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STUDIES ON THE DYNAMIC BUCKLING OF CIRCULAR PLATE IRRADIATED BY LASER BEAM *

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Abstract: *The dynamic buckling of thin copper plate induced by laser beam, was analyzed with the numerical integration and disturbance methods of controlling equation. The buckling and post-buckling of thin plate were shown, with the consideration of the temperature distribution, inertia effect and initial deflection. At last, the buckling criterion about the circular plate was obtained and used to investigate the relation between the critical laser intensity and the ratio of thickness and diameter of the plate. The results fit the experimental observation and the FEM simulation very well, and benefit to the understanding of failure phenomenon of structures irradiated by laser beam.*

Key words: laser; buckling; thermal shock

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Introduction

Dynamic buckling is an important failure phenomenon of the structure, which is irradiated by strong laser beam. Buckling will result in the catastrophic reduction of enduring loading ability of structures and cause some other failures. In the so-called "reverse plugging effect", which was reported firstly by our group[1], the thermal dynamic buckling of thin plate plays a key role.

In the last forty years, the dynamic buckling and instability of elastic structure has gripped a large number of researcher's attention[2-5]. The first landmark research was done by Knoing and Taub, who investigated the stability characteristics of a cylinder with flaws loaded by recurrent force. In 1960s, an argument was accepted commonly, that is the stability of general structure should be analysed from the viewpoint of dynamics, by using the Lyapunov's theory. Some important conclusions about the dynamical buckling of thin circle plate can be found in the works and papers of Edstrom, Zizicas and Birkgam.

In this paper, a general method is used to analyse the instability of thin plate, that is applicable in both the quasi-static and dynamic condition, with or without the effect of flaws.

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First, the temperature distribution in the structure is obtained by integral transformation. Then, the basic controlling equations of the thermal deformation of plates are deduced with the virtual work principle. At last, a numerical integral method and linear perturbation theory are adopted to solve the unstable problem. The result fits the experimental observations and FEM simulations very well.

1 Temperature Distribution

In the failure of structures induced by strong laser beam, thermal dynamic buckling was often found, such as, a cylinder which is irradiated by laser beam, lost drastically the ability to endure the compression force along the axis direction, when the laser intensity is up to a critical value. In the reverse plugging of thin plate, a newly found failure mode owing to the irradiation of laser beam, buckling is an important stage and linkage between the thermal bending of thin plate and deformation localization. It triggers the localization and failure in the fringe of laser irradiation region in the plate and makes the fragment to move toward the light source.

Fig.1 shows the simple sketch map of thin plate buckling resulting from the laser beam irradiation. The diameter and thickness of the round thin plate are a, d respectively. The effect of laser beam can be considered as the second-type (thermal conductivity) thermal boundary condition. Outside the region irradiated, the thermal insulation condition is used to simplify the thermal analysis, and is acceptable. At the same time, the freely supported condition is selected in the edge of the plate.

The controlling equation, boundary conditions and initial conditions are as follows:

$$\frac{\kappa}{\rho c}\frac{\partial^2 T}{\partial z^2} = \frac{\partial T}{\partial t}, \tag{1}$$

$$t = 0, T = 0, \tag{2}$$

$$\begin{cases} \kappa \dfrac{\partial R}{\partial z}\bigg|_{z=\frac{d}{2}} = q, \\ \kappa \dfrac{\partial T}{\partial z}\bigg|_{z=-\frac{d}{2}} = q. \end{cases} \tag{3}$$

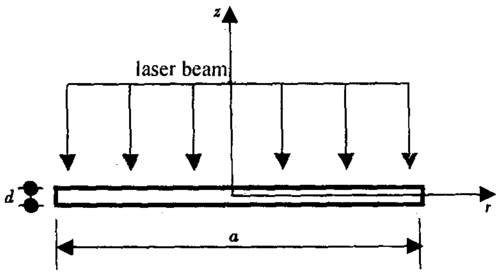

Fig.1 The sketch map of laser-induced buckling

By using the Laplace transformation, the solution of temperature field, which meets the above equations, can be gotten

$$T = \frac{2q(d/2)}{\kappa}\left[\frac{\beta t}{\pi^2} + \frac{3z^2 + 6hz - h^2}{24h^2} - \frac{2}{\pi^2}\sum_{n=1}^{\infty}\frac{(-1)^n}{n^2}e^{-n^2\beta t}\cos\frac{n\pi}{2}\left(\frac{z}{h}+1\right)\right], \tag{4}$$

$$\beta = \frac{\kappa}{\rho c}\frac{\pi^2}{4h^2}, \tag{5}$$

If the intensity of laser beam is non-uniform in the upper face of the plate, such as Gaussian distribution or double-Gaussian distribution, a more complex solution could be obtained by solving a 2-D thermal conductivity problem, which has a form of series of Bessel function.

2 Controlling Equation of Thin Plate

u, v, w represent the radius displacement, circumferential displacement of neutral surface and deflection of the plate, respectively. The displacement field of plate can be described as

$$\begin{cases} u_r = u - z\varphi(r,t), \\ u_\theta = 0, \\ u_z = w(r,t). \end{cases} \tag{6}$$

So, the relative strain field can be written as follows, if the strain of z direction is ignored

$$\varepsilon_{11} = \frac{\partial u}{\partial r} + \frac{1}{2}\left(\frac{\partial w}{\partial r}\right)^2 - z\frac{\partial^2 w}{\partial r^2}, \tag{7}$$

$$\varepsilon_{22} = \frac{u}{r} - \frac{z}{r}\frac{\partial w}{\partial r}, \tag{8}$$

$$\varepsilon_{12} = 0. \tag{9}$$

When an initial deflection w_0 exists, which is considered as a kind of defect, the strain fields can be got by simple derivation

$$\varepsilon_{11} = \frac{\partial u}{\partial r} + \frac{1}{2}\left(\frac{\partial w}{\partial r}\right)^2 - \frac{1}{2}\left(\frac{\partial w_0}{\partial r}\right)^2 - z\frac{\partial^2 w}{\partial r^2} + z\frac{\partial^2 w_0}{\partial r^2}, \tag{10}$$

$$\varepsilon_{22} = \frac{u}{r} - \frac{z}{r}\frac{\partial w}{\partial r} + \frac{z}{r}\frac{\partial w_0}{\partial r}, \tag{11}$$

$$\varepsilon_{12} = 0, \tag{12}$$

where

$$\tilde{\varepsilon}_{11} = \frac{\partial u}{\partial r} + \frac{1}{2}\left(\frac{\partial w}{\partial r}\right)^2 - \frac{1}{2}\left(\frac{\partial w_0}{\partial r}\right)^2, \tag{13}$$

$$\tilde{\varepsilon}_{22} = \frac{u}{r}. \tag{14}$$

Using the general Hooke law, the corresponding stresses fields are

$$\sigma_{11} = \frac{E(\varepsilon_{11} + \nu\varepsilon_{22})}{1-\nu^2} - \frac{E\alpha\theta}{1-\nu}, \tag{15}$$

$$\sigma_{22} = \frac{E(\varepsilon_{22} + \nu\varepsilon_{11})}{1-\nu^2} - \frac{E\alpha\theta}{1-\nu}, \quad \sigma_{12} = 0. \tag{16}$$

From Eqs. (10) – (12), (15) and (16), the elastic deformation energy of the plate can be depicted as

$$\Pi = \frac{\pi E d^3}{12(1-\nu^2)}\int_0^a \left\{\frac{12}{d^2}e_1^2 + [\nabla^2(w-w_0)]^2 + 2(1-\nu)\frac{\partial^2(w-w_0)}{\partial r^2}\frac{\partial(w-w_0)}{\partial r}\Big/ r\right\} r\,\mathrm{d}r - \frac{2E\alpha\pi}{1-\nu}\int_0^a [e_1\bar{\theta} - \nabla^2(w-w_0)\tilde{\theta}]\,r\,\mathrm{d}r - 2\pi\int_0^a (p - \rho d\ddot{w})\,w r\,\mathrm{d}r, \tag{17}$$

where

$$\bar{\theta} = \int_{-d/2}^{d/2}\theta(r,z)\,\mathrm{d}z, \tag{18}$$

$$\tilde{\theta} = \int_{-d/2}^{d/2}\theta(r,z)\,z\,\mathrm{d}z, \tag{19}$$

$$e_1 = \tilde{\varepsilon}_{11} + \tilde{\varepsilon}_{22}. \tag{20}$$

So, the controlling equations of the plate deformation is obtained by the virtual displacement principle

$$D\nabla^4(w-w_0)+K^2\nabla^2 w+\frac{E\alpha}{1-\nu}\nabla^2\bar{\theta}=p-\rho d\ddot{w}, \tag{21}$$

$$\frac{12D}{d^2}e_1-E\alpha\bar{\theta}/(1-\nu)=K^2, \tag{22}$$

$$D=\frac{Eh^3}{12(1-\nu^2)}, \tag{23}$$

where D is the bending stiffness of plates. Then, the freely supported boundary condition, expressed by displacement and deflection, can be rewritten as

$$\begin{cases} u=0, \\ w=0, \\ D\left[\dfrac{\partial^2(w-w_0)}{\partial r^2}+\dfrac{\nu}{r}\dfrac{\partial(w-w_0)}{\partial r}\right]+\dfrac{E\alpha}{1-\nu}\bar{\theta}=0. \end{cases} \tag{24}$$

3 Deflection and Instability

In order to simplify the analysis, the followings non-dimensional parameters are introduced as

$$w'=\frac{w}{d},\quad w_0'=\frac{w_0}{d},\quad \xi=\frac{r}{d},\quad C_0=\frac{d}{a}, \tag{25}$$

$$\bar{\theta}'=\frac{\bar{\theta}}{dT^*},\quad \alpha'=\alpha T^*,\quad \tilde{\theta}'=\frac{\tilde{\theta}}{d^2T^*}. \tag{26}$$

Then, the renew controlling equations are

$$\frac{1}{12(1-\nu^2)}C_0^2\frac{1}{\xi}\frac{\partial}{\partial\xi}\left\{\xi\frac{\partial}{\partial\xi}\left[\frac{1}{\xi}\frac{\partial}{\partial\xi}\left(\xi\frac{\partial(w'-w_0')}{\partial\xi}\right)\right]\right\}-\frac{K^2}{Ed}\frac{1}{\xi}\frac{\partial}{\partial\xi}\left(\xi\frac{\partial w'}{\partial\xi}\right)+\frac{\alpha'}{1-\nu}\frac{1}{\xi}\frac{\partial}{\partial\xi}\left(\xi\frac{\partial\tilde{\theta}'}{\partial\xi}\right)+\frac{d}{EC_0^2}\rho d\ddot{w}'=0, \tag{27}$$

$$\frac{1}{1-\nu^2}e_1-\frac{\alpha'\bar{\theta}'}{1-\nu}=\frac{K^2}{E\alpha}. \tag{28}$$

According to the series expansion method[4-5], presented by Poincare et al., we can get (the deflection of center of the plate is denoted by δ)

$$\frac{1}{12(1-\nu^2)}C_0^2(w'-w_0')=\frac{4K^2\delta}{Ed^2}\left(\frac{A_2}{64}\xi^4+\frac{A_4}{144}\xi^6\right)-\frac{\alpha'}{1-\nu}\sum_j\frac{\tilde{\theta}_j'}{(j+2)^2}\xi^{j+2}-\frac{\rho d\delta}{EC_0^2}\left(\frac{1}{64}\xi^4+\frac{A_2\xi^6}{576}+\frac{A_4\xi^8}{2\,304}\right)+\frac{C_1}{4}\xi^2(\lg\xi-1)+\frac{C_2}{4}\xi^2+C_3\lg\xi+C_4, \tag{29}$$

$$\frac{K^2}{Ed}=\left[\left(\frac{\delta}{d}\right)^2-\left(\frac{\delta_0}{d}\right)^2\right]\lambda_1-\frac{2\alpha'}{1-\nu}\sum_i\frac{\bar{\theta}_i'}{i+2}, \tag{30}$$

where

$$C_1=C_3=0,\quad A_2=\frac{-2(3+\nu)}{5+\nu},\quad A_4=\frac{1+\nu}{5+\nu}, \tag{31}$$

$$C_2=\frac{-2}{1+\nu}\left\{\frac{4K^2\delta}{Ed^2}\left[\frac{A_2}{16}(3+\nu)+\frac{A_4}{24}(5+\nu)\right]\right\}-\frac{2\alpha}{1+\nu}\sum_j\frac{\tilde{\theta}_j'}{j+2}+\frac{2\rho d}{1+\nu}\frac{\ddot{\delta}}{EC_0^2}\left[\frac{3+\nu}{16}+\frac{5+\nu}{96}A_2+\frac{\nu+7}{288}A_4\right], \tag{32}$$

$$C_4 = -\frac{K^2\delta}{ED^2}\left(\frac{A_2}{16}+\frac{A_4}{36}\right)+\frac{\alpha}{1-\nu}\sum_j\frac{\tilde{\theta}'_j}{(j+2)^2}+\frac{\rho d\ddot{\delta}}{EC_0^2}\left(\frac{1}{64}+\frac{A_2}{576}+\frac{A_4}{2\,304}\right)-\frac{C_2}{4}. \quad (33)$$

If the temperature field and temperature moment are expanded with the following series forms:

$$\bar{\theta}' = \sum_i \bar{\theta}'_i \xi^i \quad (i = 0,2,4,\cdots), \quad (34)$$

$$\tilde{\theta}' = \sum_j \tilde{\theta}'_j \xi^j \quad (j = 0,2,4,\cdots). \quad (35)$$

And only the deflection in the plate center point is discussed, then

$$f_1\frac{\ddot{\delta}}{d}+f_2\left(\frac{\delta}{d}\right)^3+f_3\frac{\delta}{d}+f_4 = 0, \quad (36)$$

where

$$f_1 = \frac{\rho a^2}{E}\left(\lambda_3-\frac{\lambda_5}{2(1+\nu)}\right), \quad f_2 = \lambda_1\left(\frac{4\lambda_4}{2(1+\nu)}-\lambda_2\right), \quad (37)$$

$$f_3 = f_2\left(\frac{\ddot{\delta}_0}{d}\right)^2-\frac{1}{12(1-\nu^2)}C_0^2-\frac{f_2}{\lambda_1}\frac{2\alpha'}{1-\nu}\sum_i\frac{\bar{\theta}'_i}{i+2}, \quad (38)$$

$$f_4 = \sum_j\frac{\alpha'[4+(1+\nu)j]}{2(1-\nu^2)(j+2)^2}\tilde{\theta}'_j+\frac{1}{12(1-\nu^2)}C_0^2\frac{\delta_0}{d}, \quad (39)$$

$$\lambda_1 = \frac{1}{1-\nu^2}C_0^2\left(A_2^2+2A_4^2+\frac{8}{3}A_2A_4\right), \quad \lambda_2 = \left(\frac{A_2}{16}+\frac{A_4}{36}\right), \quad (40)$$

$$\lambda_3 = \frac{1}{64}+\frac{A_2}{576}+\frac{A_4}{2\,304}, \quad \lambda_4 = \frac{A_2}{16}(3+\nu)+\frac{A_4}{24}(5+\nu), \quad (41)$$

$$\lambda_5 = \frac{\nu+3}{16}+\frac{5+\nu}{96}A_2+\frac{7+\nu}{288}A_4. \quad (42)$$

The initial condition can be written as

$$t = 0, \quad \frac{\dot{\delta}}{d} = 0, \quad \frac{\delta}{d} = \frac{\delta_0}{d}. \quad (43)$$

We suppose that

$$\frac{\delta}{d} = \delta_0^* + \delta^*, \quad (44)$$

where δ_0^*, δ^* denote the stable and perturbation solutions respectively, so

$$f_1\ddot{\delta}^* + 3f_2\delta_0^{*2}\delta^* + f_3\delta^* = 0. \quad (45)$$

If δ^* could be expressed as

$$\delta^* = \Delta\exp(\alpha t). \quad (46)$$

In case of $\delta_0^* = \delta'_0$, the critical condition of instability ($\alpha = 0$) is

$$3\frac{\delta_0}{d}f_2 + f_3 = 0. \quad (47)$$

4 Results and Discussions

The material constants, structure dimensions and laser parameters, which are used in this paper, are given in the Eq.(48)

$$q = 10^7 \mathrm{W/cm^2}, \quad \rho = 8\,900 \mathrm{Kg/m^2}, \quad E = 10^{11} \mathrm{Pa}, \quad \nu = 0.163,$$
$$\kappa = 1.09 \mathrm{W/cm \cdot K}, \quad c = 250 J/\mathrm{Kg \cdot K}, \quad \alpha = 2 \times 10^{-5} \mathrm{K^{-1}},$$
$$a = 10\mathrm{mm}, \quad d = 0.2\mathrm{mm}. \tag{48}$$

Based on the solution, given in Eq. (4), the temperature fields at different time are drawn in Fig. 2. It shows that, with the increment of irradiation time of laser beam, the temperature of upper face of the plate rises rapidly, and the heat is conducted along the thickness direction continuously.

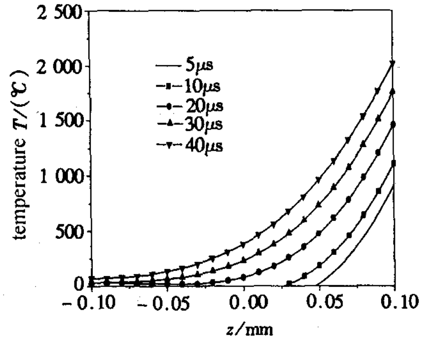

Fig. 2 Temperature distribution along the thickness of the plate

By using the two-rank, two-order semi-implicit Runge-Kutta integral method, the controlling equation about the deflection in center point of can be solved numerically, and shown in Fig. 3. The relation curves between deflection and laser irradiation time, represent the buckling and post-buckling process of the thin plate. From these curves, one can obtain the effects of various factors, such as defects, inertial etc. The following conclusions can be drawn:

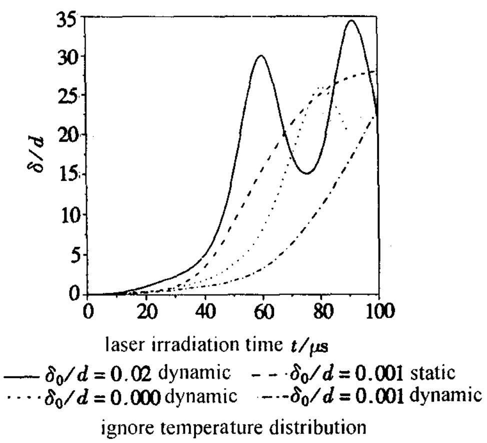

Fig. 3 The deflection *vs*. irradiation time of laser beam

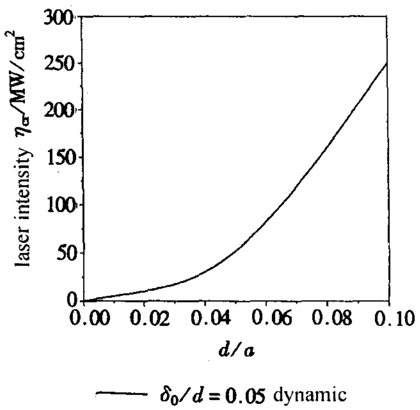

Fig. 4 The critical power density of laser beam *vs*. ration of thickness and diameter

1) The initiation of thermal buckling of the thin plate is retarded by the inertial effect.

2) The buckling and post-buckling of the plate is very sensitive to the magnitude of the initial deflection. With the enlargement of the defect, the occurrence of instability will be moved up.

3) The non-uniform of temperatures along the thickness will benefit to the happening of buckling. If the temperature moment is ignored, the prediction on the thermal dynamic buckling of the round plate will be unacceptable.

It must be noted, the definition about the dynamic buckling and determination of critical value are difficult. In this paper, the viewpoint of Budiansky et al. is adopted. We suppose that the initial point of buckling should exist where the slope of the deflection-irradiation time curve rises abruptly. However, the method is not very strict from the mathematics, though it is applied widely in numerical analysis about instability problem.

As is mentioned in the last section, linear perturbation method can be introduced to obtain theoretically the critical value of instability. By using the above method, the relationship between the critical laser beam intensity and ratio of thickness and diameter of the plate corresponding to the beginning of buckling, can be obtained and shown in Fig.4. From the figure, we can find that the critical laser power density is nearly linear to the ratio of thickness and diameter to the plate. At last, the result obtained in this paper fit well the experimental studies[1] and FEM numerical investigation[6].

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