c}$$

$$\Phi_{2,t} = -\frac{\delta H}{\delta \zeta_2} = -\frac{1}{2}(\nabla \Phi_2)^2 - g\left(\zeta_1 + \frac{\zeta_2}{\sigma}\right) + \frac{1}{2\sigma^2}\zeta_2^2(\nabla^2 \Phi_2)^2 - \frac{1}{2}\zeta_{1,t}^2. \tag{22d}$$

Nondimensionize Eq. (22) and only consider two-dimensional flow. Let

$$\nabla \Phi_1 \to u_1, \nabla \Phi_2 \to u_2, \tag{23}$$

and

$$\eta_1 = \zeta_1 = 1 + \zeta_1, \ \eta_2 - \eta_1 = \zeta_2/\sigma = r(1 + \zeta_2), \tag{24}$$

where $r = h_2/h_1$ is the thickness ratio of upper and lower fluids at rest. Under the linearization approximation, Eq. (22) turns into

$$\left.\begin{aligned} &\frac{\partial \zeta_1}{\partial t} + \frac{\partial u_1}{\partial x} = 0, \quad \frac{\partial \zeta_2}{\partial t} + \frac{\partial u_2}{\partial x} = 0, \\ &\frac{\partial u_1}{\partial t} + \frac{\partial \zeta_1}{\partial x} + \sigma\frac{\partial \zeta_2}{\partial x} = 0, \quad \frac{\partial u_2}{\partial t} + \frac{\partial \zeta_2}{\partial x} + r\frac{\partial \zeta_1}{\partial x} = 0. \end{aligned}\right\} \tag{25}$$

The corresponding characteristic equation is

$$(C^2 - 1)(C^2 - r) - \sigma r = 0, \tag{26}$$

which has solution

$$C_\pm^2 = \frac{1}{2}[(1 + r) \pm \sqrt{(1 - r^2) + 4\sigma r}], \tag{27}$$

C_+ and C_- are the velocities of linear gravity waves, corresponding to the fast mode (surface mode) and slow mode (internal mode) respectively (see Ref. [10]).

For the two-fluid system considered herein, Dai Shiqiang derived the generalized Boussinesq equation, starting from the shallow water approximation and taking vertical average of velocities[10]. The equations in Eq. (22) are slightly different from those presented by Dai Shiqiang, while the linearized form Eq. (25) agrees with those in Ref. [10]. The dispersion term derived with the shallow water approximation for the Hamiltonian system contains only the derivatives with respect to space variables, which is different from that in the Boussinesq equation, which is the derivative with respect to space and time variables.

2 Conclusion

In the present paper, for a two-fluid system, the appropriate canonical variables (i. e., generalized displacements and momenta) have been chosen for the first time, that is, the kinetic thickness of the lower fluid and the reduced kinetic thickness of upper fluid have been taken as the generalized displacements and the velocity potentials at interface and free surface as the generalized momenta respectively. The results show that the procedure presented above is an effective way to enter the Hamiltonian system. Exactly speaking, one should first establish Hamilton's principle for the system, then define the appropriate canonical variables and finally obtain the Hamiltonian canonical equations via the Legendre transformation. The results herein are the extention of those in Refs. [3], [4] and [9].

References

[1] Luke J C. A variational principle for a fluid with a free surface [J]. *J Fluid Mech*, 1967, **27**:395 ~ 397

[2] Whitham G B. Variational methods and application to water waves[J]. *Proc Roy Soc A*, 1967, **299**(1):6 ~ 25

[3] Zakharov V E. Stability of periodic wave of finite amplitude on the surface of a deep fluid [J]. *J Appl Mech Tech Phys*, 1968, **2**(2):190 ~ 194

[4] Miles J W. On Hamilton's principle for surface waves [J]. *J Fluid Mech*, 1977, **83**:153 ~ 158

[5] Milder D M. A note regarding "On Hamilton's principle for surfaces waves"[J]. *J Fluid Mech*, 1977, **83**:159 ~ 161

[6] Benjamin T B, Olver P J. Hamiltonian structure, symmetries and conservation laws for water wave [J]. *J Fluid Mech*, 1982, **125**:137 ~ 185

[7] Zhang Baoshan, Lu Dongqiang, Dai Shiqiang, et al.. Theory of Hamiltonian system for nonlinear waves and its applications [J]. *Advan in Mech*, 1998, **28** (4):521 ~ 531 (in Chinese)

[8] Craig W, Groves M D. Hamiltonian long-wave approximations to the water waves problems [J]. *Wave Motion*, 1994, **19**:367 ~ 389

[9] Lu Dongqiang, Dai Shiqiang, Zhang Baoshan. Infinite-dimensional Hamiltonian structure for nonlinear water waves [A]. In: Cheng Changjun, Dai Shiqiang, Liu Yulu eds. *Modern Mathematics and Mechanics* (*MMM*-Ⅶ) [C]. Shanghai: Shanghai University Press, 1997, 387 ~ 390 (in Chinese)

[10] Dia Shiqiang. The interaction of two pairs of solitary waves is a two-fluid system [J]. *Scientia Sinica* (*Ser. A*), 1984, **27**(5):507 ~ 520