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APPLICATIONS OF WAVELET GALERKIN FEM TO BENDING OF BEAM AND PLATE STRUCTURES*

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Abstract

In this paper, an approach is proposed for taking calculations of high order differentials of scaling functions in wavelet theory in order to apply the wavelet Galerkin FEM to numerical analysis of those boundary-value problems with order higher than 2. After that, it is realized that the wavelet Galerkin FEM is used to solve mechanical problems such as bending of beams and plates. The numerical results show that this method has good precision.

Key words applications of wavelet theory, scaling functions, operation of high-order derivations, Galerkin FEM, bending of beams and plates

I. Introduction

It has been found that the wavelet theory is a powerful mathematical tool developed in recent years. As a new mathematical tool, it has been extensively applied in the analysis of signal process, partten recognition, function approximation, and solving differential equation (s), etc.. Since a small signal in a signal process can be captured by the wavelet theory, its applications have been paid much attention both in theory and in engneering[1~5]. Recently, the wavelet theory has been generalized to find a numerical solution of a differential equation. For example, the wavelet theory combined with the Galerkin FEM method is successfully used to solve twopoint boundary-value problem with the differential equation of order two. In this method, the domain is auto-discretized, and the admissible function of the Galerkin method is taken as the scaling functions of a wavelet theory. It is found that this method has high precision and fast convegence. However, its applications in this area have been limited to the case of the boundary-value problems with the order of differential equations not higher than 2, i. e., the Laplace equations and the Poisson's equations in electromagnetic fields, since it is not easy to obtain the derivations of order 2 or higher to the scaling functions. Due to the order of differential equations in mechanics of beam and plate structures is 4 generally, it is a key step to find a way to perform the calculations of high-order derivations and the terms of multiple of one derivative function with others for applying the wavelet FEM in mechanics of structures.

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Here, we will introduce a way to carry out the calculations of high order derivations of the Daubechies' scaling functions and their multiple terms such that the application of the wavelet FEM in structural mechanics is made possible. In order to show the possibility, some examples of applying wavelet FEM to beam and plate bending are exhibited numerically. It is shown that this method has high precision and reliability.

II. Basic Concepts of Wavelet Theory

2.1 Scaling function and wavelet function

Definition 1 A function $\phi(x)\in L^2$ is said to be a scaling or dilation function if it has the following properties or characteristics:

(1) there is a set of sequence $\{p(k)\}_{k\in Z}$ such that

$$\phi(x) = \sum_k p(k)\phi(2x-k) \tag{2.1}$$

is held;

(2) a set of bases of scaling functions, $\{\phi_{n,k}(x)\}_{n,k\in Z}$, satisfies the orthonomarlized condition

$$\int_{-\infty}^{\infty}\phi_{n,k}(x)\phi_{n,l}(x)\,dx = \delta_{kl} \tag{2.2}$$

For a function $f(x)\in L^2$ let the scaling transformation

$$A_n f(x) = \sum_k a_{n,k}\phi_{n,k}(x) \tag{2.3}$$

in which

$$a_{n,k} = \langle f(x),\phi_{n,k}(x)\rangle = \int_{-\infty}^{\infty} f(x)\phi_{n,k}(x)\,dx \tag{2.4}$$

$$\phi_{n,k}(x) = 2^{n/2}\phi(2^n x-k) \tag{2.5}$$

(3) the set of function space

$$V_n = \left\{\sum_k a_k\phi_{n,k}(x);\, n,k\in Z, a_k\in R\right\} \tag{2.6}$$

has the relationship

(i) $V_n \subset V_{n+1}$;

(ii) $A_n f(x)\in V_n \leftrightarrow A_n f(2x)\in V_{n+1}$;

(iii) $A_n f(x)\in V_n \leftrightarrow A_n f(x+2^{-n})\in V_n$;

(iv) $\lim_{n\to\infty} V_n = \bigcup_n V_n$ is dense in $L^2(R)$;

(v) $\lim_{n\to-\infty} \bigcap_n V_n = \{\emptyset\}$;

(vi) the set $\{\phi(x-k)\}$ forms a Riese or unconditional bases for V_0, that is, there exist constants A and B, with $0 < A \leqslant B < \infty$ such that

$$A\sum_{k\in Z}|c_k|^2 \leqslant \left\|\sum_{k\in Z}c_k\phi(x-k)\right\|_2^2 \leqslant B\sum_{k\in Z}|c_k|^2$$

for any sequence $\{c_k\}\in l_2$ the space of all square summable sequences. Here, the coefficients $p(k)$ are called filter coefficients and it is often the case that only a finite number of these are non-zero.

Definition 2 If there is a sequence $\{q(k)\}_{k\in Z}$ such that the function

$$\psi(x) = \sum_k q(x)\phi(2x - k) \tag{2.7}$$

satisfies the conditions of the form

(1) the function sequence

$$\psi_{n,k}(x) = 2^{n/2}\psi(2^n x - k) \tag{2.8a}$$

is orthogonal, i. e.,

$$\int_{-\infty}^{\infty} \psi_{n,k}(x)\psi_{m,l}(x)\,dx = \delta_{mn}\delta_{kl}, \qquad m,n,k,l \in Z \tag{2.8b}$$

or is semi-orthogonal characteristic

$$\int_{-\infty}^{\infty} \psi_{n,k}(x)\psi_{m,l}(x)\,dx = \delta_{mn}, \qquad m,n,k,l \in Z \tag{2.9}$$

or bi-orthogonal behavior

$$\int_{-\infty}^{\infty} \tilde{\psi}_{n,k}(x)\psi_{m,l}(x)\,dx = \delta_{mn}\delta_{kl}, \qquad m,n,k,l \in Z \tag{2.10}$$

in which $\tilde{\psi}_{n,k}(x)$ is a dual of function $\psi_{n,k}(x)$;

(2) For arbitrary coefficients b_k, the set of function space W_n, spanned by $\psi_{n,k}(x)$

$$W_n = \left\{\sum_k b_k\psi_{n,k}(x);\, n,k \in Z,\, b_k \in R\right\}$$

has the following properties

(i) $V_{n+1} = V_n \oplus W_n$;

(ii) for different integers m and n, W_n is orthogonal to W_m;

(iii) $\oplus_{n=-\infty}^{\infty} W_n = L^2(R)$;

then the function $\psi(x)$ is called a basic (or mother) wavelet function while the functions $\psi_{n,k}(x)$ are the bases of the wavelet function.

2.2 Decomposition and reconstruction

After a scaling function $\phi(x)$ and the corresponding wavelet function $\psi(x)$ defined in definitions 1 and 2, respectively, one can find the scaling transform

$$A_n f(x) = \sum_k a_{n,k}\phi_{n,k}(x) \tag{2.11}$$

$$a_{n,k} = \langle f(x), \phi_{n,k}(x)\rangle = \int_{-\infty}^{\infty} f(x)\phi_{n,k}(x)\,dx \tag{2.12}$$

in which the operator

$$A_n: L^2(R) \ni f(x) \mapsto A_n f(x) \in V_n \tag{2.13}$$

and the wavelet transform

$$D_n f(x) = \sum_k b_{n,k}\psi_{n,k}(x) \tag{2.14}$$

$$b_{n,k} = \langle f(x), \psi_{n,k}(x)\rangle = \int_{-\infty}^{\infty} f(x)\psi_{n,k}(x)\,dx \tag{2.15}$$

Here, the operator

$$D_n: L^2(R) \ni f(x) \mapsto D_n f(x) \in W_n \tag{2.16}$$

The process of finding the coefficients $a_{n,k}$ and $b_{n,k}$ is called decomposition of a function or signal process $f(x)$ while the summation of $A_n f(x)$ and $D_n f(x)$ is referred to as reconstruction under the meaning of scaling function and wavelet function, respectively. When n is large enough, one can show

$$f(x) \approx A_n f(x) = \sum_k a_{n,k}\phi_{n,k}(x) \tag{2.17}$$

The part of $D_n f(x)$ reflects the high frequency part of $f(x)$ associated with symbol n. In fact, from the definition 2, we have

$$f(x) \doteq \sum_n \sum_k b_{n,k}\psi_{n,k}(x) \tag{2.18}$$

which decomposes a function $f(x)$ into the parts of different frequencies. The symbol k characterizes the position of the function. Hence, $b_{n,k}$ behaves the characteristics of both frequency and position of a function or signal $f(x)$.

III. Daubechies' Wavelet Function and Its High Order Derivations

3.1 Scaling function and wavelet function

In the way of constructing Daubechies' wavelet, the coefficients $p(k)$ and $q(k)$ are taken as even-integer terms with non-zero, which is identified by symbol N. That is,

$$\phi_N(x) = \sum_{k=0}^{2N-1} p_N(k)\phi_N(2x-k) \tag{3.1a}$$

$$\psi_N(x) = \sum_{k=0}^{2N-1} q_N(k)\phi_N(2x-k) \tag{3.1b}$$

The coefficients are selected to satisfy the following conditions:
Orthogonal Conditions

$$\int_{-\infty}^{\infty} \phi_N(x)\phi_N(x-k)\,dx = \delta_{0k} \qquad k \in Z \tag{3.2a}$$

$$\int_{-\infty}^{\infty} \phi_N(x)\psi_N(x)\,dx = 0 \tag{3.2b}$$

Normalized Condition

$$\int_{-\infty}^{\infty} \phi_N(x)\,dx = 1 \tag{3.2c}$$

Additional Conditions

In the Daubechies' wavelet theory, it is set that the scaling functions can be used to exactly represent polynomials of order up to but not greater than N. That is

$$\sum_{r=0}^{N} \alpha_r x^r = \sum_{k=-\infty}^{\infty} c_k\phi_N(x-k) \tag{3.3a}$$

which leads to

$$\int_{-\infty}^{\infty} x^r\psi_N(x)\,dx = 0 \qquad (r = 0,1,2,\cdots,N) \tag{3.3b}$$

If we choose

$$q_N(k) = (-1)^k p_N(1-k) \tag{3.4}$$

the Eq. (3.2b) is auto-satisfied. Substitution of Eqs. (3.1) into Eqs. (3.2) and (3.3) leads to the nonlinear algebraic equations on unknowns $p_N(k)$. Table 1 shows the values of $p_N(k)$ for $N = 2, 3, 4, 5$. Once the values of $p_N(k)$ are gained, the values of $\phi_N(k)$ at the integer points can be obtained from Eqs. (3.1a) and (3.2c). Then repeat to use Eq. (3.1a), one can get the values of $\phi_N(x)$ at the dyadic points $k2^{-n}$. Thus, the scaling function $\phi_N(x)$ is constructed out numerically. After that, the wavelet function $\psi_N(x)$ can be numerically generated from Eq. (3.1b). Fig. 1 plots the curves of the Daubechies's scaling functions.

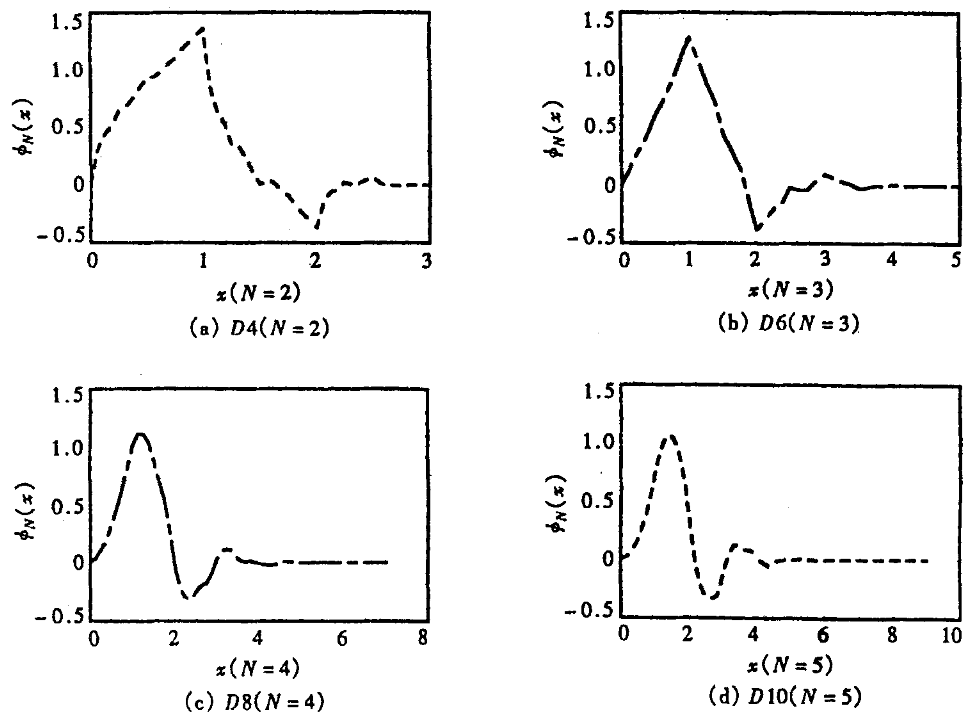


Fig. 1 Plots of Daubechies' scaling functions

Table 1 Filter coefficients $p_N(k)$ of the Daubechies' scaling function

k	$p_N(k)$			
	$N=2$	$N=3$	$N=4$	$N=5$
0	0.6830127	0.4704672	0.3258034	0.2264190
1	1.1830127	1.1411169	1.0109457	0.8539435
2	0.3169873	0.6503650	0.8922001	1.0243269
3	-0.1830127	-0.1909344	-0.0395750	0.1957669
4		-0.1208322	-0.2645072	-0.3426567
5		0.0498175	0.0436163	-0.04560113
6			0.0465036	0.1097026
7			-0.0149869	-0.00882680
8				-0.01779197
9				0.0047174

3.2 Operation of derivations of Daubechies' scaling functions

Since a scaling function is, in general, expressed as numerical form, we have to take some special way to perform calculations of its differentials in order to retain a desirable precision. Taking derivations of the dilation function of Eq. (3.1a) with d order, we symbolly denote

$$\phi_N^{(d)}(x) = \sum_{k=0}^{2N-1} a_k^{(d)} \phi_N^{(d)}(2x - k) \tag{3.5}$$

in which

$$a_k^{(d)} = 2^d p_N(k) \tag{3.6}$$

Due to $\mathrm{supp}\phi_N(x) = [0,2N-1]$, one can get $\mathrm{supp}\phi_N^{(d)}(x) \subseteq [0,2N-1]$. Hence, the values of Eq. (3.5) at the integer points of the supported region $[0,2N-1]$ may be written by

$$\begin{aligned}
\phi_N^{(d)}(0) &= a_0^{(d)} \phi_N^{(d)}(0) \\
\phi_N^{(d)}(1) &= a_0^{(d)} \phi_N^{(d)}(2) + a_1^{(d)} \phi_N^{(d)}(1) + a_2^{(d)} \phi_N^{(d)}(0) \\
&\cdots\cdots \\
\phi_N^{(d)}(2N-2) &= a_{2N-3}^{(d)} \phi_N^{(d)}(2N-1) + a_{2N-2}^{(d)} \phi_N^{(d)}(2N-2) \\
&\quad + a_{2N-1}^{(d)} \phi_N^{(d)}(2N-3) \\
\phi_N^{(d)}(2N-1) &= a_{2N-1}^{(d)} \phi_N^{(d)}(2N-1)
\end{aligned}$$

which can be compactly formulated by matrix form

$$[\mathbf{A}]^{(d)} [\mathbf{\Phi}]^{(d)} = \mathbf{0} \tag{3.7}$$

where $[\mathbf{\Phi}]^{(d)} = [\phi_N^{(d)}(0), \phi_N^{(d)}(1), \cdots, \phi_N^{(d)}(2N-1)]^T$, $[\mathbf{A}]^{(d)}$ is a square matrix whose elements are the coefficients of above aglebraic equations with unknows $[\mathbf{\Phi}]^{(d)}$. It is obvious that $[\mathbf{A}]^{(d)}$ is singular. In order to find the non-zero solution of Eq. (3.7), here, we add some conditions as follows. Let

$$x^d = \sum_{k=-\infty}^{\infty} c_k \phi_N(x - k) \tag{3.8}$$

Then, we have

$$\begin{aligned}
c_k &= \langle x^d, \phi_N(x - k) \rangle \\
&= \sum_{i=0}^{d} C_d^i k^{d-i} \langle x^i, \phi_N(x) \rangle = \sum_{i=0}^{d} C_d^i k^{d-i} A_i
\end{aligned} \tag{3.9}$$

in which

$$A_i = \langle x^i, \phi_N(x) \rangle \qquad (i = 0,1,2,\cdots,d) \tag{3.10}$$

Considering the dilation Eq. (3.1a) and Eq. (3.10), one can find the recurrence formulas of the form

$$A_i = 2^{-(i+1)} \sum_{k=0}^{2N-1} \sum_{j=0}^{i} C_i^j k^{i-j} p_N(k) A_j \tag{3.11}$$

from which we can get $A_0, A_1, \cdots, A_d$. After that, c_k can be determined by Eq. (3.9). Taking the operation of d order derivations of Eq. (3.8) with respect to x, then calculating the values of them at one integer point, such as $x=0$ point without lossing generality, in the supported region, we get

$$\sum_{k=0}^{2N-1} c_k \phi_N^{(d)}(x) = d! \tag{3.12}$$

Adding Eq. (3.12) into Eq. (3.7), we can find the values of $\phi_N^{(d)}(x)$ at all integer points in the supported region, that is, we can get $\phi_N^{(d)}(j), j \in Z$. According to Eq. (3.5), one can write

$$\phi_N^{(d)}\left(\frac{x}{2}\right) = \sum_{k=0}^{2N-1} 2^d p_N(k) \phi_N^{(d)}(x-k) \tag{3.13}$$

which can give the values of $\phi_N^{(d)}(x)$ at the dyadic points $\{i/2^n, i, n \in Z\}$.

Following this way proposed here, once $\phi_N^{(d)}(x)$ is numerically generated, the reconstruction of d order derivations of a function $f(x) \in L^2(R)$ can be conducted.

IV. Applications in Bending of Beams and Plates

In conventional Galerkin method, the admissible function should be chosen to satisfy the boundary conditions of the problem. However, in wavelet Galerkin FEM, the admissible functions are employed as the base functions of a scaling function. Due to independence of them on the boundary conditions, the algebraic equations to determine unknowns consist of two parts, one is generated from the differential equation(s) while another one is from the boundary conditions.

4.1 Application to bending of beams

From the Euler-Bernonlli beam theory, after the quantities are non-dimensionlized, we can write the governing equation of beam bending in the dimensionless form

$$\frac{d^4 u}{dx^4} = q(x), \qquad 0 < x < 1 \tag{4.1}$$

with boundary conditions

$$\text{Rigidly Clamped Support:} \quad u = \frac{du}{dx} = 0 \quad x = 0 \text{ or/and } x = 1 \tag{4.2a}$$

$$\text{Simple Support:} \quad u = \frac{d^2 u}{dx^2} = 0 \quad x = 0 \text{ or/and } x = 1 \tag{4.2b}$$

$$\text{Free End:} \quad \frac{d^2 u}{dx^2} = \frac{d^3 u}{dx^3} = 0 \quad x = 0 \text{ or/and } x = 1 \tag{4.2c}$$

Let the solution of Eq. (4.1) be

$$u(x) \approx A_m u(x) = \sum_k a_{m,k} \phi_{m,k}(x) \tag{4.3}$$

Substituting Eq. (4.3) into Eq. (4.1), and integrating the resulted equations multipled with the weighted functions $\phi_{m,j}(x)$ by the Galerkin method, we have

$$\int_0^1 \phi_{m,j}(x) \left[\frac{d^4}{dx^4} \left(\sum_k a_{m,k} \phi_{m,k}(x) \right) - q(x) \right] dx = 0 \tag{4.4}$$

Denote

$$k_{jk} = \int_0^1 \phi_{m,j}(x) \frac{d^4}{dx^4} \phi_{m,k}(x) dx \tag{4.5}$$

$$f_j = \int_0^1 \phi_{m,j}(x) q(x) dx \tag{4.6}$$

$$[\mathbf{K}] = [k_{jk}], [\mathbf{F}] = [f_j], [\mathbf{a}] = [a_{m,k}]$$

Then Eq. (4.4) can be reduced into the matrix form

$$[\mathbf{K}][\mathbf{a}] = [\mathbf{F}] \tag{4.7}$$

Four equations from the boundary conditions have to be added into the system equations for unknowns [**a**]. For example, to a simply supported beam, the added equations are

$$\left.\begin{aligned} u(0) &= \sum_k a_{m,k}\phi_{m,k}(0) = 0, \ u''(0) = \sum_k a_{m,k}\phi''_{m,k}(0) = 0 \\ u(1) &= \sum_k a_{m,k}\phi_{m,k}(1) = 0, \ u''(1) = \sum_k a_{m,k}\phi''_{m,k}(1) = 0 \end{aligned}\right\} \tag{4.8}$$

Solving the solution of Eq. (4.7) with conditions of Eqs. (4.8), one can get the wavelet Galerkin FEM solution to the original boundary-value problem of beam bending.

4.2 Application to bending of plates

For the simplicity, here, we focus our attention on bending of rectangular plates. Taking a Cartesian coordinate system in which x- and y- axes parallal to two sides of plate, respectively. By means of dimensionless of quantities, similar to process of beam bending, we can write the deflection equation of plates

$$\nabla^4 u(x,y) = q(x,y), \qquad 0 < x,y < 1 \tag{4.9}$$

For simply supported conditions, we have

$$\left.\begin{aligned} & u|_{x=0} = u|_{x=1} = 0, \ u|_{y=0} = u|_{y=1} = 0 \\ & \left.\frac{\partial^2 u}{\partial x^2}\right|_{x=0} = \left.\frac{\partial^2 u}{\partial x^2}\right|_{x=1} = 0, \ \left.\frac{\partial^2 u}{\partial y^2}\right|_{y=0} = \left.\frac{\partial^2 u}{\partial y^2}\right|_{y=1} = 0 \end{aligned}\right\} \tag{4.10}$$

In wavelet theory of 2-dimension, the bases of scaling function $\phi_{m,j,k}(x,y)$ are taken as

$$\phi_{m,j,k}(x,y) = \phi_{m,j}(x)\phi_{m,k}(y) \tag{4.11}$$

There is no defficulty to show that the functions $\phi_{m,j,k}(x,y)$ satisfy orthonormal conditions of bases of scaling functions. Let

$$u(x,y) \approx A_m u(x,y) = \sum_{j,k} a_{m,j,k}\phi_{m,j,k}(x,y) \tag{4.12}$$

Substituting Eq. (4.12) into (4.9), taking the weighted functions as $\phi_{m,j',k'}(x)$, and integrating the resulted equation, one can get the system of algebraic equations with unknows $a_{m,j,k}$ in matrix form

$$[\mathbf{K}][\mathbf{a}] = [\mathbf{F}] \tag{4.13}$$

in which the elements of coefficient matrix $[\mathbf{K}]$ and the column matrix $[\mathbf{F}]$ of inhomogeneous term are respectively

$$k_{j'k',jk} = \int_0^1\int_0^1 \phi_{m,j',k'}(x,y)\,\nabla^4\phi_{m,j,k}(x)\,dxdy \qquad (4.14a)$$

$$f_{j',k'} = \int_0^1\int_0^1 q(x,y)\,\phi_{m,j',k'}(x,y)\,dxdy \qquad (4.14b)$$

with additional conditions from boundary conditions

$$u(0,y) = \sum_{j,k} a_{m,j,k}\phi_{m,j,k}(0,y) = 0 \qquad (4.15a)$$

$$u(1,y) = \sum_{j,k} a_{m,j,k}\phi_{m,j,k}(1,y) = 0 \qquad (4.15b)$$

$$u(x,0) = \sum_{j,k} a_{m,j,k}\phi_{m,j,k}(x,0) = 0 \qquad (4.15c)$$

$$u(x,1) = \sum_{j,k} a_{m,j,k}\phi_{m,j,k}(x,1) = 0 \qquad (4.15d)$$

$$\frac{\partial^2 u(0,y)}{\partial x^2} = \sum_j\sum_k a_{m,j,k}\phi''_{m,j}(0)\phi_{m,k}(y) = 0 \qquad (4.15e)$$

$$\frac{\partial^2 u(1,y)}{\partial x^2} = \sum_j\sum_k a_{m,j,k}\phi''_{m,j}(1)\phi_{m,k}(y) = 0 \qquad (4.15f)$$

$$\frac{\partial^2 u(x,0)}{\partial y^2} = \sum_j\sum_k a_{m,j,k}\phi_{m,j}(x)\phi''_{m,k}(0) = 0 \qquad (4.15g)$$

$$\frac{\partial^2 u(x,1)}{\partial y^2} = \sum_j\sum_k a_{m,j,k}\phi_{m,j}(x)\phi''_{m,k}(1) = 0 \qquad (4.15h)$$

From Eqs. (4.13) and (4.15), we can obtain the coefficients $a_{m,j,k}$'. Further, the approximate solution of Eq. (4.11) is got. For other boundary conditions, its wavelet Galerkin solution may be obtained similarly.

V. Numerical Results

Here, the scaling function of Daubechies' wavelet theory is constructed in the supported region $[0, 2N-1]$ for resolution level $m=0$. Fig. 1 plots the scaling functions of Daubechies' scaling functions $\phi_N(x)$ for $N=2, 3, 4, 5$ (this is, it is identified by $D4$, $D6$, $D8$, and $D10$, respectively). In order to show the efficience of calculation of high order derivations of the scaling functions, Fig. 2 exhabites the comparison of the numerical and exact results of one order differential of $f(x) = 0.5\sin 2x$ with respect to x. Afterward, the bending of beams with different supported ends and under distributed load $q(x) = 1.0$ or $q(x) = x$ is solved by wavelet FEM introduced in previous section. Fig. 3 and Fig. 4 shows the deflection curves of a simply supported beam (S-S), and a beam with one clamped end and another simply supported end (C-S) under these two kinds of transverse loads, respectively. In these figures, the wavelet FEM solutions are got from the scaling function of Daubechies' wavelet theory with $m=0$ and $N=5$. Fig. 5 displays the

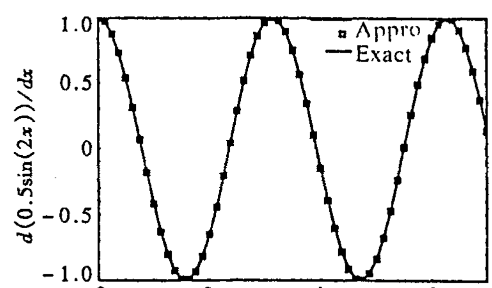
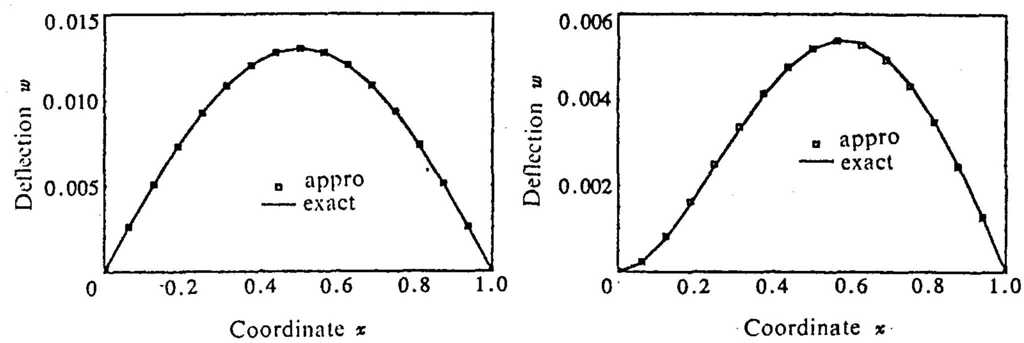


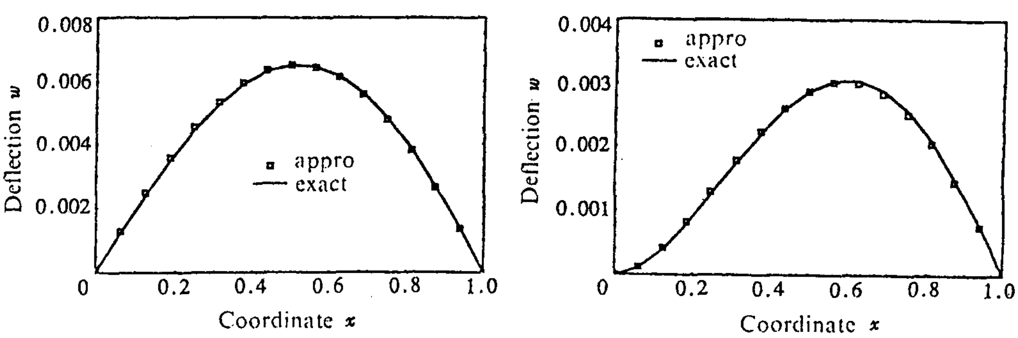
Fig. 2 Simulation of derivative function of 0.5sin2x with respective to x by means of the wavelet theory

comparison of deflection curves of simply supported plates under distributed load $q(x,y) = 4\pi^4\sin(\pi x)\sin(\pi y)$ from wavelet FEM solutions for $N=3$, 4, 5 and the exact solution of the problem. From the numerical results, it is found that the wavelet FEM solutions to bending problems of beam and plate structures have good precision, and they approach to exact solutions of the problems with increase of supported region of the scaling functions or symbol N. In general, it is enough for the precision of the wavelet FEM solutions when $N=5$.



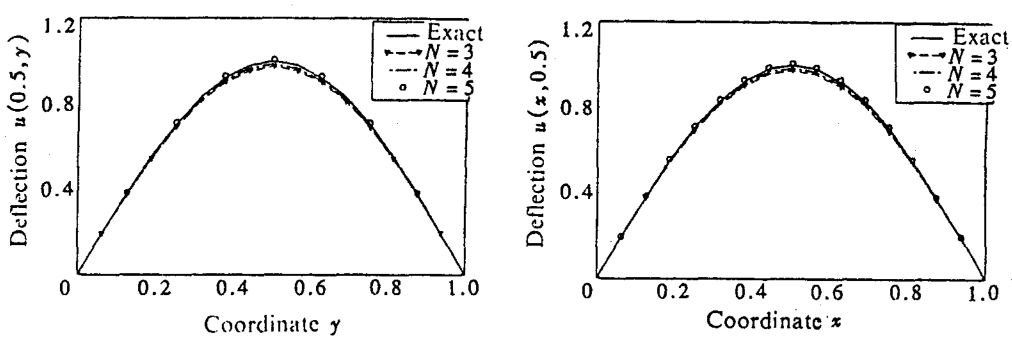
(a) S-S supported beam (b) C-S supported beam

Fig. 3 Comparison of approximate solutions from wavelet Galerkin FEM and exact solution for deflection of beams under load $q(x)=1$



(a) S-S supported beam (b) C-S supported beam

Fig. 4 Comparison of approximate solutions from wavelet Galerkin FEM and exact solution for deflection of beams under load $q(x)=x$



(a) Deflection curve at the central line along y-direction ($x=0.5$) (b) Deflection curve at the central line along x-direction ($y=0.5$)

Fig. 5 Comparison of approximate solutions based on Wavelet Galerkin FEM with exact solution for deflection of simply supported plate under load ($q(x,y)=4\pi^4\sin\pi x\sin\pi y$)

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References

[1] R. L. Motard and B. Joseph, *Wavelet Applications in Chemical Engineering*, Kluwer Academic Publishers, Boston (1994).

[2] J. R. Williams and K. Amaratunga, Introduction to vavelets in engineering, *Internat. J. Numer. Methods Engrg.*, **37**, 14 (1994), 2365~2388.

[3] I. Daubechies, Orthonormal bases of compactly supported wavelets, *Comm. Pure Appl. Math.*, **41**, 7 (1988), 909~996.

[4] K. Amaratunga and J. William, Wavelet-Galerkin solution for one-dimensional partial differential equations, *Internat. J. Numer. Methods in Engrg.*, **37**, 16 (1994), 2703~2716.

[5] J. Ko, A. J. Kurdila and M. Pilant, A class of wavelet-based finite element methods for computational mechanics, *Proc. 35th Structures, Structural Dynamics and Materials Conference*, Hilton Head, South Carolina, May (1994), 665~675.

[6] Y. H. Zhou and J. Z. Wang, Generalinzed Gaussian method and its application in solving nonlinear boundary-value problems of ordinary differential equations, *Proc. 7th National Modern Math. Mech. of China*, Shanghai, Nov. (1997), 464~467.(in Chinese)