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PROBLEM OF HYDRODYNAMIC PRESSURE ON SUDDENLY STARTING VESSEL

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Abstract

In this paper, Lagrangian method is applied to discuss the problem of the hydrodynamic pressure on a suddenly starting vessel. The free surface profile and the coefficients of the hydrodynamic pressure on the vessel wall are obtained. And it is verified that the singularity of the pressure near the free surface is only logarithmic.

Key words Lagrangian method, vessel, hydrodynamic pressure, water waves

I. Introduction

During earthquakes the fluid in a reservoir exerts an impulsive force on the dam, and in an accelerating spacecraft the liquid propellant also engenders an impulsive force on the tank. These rocking problems of fluid due to an accelerating rigid wall may sum up the problem of the hydrodynamic pressure on a suddenly starting vessel.

The study of this problem originated in the 1920s. At that time, because the dam was designed in the earthquake regions of America, the hydrodynamic pressure exerted on a dam during earthquake must be estimated. In 1933 Westergaard[1] first got an analytic solution by a linearized theory neglecting effect of the free surface of the fluid. In the same year a remarkably simple momentum-balance method was applied to solve this problem by Von Kármán[2]. His solution was very close to Westergaard's result. A nonlinearized method considering the effect of free surface was not developed to treat the problem until 1983 by Chwang[3]. By the same method Chwang and Wang (1984)[4] solved the problem of the dynamic fluid pressure on a suddenly starting vessel. However, the coefficient of the hydrodynamic pressure becomes rather great adjecent to the free surface; also, the character of singularity was not clarified.

In the above literature referred to it should be mentioned that these problems are solved adopting the Eulerian method which uses Eulerian coordinates. Up to 1988 Tao Ming-de and Cen Yun[5] treated a problem of an accelerating vertical plate towards fluid by the Lagrangian method, however, they have not explained the charater of singularity. In this paper, the Lagrangian method is applied to study the problem of suddenly starting vessel. The distribution of the dynamic fluid pressure exerted on the wall is determined. It is verified that the singularity in pressure adjecent to the free surface is only logarithmic. Due to the use of the Lagrangian method, we are able to calculate the coefficient of the hydrodynamic pressure on the wall from the bottom to the free surface of the fluid at any time, which is only calculated up to the intial position of the free surface of the fluid by the Eulerian method. And the former result is greater than the latter one. But the free surface profiles obtained by the Lagrangian method is the same as that given by the Eulerian method.

The great advantage of the use of Lagrangian method in the problem of transient water wave is that the nonlinear boundary condition on the free surface is exactly satisfied, for the equation of the free surface in the conventional Eulerian method, which is an unknown function of the space and time coordinates, is now a fixed curve in the space of the independent Lagrangian variables. This simplification is achieved at the expense of increased complexity of the governing equations but the difficulty of having to solve equations in an unknown domain, whose precise definition is implicitly wrapped up in the problem, is avoided.

II. Governing Equations and Boundary Conditions

Let us consider a rigid vessel of two dimensions with vertical side walls and horizontal bottom that is symmetrical with respect to y axis (see Fig. 1). The undisturbed fluid depth is h and L is the length of the rectangular vessel. The walls of the vessel are assumed to be given an impulsive acceleration in the x direction, and the fluid is assumed to be incompressible and ideal. Let (a, b) represent the Cartesian coordinates (X, Y) of a particle at $t=0$ and these Lagrangian coordinates label the particle in the subsequent motion. Then its displacement $X(a, b, t)$, $Y(a, b, t)$ at time $t>0$ satisfies the Lagrangian momentum equations

$$X_{tt}X_a+(Y_{tt}+g)Y_a=-\frac{1}{\rho}\frac{\partial p}{\partial a}, \qquad (2.1a)$$

$$X_{tt}X_b+(Y_{tt}+g)Y_b=-\frac{1}{\rho}\frac{\partial p}{\partial b}, \qquad (2.1b)$$

and the continuous equation

$$\begin{vmatrix} X_a & Y_a \\ X_b & Y_b \end{vmatrix}=1 \qquad (2.2)$$

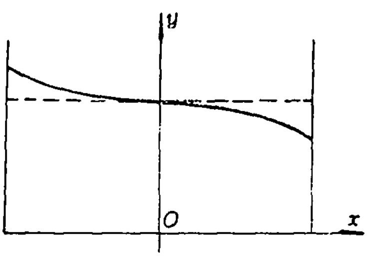

Fig. 1

where p is the pressure, g is the gravitational constant, ρ is the density and subscripts denote partial derivatives. If p is eliminated from (2.1a, b) we obtain

$$(X_aX_{bt}+Y_aY_{bt})-(X_bX_{at}+Y_bY_{at})=0 \qquad (2.3)$$

where it is assumed that the motion is irrotational.

Based on Pohle's[6] theory, for small values of the time we may expand X, Y and p by power series in t. Then we have

$$X=a+X^{(2)}(a,b)t^2+X^{(3)}(a,b)t^3+X^{(4)}(a,b)t^4+\cdots, \qquad (2.4a)$$

$$Y=b+Y^{(2)}(a,b)t^2+Y^{(3)}(a,b)t^3+Y^{(4)}(a,b)t^4+\cdots, \qquad (2.4b)$$

$$p=p^{(0)}(a,b)+p^{(1)}(a,b)t+p^{(2)}(a,b)t^2+p^{(3)}(a,b)t^3+\cdots, \qquad (2.4c)$$

where $X^{(i)}$, $Y^{(i)}$ and $p^{(i)}$ related to a and b are undetermined coefficients. It is assumed that the vessel is initially at rest for (2.4a,b). Substituting (2.4a,b) into (2.3), we have

$$X_a^{(2)}+Y_b^{(2)}=0,\ X_b^{(2)}-Y_a^{(2)}=0,\ X_a^{(3)}+Y_b^{(3)}=0,\ X_b^{(3)}-Y_a^{(3)}=0, \qquad (2.5a\text{–}d)$$

$$X_a^{(4)}+Y_b^{(4)}=X_b^{(2)}Y_a^{(2)}-X_a^{(2)}Y_b^{(2)},\ X_b^{(4)}-Y_a^{(4)}=0, \qquad (2.5e,f)$$

...

Substituting (2.4) into (2.1) we have

$$2X^{(2)}+\frac{1}{\rho}p_a^{(0)}=0,\quad 2Y^{(2)}+g+\frac{1}{\rho}p_b^{(0)}=0\,, \tag{2.6a,b}$$

$$6X^{(3)}+\frac{1}{\rho}p_a^{(1)}=0,\quad 6Y^{(3)}+\frac{1}{\rho}p_b^{(1)}=0\,, \tag{2.6c,d}$$

$$2X^{(2)}X_a^{(2)}+12X^{(4)}+(2Y^{(2)}+g)Y_a^{(2)}+\frac{1}{\rho}p_a^{(2)}=0, \tag{2.6e}$$

$$2X^{(2)}X_b^{(2)}+12Y^{(4)}+(2Y^{(2)}+g)Y_b^{(2)}+\frac{1}{\rho}p_b^{(2)}=0\,. \tag{2.6f}$$

...

On the free surface since the pressure $p\equiv 0$, for $b=h$, we have from (2.4c)

$$p^{(i)}(a,h)=0 \qquad (i=0,1,2). \tag{2.7}$$

Thus we obtain by (2.6a) and (2.5a)

$$X^{(2)}(a,h)=0 \tag{2.8}$$

and

$$Y_b^{(2)}(a,h)=0\,. \tag{2.9}$$

At the bottom $b=0$, $Y\equiv 0$. Therefore we have by (2.4b)

$$Y^{(i)}(a,0)=0 \qquad (i=2,3,4)\,. \tag{2.10}$$

And we have from (2.5b)

$$X_b^{(2)}(a,0)=0\,. \tag{2.11}$$

In this paper, we only consider the case of a constant acceleration a_1. On the side walls, the normal velocity of the fluid must be the same as that of the vessel. Thus

$$X_t(\pm L/2,\ b,t)=a_1 t\,,$$

by which we have

$$2X^{(2)}(\pm L/2,b)=a_1\,,\ 3X^{(3)}(\pm L/2,b)=0,\ 4X^{(4)}(\pm L/2,b)=0\,. \tag{2.12a-c}$$

Substituting (2.12a, b, c) into (2.6a, c, e), respectively, we have

$$a_1+\rho^{-1}p_a^{(0)}(\pm L/2,b)=0,\quad p_a^{(1)}(\pm L/2,b)=0\,, \tag{2.13a,b}$$

$$a_1 X_a^{(2)}(\pm L/2,b)+(1/\rho)p_a^{(2)}(\pm L/2,b)=0\,. \tag{2.13c}$$

And substituting (2.10) and (2.11) into (2.6b, d, f), we have

$$p_b^{(0)}(a,0)=-\rho g,\quad p_b^{(1)}(a,0)=0,\quad p_b^{(2)}(a,0)=-\rho g Y_b^{(2)}(a,0)\,. \tag{2.14a,b,c}$$

III. Solution of Displacement

Eliminating $Y^{(2)}$ from (2.5a ,b), we obtain the Laplace equation

$$\nabla^2 X^{(2)}=0\,.$$

Then the solution of coefficient $X^{(2)}$, satisfying the above equation and the boundary conditions (2.8), (2.11) and (2.12a), is obtained by the Fourier-series method as

$$X^{(2)}(a,b)=a_1\sum_{k=0}^{\infty}(-1)^k\frac{\text{ch}\mu_k\bar{a}\cos\mu_k\bar{b}}{\mu_k\text{ch}0.5\mu_k\bar{L}}, \tag{3.1}$$

where

$$\mu_k=\frac{\pi(2k+1)}{2},\quad \bar{a}=\frac{a}{h},\quad \bar{b}=\frac{b}{h},\quad \bar{L}=\frac{L}{h}.$$

Substituting Eq. (3.1) into Eq (2.5a), we have the coefficient

$$Y^{(2)}(a,b)=a_1\sum_{k=0}^{\infty}(-1)^k\frac{\text{sh}\mu_k\bar{a}\sin\mu_k\bar{b}}{\mu_k\text{ch}0.5\mu_k\bar{L}} \tag{3.2}$$

It is deduced from (2.5c, d) that $X^{(3)}$ is a conjugate harmonic function with respect to $Y^{(3)}$. And we have by (2.10) and (2.5d)

$$X_b^{(3)}(a,0)=0. \tag{3.3}$$

From (2.7) and (2.6c) we have

$$X^{(3)}(a,h)=0. \tag{3.4}$$

Thus we obtain from (3.3), (3.4) and (2.12b)

$$X^{(3)}(a,b)\equiv Y^{(3)}(a,b)\equiv 0. \tag{3.5}$$

Therefore the Lagrangian equations of the disturbed free surface are

$$X/h=a/h+O(\epsilon^4)=\bar{a}+O(\epsilon^4) \tag{3.6a}$$

and

$$\frac{Y}{h}=1-2\epsilon^2\sum_{k=0}^{\infty}\frac{\text{sh}\mu_k\bar{a}}{\mu_k\text{ch}0.5\mu_k\bar{L}}+O(\epsilon^4), \tag{3.6b}$$

where ϵ is defined by $\epsilon=t\sqrt{a/2h}$.

IV. Solution of Pressure

The coefficient $p^{(0)}$ of the pressure expansion can be obtained by integrating (2.6b),

$$p^{(0)}(a,b)=\rho g(h-b)+2\rho a_1 h\sum_{k=0}^{\infty}(-1)^{k+1}\frac{\text{sh}\mu_k a/h\cos\mu_k b/h}{\mu_k^2\text{ch}\mu_k L/2h}, \tag{4.1}$$

where the first and second terms represent the hydrostatic and hydrodynamic pressure, respectively. From (2.6d) and (3.5) we can easily have

$$p^{(1)}(a,b)\equiv 0. \tag{4.2}$$

Eliminating $X^{(4)}$ and $Y^{(4)}$ from (2.5e) and (2.6e, f), we obtain the Poisson equation

$$p_{aa}^{(2)}+p_{bb}^{(2)}=-16\rho\,(X_b^{(2)}Y_a^{(2)}-X_a^{(2)}Y_b^{(2)}), \tag{4.3}$$

subject to the boundary conditions (2.7), (2.13c) and (2.14c). We may solve this inhomogeneous boundary value problem by means of Green's function method. Green's function corresponding to the equation may be given by eigenfunction expansion method as (see Appendix A)

$$K(x,y;\xi,\eta)=\begin{cases}\dfrac{2}{h}\sum\limits_{k=0}^{\infty}\dfrac{\cos\nu_k y\cos\nu_k\eta}{\nu_k\mathrm{sh}\nu_k L}\mathrm{ch}\nu_k\left(\xi-\dfrac{L}{2}\right)\mathrm{ch}\nu_k\left(x+\dfrac{L}{2}\right), & \text{for } -\dfrac{L}{2}<x<\xi, \\ \dfrac{2}{h}\sum\limits_{k=0}^{\infty}\dfrac{\cos\nu_k y\cos\nu_k\eta}{\nu_k\mathrm{sh}\nu_k L}\mathrm{ch}\nu_k\left(\xi+\dfrac{L}{2}\right)\mathrm{ch}\nu_k\left(x-\dfrac{L}{2}\right), & \text{for } \xi<x<\dfrac{L}{2}\end{cases} \tag{4.4}$$

where $\nu_k=\mu_k/h$. By (4.4)$p^{(2)}(a,b)$may be represented

$$\begin{aligned}p^{(2)}(a,b)=&\int_{-L/2}^{L/2}\int_0^h K(a,b;\xi,\eta)[16\rho(X_b^{(2)}(\xi,\eta)Y_a^{(2)}(\xi,\eta)\\&-X_a^{(2)}(\xi,\eta)Y_b^{(2)}(\xi,\eta))]d\xi d\eta\\&+\int_0^h K\left(a,b;\frac{L}{2},\eta\right)\left[-\rho a_1 X_a^{(2)}\left(\frac{L}{2},\eta\right)\right]d\eta\\&+\int_0^h K\left(a,b;-\frac{L}{2},\eta\right)\left[-\rho a_1 X_a^{(2)}\left(-\frac{L}{2},\eta\right)\right]d\eta\\&-\int_{-L/2}^{L/2}K(a,b;\xi,0)[-\rho g Y_b^{(2)}(\xi,0)]d\xi\,.\end{aligned}$$

The hydrodynamic pressure coefficient on the left-hand side wall is defined by

$$C_p=\frac{p_D(-L/2,\,b)}{\rho a_1 h},$$

where p_D is the hydrodynamic pressure. By sorting out the above expression we have

$$\begin{aligned}C_p(\bar{b})=&2\sum_{k=0}^{\infty}(-1)^k\frac{\mathrm{th}0.5\mu_k\bar{L}\cos\mu_k\bar{b}}{\mu_k^2}\\&+\epsilon^2\Bigg\{64\int_{-\bar{L}/2}^{\bar{L}/2}d\bar{\xi}\int_0^1 d\bar{\eta}\left(\sum_{k=0}^{\infty}\frac{\cos\mu_k\bar{b}\cos\mu_k\bar{\eta}}{\mu_k\mathrm{sh}\mu_k\bar{L}}\mathrm{ch}\mu_k\left(\bar{\xi}-\frac{\bar{L}}{2}\right)\right)\\&\left[\left(\sum_{k=0}^{\infty}(-1)^k\frac{\mathrm{sh}\mu_k\bar{\xi}\cos\mu_k\bar{\eta}}{\mathrm{ch}0.5\mu_k\bar{L}}\right)^2+\left(\sum_{k=0}^{\infty}(-1)^k\frac{\mathrm{ch}\mu_k\bar{\xi}\sin\mu_k\bar{\eta}}{\mathrm{ch}0.5\mu_k\bar{L}}\right)^2\right]\\&+4\int_0^1 d\bar{\eta}\left(\sum_{k=0}^{\infty}(-1)^k\mathrm{th}\frac{1}{2}\mu_k\bar{L}\cos\mu_k\bar{\eta}\right)\left(\sum_{k=0}^{\infty}\frac{\cos\mu_k\bar{b}\cos\mu_k\bar{\eta}}{\mu_k\mathrm{sh}\mu_k\bar{L}}(\mathrm{ch}\mu_k\bar{L}-1)\right)\\&+\frac{4}{\alpha}\int_{-\bar{L}/2}^{\bar{L}/2}d\bar{\xi}\left(\sum_{k=0}^{\infty}\frac{\mathrm{sh}\mu_k\bar{\xi}}{\mathrm{ch}0.5\mu_k\bar{L}}\right)\left(\sum_{k=0}^{\infty}\frac{\cos\mu_k\bar{b}}{\mu_k\mathrm{sh}\mu_k\bar{L}}\mathrm{ch}\mu_k\left(\bar{\xi}-\frac{\bar{L}}{2}\right)\right)\Bigg\},\end{aligned}\tag{4.5}$$

where α is defined by $\alpha=a_1/g$.

V. Numerical Results and Conclusion

For $\bar{L}=1$, $\alpha=0.2$, the free surface profile given by (3.6) is plotted in Fig. 2 for several values of the dimensionless time parameter ϵ , and the hydrodynamic pressure coefficient C_p on the left-hand side wall is plotted in Fig. 3 for several values of the non-dimensional time parameter ϵ . The expression (3.6) and (4.5) is given the main conclusion of this paper.

As shown in Fig. 2, for non-vanishing values of ϵ, equation (3.6b) has logarithmic singularities at $\bar{a}=-\bar{L}/2$. From Fig. 3 we note that C_p has also logarithmic singularics (see Appendix B) as a result of the fact that the boundary values is discontinuous at $\bar{a}=\pm\bar{L}/2$ and $\bar{b}=1$. We note from (2.8) that $X^{(2)}$ vanishes on the free surface. However, from (2.12a) $X^{(2)}$ is a nonzero value $a_i/2$. Hence $X^{(2)}$ is discontinuous at the point between the free surface and the side wall. The physical mechanism of the flow adjecent to a singular point is rather complicated.

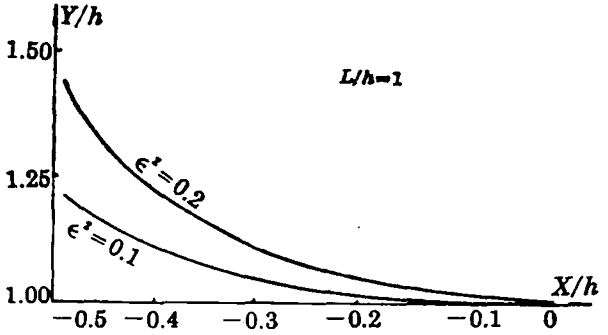

Fig. 2

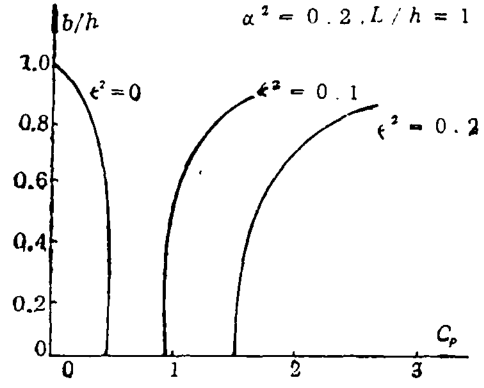

Fig. 3

Appendix A

Let Green's function $K(x,y;\xi,\eta)$ satisfy

$$\nabla^2\cdot K=-\delta(x-\xi)\delta(y-\eta), \quad \text{(A.1a)}$$

$$\partial K/\partial x=0, \quad (x=\pm L/2), \quad \text{(A.1b)}$$

$$\partial K/\partial y=0, \quad (y=0), \quad \text{(A.1c)}$$

$$K=0, \quad (y=h). \quad \text{(A.1d)}$$

The operator L is defined by $L=-\partial^2/\partial y^2$. Then by (A.1 a) we get

$$\nabla^2 K=\partial^2 K/\partial x^2-LK=-\delta(x-\xi)\delta(y-\eta) \quad \text{(A.2)}$$

Let $M_i(y)$ be the eigenfunction of the operator L, thus $M_i(y)$ satisfies

$$LM_i=\lambda_i M_i, \quad \text{(A.3)}$$

and

$$(M_i,M_j)=\delta_{ij}. \quad \text{(A.4)}$$

From (A.1c, d) we can obtain

$$M_i(h)=M_i'(0)=0. \quad \text{(A.5)}$$

Thus the eigenfunction M_i, satisfying the equation (A.3) subject to (A.5), is obtained as

$$M_i=\sqrt{2/h}\cos\sqrt{\lambda_i}\,y, \quad \text{(A.6)}$$

where

$$\sqrt{\lambda_i}=(i+1/2)\pi/h, \quad (i=0,1,2,\cdots). \quad \text{(A.7)}$$

Let K be expanded the series of the eigenfunction

$$K(x,y;\xi,\eta)=\sum_{i=0}^{\infty}N_i(x)M_i(y). \quad \text{(A.8)}$$

Multiplying equation (A.2) by M_i, integrating it, and considering the self-adjoint operator L and the orthogonal function M_i, we have

$$N_i'' - \lambda_i N_i = -M_i(\eta)\delta(x-\xi). \tag{A.9}$$

By (A.1b) we get

$$N_i'(-L/2) = N_i'(L/2) = 0.$$

Thus we have

$$N_i(x) = \begin{cases} A\dfrac{\mathrm{ch}\nu_i(x+L/2)}{\mathrm{ch}0.5\nu_i L}, & \text{for } -\dfrac{L}{2}<x<\xi, \\ B\dfrac{\mathrm{ch}\nu_i(x-L/2)}{\mathrm{ch}0.5\nu_i L}, & \text{for } \xi<x<\dfrac{L}{2}. \end{cases} \tag{A.10}$$

Since N_i is continuous at $x=\xi$ and N_i' is discontinuous, we can obtain

$$A = \frac{M_i(\eta)}{h\nu_i}\cdot\frac{\mathrm{ch}0.5\nu_i L\,\mathrm{ch}\nu_i(\xi-L/2)}{\mathrm{sh}\nu_i L},$$

$$B = \frac{M_i(\eta)}{h\nu_i}\cdot\frac{\mathrm{ch}0.5\nu_i L\,\mathrm{ch}\nu_i(\xi+L/2)}{\mathrm{sh}\nu_i L}.$$

Substituting A and B into (A.10), then we get (4.4).

Appendix B

Let $\bar{b}$ approach to one, thus we could make a semicircle R_δ with the center at $(-\bar{L}/2, \bar{b})$ and the radius $\delta=(1-\bar{b})/2\ll 1$ in the domain. A transform is defined by

$$\xi = r\cos\theta - \bar{L}/2,\quad \bar{\eta} = \bar{b} + r\sin\theta,\quad (0<r<\delta \text{ and } |\theta|<\pi/2).$$

Let us estimate the double integral in (4.5). In the semicircle domain R_δ the third summation term of (4.5) is much smaller than the second one therefore, we only estimate the former two summation terms Σ_1 and Σ_2.

The principal part of Σ_1 is

$$\sum_{k=0}^{\infty} \frac{\mathrm{ch}\mu_k(\xi-\bar{L}/2)}{\mu_k \mathrm{sh}\mu_k \bar{L}}. \tag{B.1}$$

When k approaches to sufficiently great, the summation (B.1) could be rewritten as

$$\sum_{k=0}^{\infty} \frac{\exp[\mu_k(\bar{L}-r\cos\theta)]}{\mu_k \exp[\mu_k \bar{L}]} = \sum_{k=0}^{\infty} \frac{1}{\mu_k}\exp[-\mu_k r\cos\theta] = F(r\cos\theta).$$

Let $u = r\cos\theta$, thus we have

$$\frac{dF}{du} = -\sum_{k=0}^{\infty} \exp[-\mu_k u] = \frac{\exp[-\pi u/2]}{1-\exp[-\pi u]},$$

hence

$$F \sim -\ln(1-\exp[-\pi u/2]) \sim -\ln(r\cos\theta).$$

The principal part of Σ_2 is

$$\sum_{k=0}^{\infty}(-1)^k\cos\mu_k\bar{\eta} = \sum_{k=0}^{\infty}(-1)^k\cos\mu_k(\bar{b}+r\sin\theta)$$

$$=\sum_{k=0}^{\infty}(-1)^k \cos\mu_k(1-2\delta+r\sin\theta)=\sum_{k=0}^{\infty}\sin\left[(2\delta-r\sin\theta)\left(k+\frac{1}{2}\right)\pi\right]. \tag{B.2}$$

If φ denotes $(2\delta-r\sin\theta)\pi$, expression (B.2) is rewritten as

$$\sum_{k=0}^{\infty}\sin\left(k\varphi+\frac{\varphi}{2}\right)=\cos\frac{\varphi}{2}\sum_{k=0}^{\infty}\sin k\varphi+\sin\frac{\varphi}{2}\sum_{k=0}^{\infty}\cos k\varphi .$$

Since

$$\sum_{k=0}^{\infty}\cos k\varphi=\frac{\sin(n+1/2)\varphi}{2\sin(\varphi/2)}\Big|_{n\to\infty}+\frac{1}{2}\sim\frac{1}{\delta},$$

$$\sum_{k=0}^{\infty}\sin k\varphi=\frac{\cos(\varphi/2)-\cos(n\varphi+\varphi/2)}{2\sin(\varphi/2)}\Big|_{n\to\infty}\sim\frac{1}{\delta},$$

the principal part of the double integral in R_δ is given

$$-\frac{1}{\delta^2}\int_0^\delta rdr\int_{-\pi/2}^{\pi/2}d\theta(\ln(r\cos\theta))$$

$$=-\frac{1}{\delta^2}\int_0^\delta\int_{-\pi/2}^{\pi/2}(r\ln r+r\ln\cos\theta)drd\theta$$

$$=-\frac{\pi}{4}\left(\ln\delta-\frac{1}{2}\right)-\frac{1}{2}\int_0^{\pi/2}\ln\cos\theta d\theta$$

Thus we can obtain its principal part from the above expression

$$-\ln\delta\sim-\ln(1-\bar{b}).$$

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