Flow through a stenosed artery subject to periodic body acceleration

V. K. Sud
Department of Biophysics, All-India Institute of Medical Sciences, New Delhi-110029, India

G. S. Sekhon
Department of Applied Mechanics, Indian Institute of Technology, New Delhi-110016, India

Abstract—*Human bodies may occasionally be subjected to high external accelerations. The response of the vascular system under such situations has recently been the subject of several theoretical and experimental studies. However, little work appears to have so far been carried out on the analysis of blood flow in stenosed artery in the presence of body accelerations. In the paper we present a model of blood flow in a partially occluded tube subject to both the pulsatile pressure gradient due to the normal heart action and the periodic body acceleration. Closed-form solutions have been obtained for the instantaneous rate of flow and for the distributions of flow velocity, acceleration and shear stress over the stenosed length. Computational results corresponding to a stenosed carotid artery are presented and discussed.*

Keywords—*Blood flow, Body acceleration, Carotid artery, Partially occluded artery, Periodic external acceleration, Stenosis*

Med. & Biol. Eng. & Comput., 1987, **25**, 638–644

1 Introduction

STUDIES ON THE initiation and growth of stenosis (or vascular lesions or artherosclerotic plaques) in the human system have been carried out from several viewpoints. The chief factors that have been extensively analysed in this regard include lipid metabolism, hypertension, diet, hormones, age and sex. It has also been argued that flow mechanism is also an important factor as the velocity gradients and the resulting shear stress between the moving blood and the depositing atherosclerotic plaques must exercise some controlling and inhibiting effect on the developing stenosis. Additionally, the arterial impedance is also affected. Some experimental studies on flow patterns in partially occluded pipes have been reported (YOUNG, 1968; FORRESTER and YOUNG, 1970*a*, *b*; LEE and FUNG, 1970; YOUNG and TSAI, 1973*a*, *b*; BACK *et al.*, 1977). A mathematical solution on the pulsatile flow in such a constricted artery has been worked out (PADMANABHAN, 1980). However, no analysis appears to have so far been made on blood flow in a stenosed artery in the presence of body accelerations, which are caused when a human being is subjected to whole-body acceleration or vibration. For example, while riding in a vehicle such as a car, tractor or train, or while flying in an aircraft or spacecraft, man may unintentionally be subjected to body acceleration. It has also been suggested that certain circulatory disorders can be corrected by strapping human beings on to vibrating tables. In this paper, we present a model of blood flow in a partially occluded tube subject to both the pulsatile pressure gradient due to the normal heart action and periodic body acceleration. Closed-form solutions have been obtained for the instantaneous rate of flow and for the distributions of flow velocity, acceleration and shear rate throughout the stenotic length.

The different parameters typical of a stenosed artery have been incorporated. Computational results corresponding to a stenosed carotid artery for flow velocities, pressure drop, flow rate and impedance are presented and discussed.

First received 12th August 1986 and in final form 14th April 1987

2 Mathematical model

Consider the flow of an incompressible viscous fluid (blood) through a partially occluded rigid tube of normal radius a. Following YOUNG (1968), the stenotic protuberance is assumed to be an axisymmetric surface generated by a cosine curve. The effective radius R of the tube at any distance z from the centre of the stenosed portion (Fig. 1) can accordingly be represented as

$$R_1 = a - \delta\left(1 + \cos\frac{\pi z}{2z_0}\right) \quad \text{for } z \text{ lying between}$$

$$z = -2z_0 \quad \text{to} \quad z = 2z_0$$

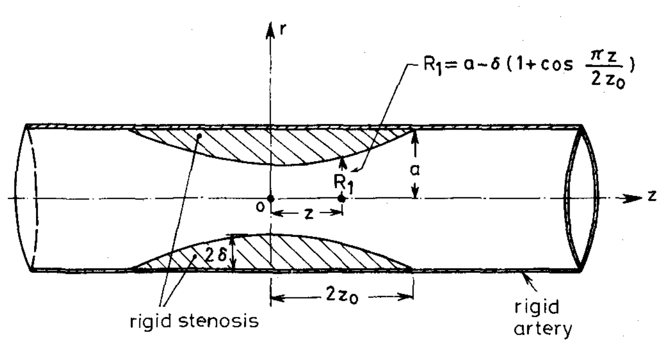


Fig. 1 *Assumed geometry of the stenosis*

otherwise

$$= a$$

where $4z_0$ is the length of the stenotic region and 2δ is the maximum protuberance of the stenotic form of the artery wall.

At $t > 0$, the system is subjected to a periodic body acceleration $F(t)$ given by

$$F(t) = a_0 \cos(\omega t + \phi)$$

where $\omega_b = 2\pi f_b$ is circular frequency f_b is the frequency in Hz. ϕ is the lead angle of $F(t)$ with respect to the heart action.

The pressure gradient at any z may be represented as follows:

$$-\frac{\partial p}{\partial z} = A_0 + A_1 \cos nt$$

where A_0 is the steady component of the pressure gradient, A_1 is the amplitude of the fluctuating component and $n = 2\pi f_p$, where f_p is the pulse frequency. Both A_0 and A_1 are functions of z. We assume that at $t < 0$ only the pumping action of the heart is present. At $t = 0$, let the instantaneous flow rate at the inlet to the stenotic artery correspond to the theoretical flow rate in a straight rigid tube subject to a pressure gradient $-\partial p/\partial z = A_0 + A_1$. As a result the flow velocity at $t = 0$ is given by (McDonald, 1974)

$$u_z(r, 0) = \frac{(R_1^2 - r^2)(A_0 + A_1)}{4\eta}$$

where η is the function of z given by $\mu_f R_1^4/a^4$. We assume:

(i) the flow is laminar
(ii) there is rotational symmetry of flow
(iii) the frequency of body acceleration f_b is so small that the wave effects can be neglected.

Fung *et al.* (1971) and Padmanabhan (1980) argued that, as Womersley's parameter $\alpha = a(\rho n/\mu_f)^{1/2}$ lies between 1 and 4 for segments of the human arterial system, the convective terms in the Navier-Stoke's equation are small compared with the corresponding viscous terms. Therefore, except for cases of acute senosis, the convective terms can be omitted. The Navier-Stokes equation of motion in a stenosed artery can therefore be expressed in cylindrical polar co-ordinates (r, θ, z) as follows:

$$\rho \frac{\partial u_z}{\partial t} = \rho a_0 \cos(\omega t + \phi) + A_0 + A_1 \cos nt + \mu_f\left(\frac{\partial^2 u_z}{\partial r^2} + \frac{1}{r}\frac{\partial u_z}{\partial r}\right) \quad (1)$$

where ρ and μ_f are the density and viscosity, respectively, of liquid flowing through the tube and u_z is the velocity of flow in the axial direction.

Eqn. 1 has to be solved subject to the following initial and boundary conditions:

(i) $$u_z(r, 0) = \frac{(R_1^2 - r^2)(A_0 + A_1)}{4\mu_f} \quad (2a)$$

(ii) $$u_z(r, t) = 0 \quad \text{at} \quad r = R \quad \text{for all } t \quad (2b)$$

(iii) $$u_z(0, t) \rightarrow \text{finite as } r \rightarrow 0 \quad (2c)$$

By applying Laplace transformation to eqn. 1, we transform it to the Laplacian plane and obtain

$$\frac{d^2\bar{u}_z}{dr^2} + \frac{1}{r}\frac{d\bar{u}_z}{dr} + \frac{i^2\rho s}{\mu_f}\bar{u}_z = \frac{\rho(A_0 + A_1)m}{4\mu_f^2}(r^2 - R_1^2) - \frac{\rho a_0(s\cos\phi - \omega\sin\phi)A_0}{\mu_f(s^2 + \omega^2)} - \frac{A_0}{\mu_f}\frac{1}{s} - \frac{A_1 s}{\mu_f(s^2 + n^2)} \quad (3)$$

where $m = a^4/R_1^4$.

The boundary conditions of eqns. 2*b* and 2*c* are also transformed to

$$\bar{u}_z(r, s) = 0 \quad \text{at} \quad r = R_1 \quad (4a)$$

and

$$\bar{u}_z(0, s) \rightarrow \text{finite value as } r \rightarrow 0 \quad (4b)$$

where the Laplace transform of $u_z(r, t)$ is given as

$$\bar{u}_z(r, s) = \int_0^\infty e^{-st} u_z(r, t)\, dt$$

The Laplace-transformed equations 3, 4*a*, and 4*b* have been solved in terms of Bessel functions with complex arguments. Following Carslaw and Jaeger (1963) and omitting details, we finally invert the obtained solution back into the physical plane.

Thus the required solution of the velocity profile is obtained as

$$\begin{aligned} u_z(r, t) = {} & \frac{A_0}{4\mu_f}(R_1^2 - r^2) + \frac{a_0}{\omega}\sin(\omega t + \phi) + \frac{A_1}{\rho n}\sin nt \\ & - \frac{a_0}{2\omega}\frac{J_0\left(\sqrt{\frac{i\omega}{\nu}}\,r\right)}{J_0\left(\sqrt{\frac{i\omega}{\nu}}\,R_1\right)}\{\sin(\omega t + \phi) + i\cos(\omega t + \phi)\} \\ & - \frac{a_0}{2\omega}\frac{J_0\left(\sqrt{-\frac{i\omega}{\nu}}\,r\right)}{J_0\left(\sqrt{-\frac{i\omega}{\nu}}\,R_1\right)}\{\sin(\omega t + \phi) - i\cos(\omega t + \phi)\} \\ & - \frac{A_1}{2\rho n}\frac{J_0\left(\sqrt{-\frac{in}{\nu}}\,r\right)}{J_0\left(\sqrt{-\frac{in}{\nu}}\,R_1\right)}\{\sin nt - i\cos nt\} \\ & - \frac{A_1}{2\rho n}\frac{J_0\left(\sqrt{-\frac{in}{\nu}}\,r\right)}{J_0\left(\sqrt{\frac{in}{\nu}}\,R_1\right)}\{\sin nt + i\cos nt\} - 2u_t(n) \end{aligned} \quad (5)$$

where for simplicity we write the unsteady term $u_t(n)$ as

$$u_t(n) = \sum_{n=1}^{\infty} \frac{\exp(-\lambda_n^2 \kappa t) J_0\left(\lambda_n \frac{r}{R_1}\right)(b_n - c_n)}{J_1(\lambda_n)\rho\kappa\lambda_n^3(\kappa^2\lambda_n^4 + \omega^2)(\kappa^2\lambda_n^4 + n^2)}$$

$$b_n = a_0 \rho\kappa\lambda_n^2(n^2 + \kappa^2\lambda_n^4)(\kappa\lambda_n^2\cos\phi + \omega\sin\phi)$$

$$\begin{aligned} c_n = {} & A_0(m - 1)[\kappa^4\lambda_n^8 + \kappa^2\lambda_n^4 + \omega^2 n^2] \\ & + A_1[(m - 1)\kappa^4\lambda_n^8 + \kappa^2\lambda_n^4 m(n^2 + \omega^2) \\ & - \omega^2\lambda_n^2\kappa^2 + \omega^2 n^2 m] \end{aligned}$$

$\kappa = \mu_f/\rho R_1^2$ and $J_1(\lambda_n)$ is a Bessel function of the first order. λ_n is the zero of the Bessel function of order 0 such that $J_0(\lambda_n) = 0$.

Also, the expression for the flow rate Q can be written as

$$Q = 2\pi \int_0^{R_1} r u_z(r, t)\, dr \tag{6}$$

Substituting the expression for $u_z(r, t)$ from eqn. 5 in the integral of eqn. 6, we arrive at the following expression for the flow rate:

$$\begin{aligned} Q = {} & \frac{\pi R_1^4}{8\mu_f} A_0 + \frac{\pi R_1^2}{\omega} a_0 \sin(\omega t + \phi) + \frac{\pi R_1^2}{\rho n} A_1 \sin nt \\ & - \frac{\pi R_1}{i\omega} a_0\{\cos(\omega t + \phi) + i \sin(\omega t + \phi)\} \\ & \times \frac{J_1\left(\sqrt{-\frac{i\omega}{\nu}} R_1\right)}{J_0\left(\sqrt{-\frac{i\omega}{\nu}} R_1\right)} \sqrt{-\frac{i\nu}{\omega}} \\ & + \frac{\pi R}{i\omega} a_0\{\cos(\omega t + \phi) - i \sin(\omega t + \phi)\} \\ & \times \frac{J_1\left(\sqrt{\frac{i\omega}{\nu}} R_1\right)}{J_0\left(\sqrt{\frac{i\omega}{\nu}} R_1\right)} \sqrt{\frac{i\nu}{\omega}} \\ & - \frac{\pi R_1}{in\rho} A_1\{\cos nt + i \sin nt\} \\ & \times \frac{J_1\left(\sqrt{-\frac{in}{\nu}} R_1\right)}{J_0\left(\sqrt{-\frac{in}{\nu}} R_1\right)} \sqrt{\frac{i\nu}{n}} \\ & + \frac{\pi R_1}{in\rho} A_1\{\cos nt - i \sin nt\} \\ & \times \frac{J_1\left(\sqrt{\frac{in}{\nu}} R_1\right)}{J_0\left(\sqrt{\frac{in}{\nu}} R_1\right)} \sqrt{\frac{-i\nu}{n}} + 2\pi R_1^2 u_t(n) \end{aligned} \tag{7}$$

The analytical solution for the velocity $u_z(r, t)$ and flow rate $Q(t)$ contain Bessel functions with complex arguments. From the point of view of the human arterial system, values of $(\rho n R^2)/\mu_f$ and $(\rho \omega R^2)/\mu_f > 2$ are significant.

For finding the solution valid under the above conditions, we use the asymptotic expansion of the Bessel functions. The asymptotic expansion of Bessel functions of argument x and order N can be written as

$$J_N(x) \sim \left(\frac{2}{\pi x}\right)^{1/2} \cos\left(x - \frac{n\pi}{2} - \frac{\pi}{4}\right)$$

Following MCLACHLAN (1955) and using the above asymptotic expression of a Bessel function with the appropriate arguments and order as required in eqn. 5, the equation for velocity profile after performing some mathematical steps reduces to:

$$\begin{aligned} u_z(r, t) = {} & \frac{A_0}{4\mu_f}(R_1^2 - r^2) + \frac{a_0}{\omega} \sin(\omega t + \phi) \\ & + \frac{A_1}{\rho n} \sin nt - \frac{a_0}{\omega} d_2 \sin(\alpha_2 + \omega t + \phi) \\ & - \frac{A_1}{\rho n} d_3 \sin(\alpha_3 + nt) + 2u_t(n) \end{aligned} \tag{8}$$

where

$$d_2 = \sqrt{\frac{R_1}{r}} \exp(\alpha_2)$$

$$d_3 = \sqrt{\frac{R_1}{r}} \exp(\alpha_3)$$

$$\alpha_2 = \sqrt{\frac{\omega}{2\nu}}(r - R_1)$$

$$\alpha_3 = \sqrt{\frac{n}{2\nu}}(r - R_1)$$

In the steady state, the above equation of velocity profile can be put into a more suitable form for computation purposes as

$$\begin{aligned} u_z(r, t) = {} & v_a + v_b \sin(\omega t + \phi + \theta_1) \\ & + v_p \sin(nt - \theta_2) \end{aligned} \tag{9}$$

where v_a, v_b and v_p are the velocity components due to steady heart action, external body acceleration and pulsating heart action, respectively. These components are

$$v_a = \frac{A_0}{4\mu_f}(R_1^2 - r^2) \tag{10}$$

$$v_b = \frac{a_0}{\omega}(1 - 2d_2 \cos\alpha_2 + d_2^2)^{1/2} \tag{11}$$

$$v_p = \frac{A_1}{\rho n}(1 - 2d_3 \cos\alpha_3 + d_3^2)^{1/2} \tag{12}$$

$$\theta_1 = \arctan\left(\frac{d_2 \sin\alpha_2}{1 - d_2 \cos\alpha_2}\right)$$

and

$$\theta_2 = \arctan\left(\frac{d_3 \sin\alpha_3}{1 - d_2 \cos\alpha_3}\right)$$

The expression for the shear stress τ is found to be

$$\begin{aligned} \tau = {} & -\frac{A_0}{2} r - \frac{a_0}{\omega}\left(\frac{\omega}{2\nu}\right)^{1/2} \mu_f d_2 \cos(\alpha_2 + \omega t + \phi) \\ & + \sin(\alpha_2 + \omega t + \phi)\left[1 - \left(\frac{\nu}{2\omega r^2}\right)^{1/2}\right] \\ & - \frac{A_1}{\rho n}\left(\frac{n}{2\nu}\right)^{1/2} \mu_f d_3 \cos(\alpha_3 + nt) \\ & + \sin(\alpha_3 + nt)\left[1 - \left(\frac{\nu}{2nr^2}\right)^{1/2}\right] \\ & - \frac{2\mu_f}{R} u_t(n)\lambda_n \end{aligned} \tag{13}$$

Similarly in the steady state the shear stress can be represented as three main components:

$$\tau = \tau_a + \tau_b \cos(\alpha_3 + \omega t + \phi - \theta_3) + \tau_p \cos(\alpha_3 + nt - \theta_4) \quad (14)$$

where τ_a, τ_b and τ_p are stress components due to steady heart action, external body acceleration and pulsating heart action, respectively. The expressions for τ_a, τ_b and τ_c are given below.

$$\tau_a = \frac{A_0}{2} r \quad (15)$$

$$\tau_b = -\mu_f a_0 \left(\frac{1}{2\omega\nu}\right)^{1/2} d_2 \left[2 + \frac{\nu}{2r^2\omega} - \frac{1}{r}\left(\frac{2\nu}{\omega}\right)^{1/2}\right]^{1/2} \quad (16)$$

$$\tau_c = -\mu_f \frac{A_1}{\rho}\left(\frac{1}{2n\nu}\right)^{1/2} d_3 \left[2 + \frac{\nu}{2r^2 n} - \frac{1}{r}\left(\frac{2\nu}{n}\right)^{1/2}\right]^{1/2} \quad (17)$$

where

$$\theta_3 = \arctan\left(1 - \frac{1}{r\sqrt{\frac{2\omega}{\nu}}}\right)$$

$$\theta_4 = \arctan\left(1 - \frac{1}{r\sqrt{\frac{2n}{\nu}}}\right)$$

Finally, the instantaneous rate of flow Q is obtained from eqn. 7 as follows:

$$Q = \frac{\pi R_1^4}{8\mu_f} A_0 + \frac{\pi R_1^2}{\omega} a_0 \sin(\omega t + \phi) + \frac{2\pi R_1^2 A_1}{\rho n} \sin nt - \sqrt{2}\frac{\pi R_1}{\omega}\left(\frac{\nu}{\omega}\right)^{1/2} a_0\{\cos(\omega t + \phi) - \sin(\omega t + \phi)\} - \sqrt{2}\frac{\pi R_1}{n\rho} A_1 \left(\frac{\nu}{\omega}\right)^{1/2}\{\cos nt - \sin nt\} \quad (18)$$

Likewise the expression of the flow rate Q in steady state can be written as

$$Q = Q_a + Q_b \sin(\omega t + \phi - \theta_5) + Q_p \sin(nt + \theta_6) \quad (19)$$

where Q_a is the average component of flow rate, Q_b the fluctuating part of flow rate due to external acceleration and Q_p is the fluctuating part of flow rate due to heart action. These are given as

$$Q_a = \frac{\pi R_1^4 A_0}{8\mu_f} \quad (20)$$

$$Q_b = \frac{\pi R_1 a_0}{\omega}\left(R_1^2 + \frac{4\nu}{\omega} + 2\sqrt{\frac{2\nu}{\omega}} R_1\right)^{1/2} \quad (21)$$

$$Q_p = \left(\frac{4\pi^2 A_1^2 R_1^2 \mu_f}{\rho^3 n^3} - \frac{4\pi^2 R_1^2 A_1^2}{\rho^2 n^2} - \frac{8\pi^2 R_1^3 A_1^2 \mu_f^{1/2}}{\sqrt{2}\rho^{5/2} n^{5/2}}\right)^{1/2}$$

where

$$\theta_5 = \arctan\left(\frac{p_1}{p_3}\right)$$

$$\theta_6 = \arctan\left(\frac{p_2}{p_4}\right)$$

$$p_1 = \frac{2\pi R_1}{\omega}\left(\frac{\mu_f}{2\rho\omega}\right)^{1/2} a_0$$

$$p_3 = \frac{\pi R_1^2}{\omega} a_0 + \frac{2\pi R_1}{\omega}\left(\frac{\mu_f}{2\rho\omega}\right)^{1/2} a_0$$

$$p_4 = \frac{2\pi R_1^2}{\rho n} A_1 - P_2$$

$$p_2 = \frac{2\pi R_1}{2\rho n}\left(\frac{\mu_f}{\rho n}\right)^{1/2}$$

At any partially occluded section, the flow rate due to body acceleration tends to be diminished. Because flow through the partially occluded portion must equal that through the nonoccluded portion, pressure drop per unit length in the former must be greater than that in the latter. In other words, pressure drop across the stenosed artery must increase. This additional pressure drop ΔP_a may be found quite accurately with the help of eqns. 20 and 21:

$$\Delta P_a = \int_{-2z_0}^{2z_0} \frac{(Q_b - Q_a)}{Q_a} \frac{A_0 a^4}{R_1^4} dz \quad (22)$$

3 Computational results

To obtain a quantitative idea of the effects of stenosis and body acceleration on blood flow in human beings, eqns. 15, 21 and 26 were evaluated, specifically for the carotid artery. The relevant data on the size of artery, the properties of blood and the action of the heart were compiled from published literature (McDonald, 1974; Milnor, 1982). Let A_0' be the value of the steady component of the pressure gradient at inlet to the stenotic artery (i.e. at $z = -2z_0$). A_0' may be calculated from known value of the average flow rate Q_a through the artery with the help of the following equation

$$A_0' = \frac{8\mu_f Q_a}{\pi a^2}$$

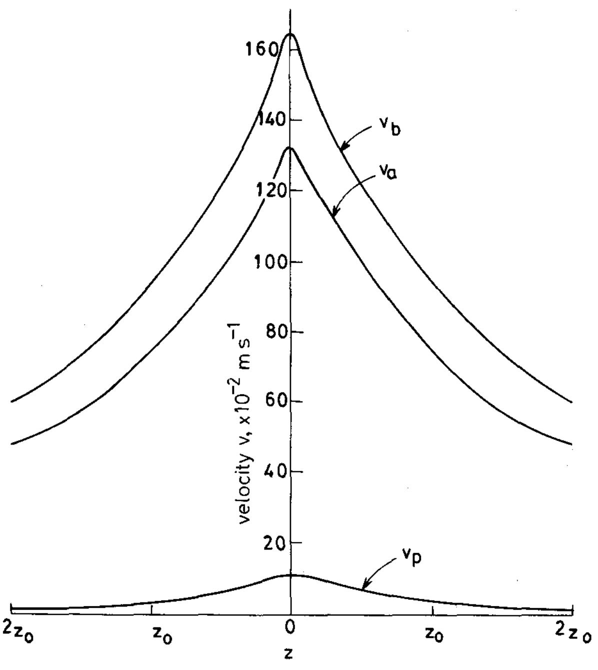


Fig. 2 *Variation of peak components along the length of the stenosis. (Carotid artery $a_0 = 0{\cdot}2$ g, $\delta = 0{\cdot}2a$, $a = 0{\cdot}4$ cm, $f_b = 0{\cdot}6$ Hz)*

v_a = component due to steady heart action
v_b = component due to body acceleration
v_p = component due to pulsating heart action

A'_1 (the fluctuating component of pressure gradient at $z = -2z_0$) was taken to be 20 per cent of A'_0. The values of μ_f and ρ for blood were taken as $0{\cdot}004$ kg m^{-1} s^{-1} and $1{\cdot}05 \times 10^3$ kg m^{-3}, respectively. Pulse frequency f_p was taken as 1·2 Hz. The expression for velocity, shear stress and rate of flow (eqns. 9, 14 and 19) can be split up into three portions—one non-fluctuating or constant, and the remaining two fluctuating or periodic. One periodic component has frequency corresponding to the heartbeat and the other has frequency corresponding to the body acceleration. The fluctuating component corresponding to the externally applied body acceleration so varies with time that its amplitude tends to a constant value after a sufficiently large interval of time.

In the following paragraphs, we present typical results.

3.1 *Variation of flow velocity along the stenosis*

Fig. 2 shows the variation of the peak velocity components v_a, v_b and v_p along the length of the stenosis. They occur at $r = 0$, i.e. points lying on the longitudinal axis of the tube. In the figure, the component v_a corresponds to the steady, non-fluctuating component of the heart action, the component v_b corresponds to the externally impressed body acceleration and the component v_p results from the pulsating component of the heart action. The maxima of the peak occurs at the neck of the stenosis, i.e. at $z = 0$, and the minima at the ends of the stenotic portion, i.e. $z = \pm 2z_0$.

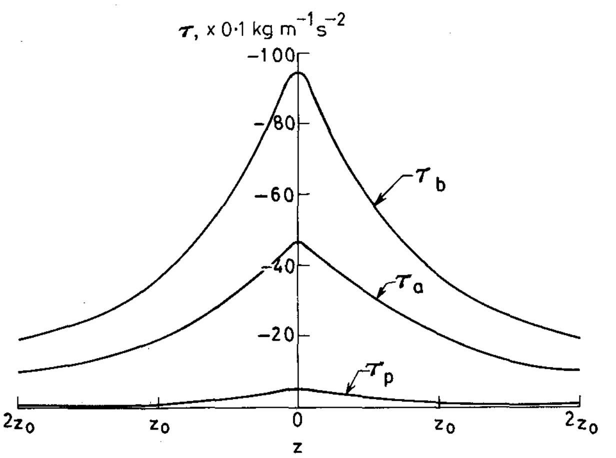


Fig. 3 *Variation of peak shear stress along the length of the stenosis. (Carotid artery, $a_0 = 0{\cdot}2$ g, $\delta = 0{\cdot}2a$, $a = 0{\cdot}4$ cm, $f_b = 0{\cdot}6$ Hz)*

τ_a = component due to steady heart action
τ_b = component due to body acceleration
τ_p = component due to pulsating heart action

3.2 *Variation of shear stress along the stenosis*

Fig. 3 shows the variation of peak shear stress components τ_a, τ_b and τ_p along the length of the stenosis. In the figure, the component τ_a corresponds to the steady, non-fluctuating component of the heart action, the component τ_b corresponds to the externally imposed body acceleration and the component τ_p results from the pulsating component of the heart action. The maximum of the peak shear stress occurs at the neck of the stenosis, i.e. at $z = 0$, and the minima at the ends of the stenotic portion, i.e. at $z = \pm 2z_0$.

3.3 *Effect of the size of stenotic protuberance*

Fig. 4 shows the effect of the size of stenotic protuberance δ on Δp pressure drop across the stenosis. It is found that greater the size of the protuberance, the greater is the pressure drop. The increase in pressure drop for a given increase in δ is small at smaller values of δ, and it becomes exponentially large at larger values of δ. Also, it is found that the pressure gradient is the largest at the neck of the stenosis and becomes smaller as the radius R_1 increases on either side of the neck (eqn. 22).

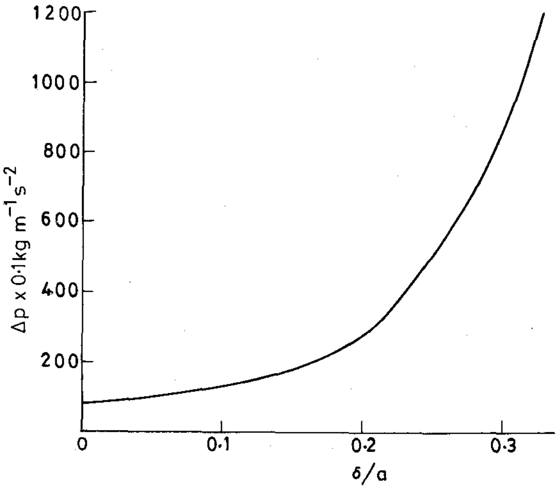


Fig. 4 *Effect of depth of occlusion δ on pressure drop ΔP. (Carotid artery, $a = 0{\cdot}4$ cm, $z_0 = 0{\cdot}4$ cm, $Q = 12{\cdot}57$ cm^3 s^{-1})*

3.4 *Effect of length of the stenosis*

When the stenotic protuberance remains constant in comparison with the nominal radius of the artery, it is found that pressure drop Δp varies in proportion to the length of the stenosis (eqn. 20).

3.5 *Effect of magnitude of the body acceleration*

The effect of the body acceleration a_0 is to increase the flow velocity and flow rate. As a result, the shear stress and the pressure drop across the stenosis are also increased. It is found that the above dependent variables vary linearly with a_0 (eqn. 21).

3.6 *Effect of frequency of body acceleration*

Fig. 5 shows the effect of frequency of the periodic body

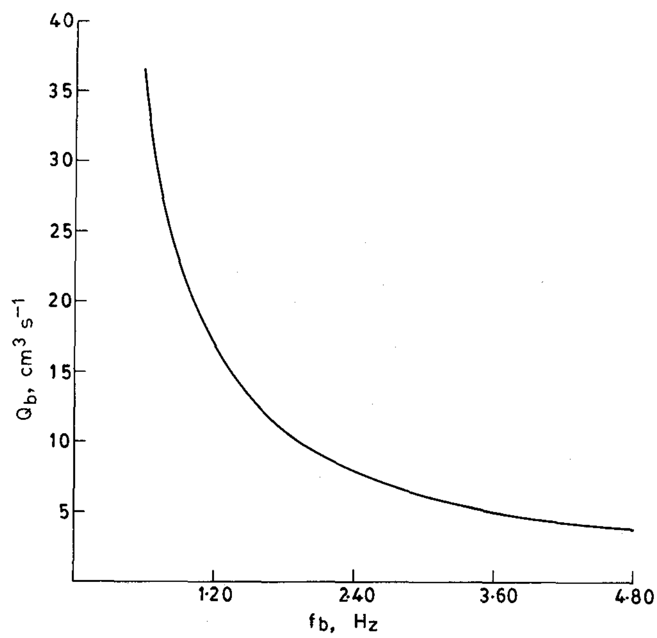


Fig. 5 *Dependence of Q_b on f_b. (Carotid artery, $a = 0{\cdot}4$ cm, $a_0 = 0{\cdot}2$ g)*

acceleration f_b on the flow component due to body acceleration. It is found that an increase in the frequency of periodic body acceleration f_b results in a decrease of the component of flow rate ascribable to body acceleration. Consequently, the shear stress, the velocities of flow and the pressure across the stenosis decrease correspondingly.

3.7 *Effect of resistive impedance*

The resistive impedance to flow of the stenosed artery may be defined as the ratio of the pressure drop across the artery to the volume rate of flow through it. The variation of resistive impedance with body acceleration is shown in Fig. 6. In the presence of body acceleration and the pulsatile pumping action of the heart, the impedance of the stenotic portion of the artery can be split into two main components—one non-fluctuating corresponding to the steady component of the pressure gradient A_0 due to the heart action, and the other fluctuating due to the externally imposed periodic body acceleration a_0. (The component of impedance due to the fluctuating component of pressure gradient as a result of the heart action can be neglected as small.) Now the pressure drop across the stenosis and the flow rate due to the steady portion of the heart action remain with time. But both the pressure drop and the flow rate vary in direct proportion to the amplitude of the body acceleration. When a_0 is small, impedance is governed by the steady heart action, and when a_0 is large, impedance is practically governed by the body acceleration. This is because, in the latter case, flow rate due to a_0 far exceeds that due to A_0.

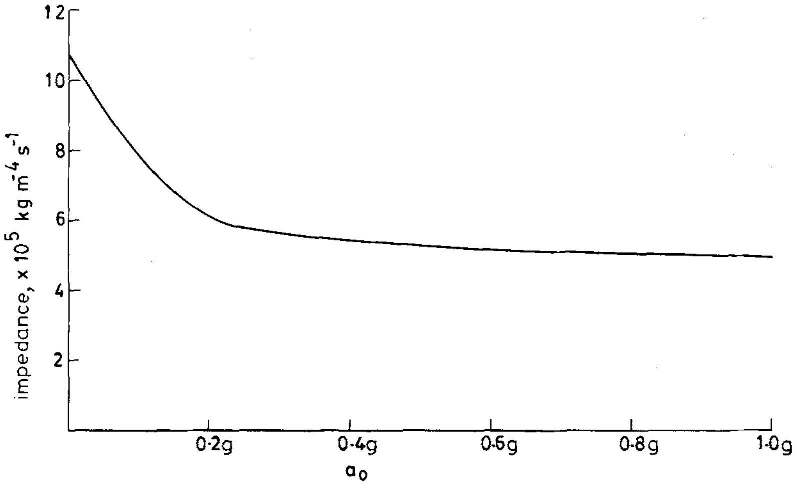


Fig. 6 *Variation of independence with acceleration. (Carotid artery, $z_0 = a, \delta = 0{\cdot}2a, a = 4\,cm, f_b = 0{\cdot}6\,Hz$)*

4 Discussion

The effect of occlusion spread over a certain length of a tube conveying a fluid is to cause an increase in the pressure drop across that length. This is because an occlusion decreases the cross-sectional area of the tube over the length where it extends. As a result the flow velocity increases, if the same volume rate of flow is maintained (Fig. 2). Because fluid velocity at the outer radius is always zero the velocity gradients and hence shear stress are increased by an occlusion. The bigger the occlusion, the greater the increase in shear stress (Fig. 3).

Now the externally imposed body acceleration tends to increase the rate of flow through the artery. As a result, it contributes its own terms to velocity gradient and shear stress. If this increased flow is maintained through the stenotic region of the artery, it results in a further increase of the pressure drop across that region. The greater the amplitude of the body acceleration or the smaller its frequency, the greater are the effects of body acceleration (Fig. 5).

From the above one can deduce the effects of occlusion and body acceleration on impedance of the stenotic artery. We find that bigger the size of the occlusion, the greater the pressure drop and therefore the greater the impedance (Fig. 4). It is found that the impedance of a moderately stenosed carotid artery caused by body acceleration is smaller than that due to the pressure gradient produced by the pumping action of the heart.

5. Conclusion

(1) Stenosis increases peak velocities, velocity gradient and resulting shear stress.
(2) The bigger the occlusion or the greater the length of the stenosis, the larger the pressure drop across the stenotic artery and greater its impedance.
(3) Body acceleration increases both the flow rate and the pressure drop across the stenosis.
(4) Impedance to body acceleration in a carotid artery is smaller than that due to pressure gradient created by heart action. Also, the impedance of the artery progressively decreases as the magnitude of body acceleration is gradually increased.

References

Back, L. D., Radbill, J. R. and Cranford, D. A. (1977) Analysis of pulsatile viscous blood flow through diseased coronary arteries of man. *J. Biomech.*, **10**, 339–353.

Carslaw, H. S. and Jaeger, J. C. (1963) *Operational methods in applied mathematics.* Dover Pub., New York, 72.

Forrester, J. H. and Young, D. F. (1970*a*) Flow through a converging-diverging tube and its applications in occlusive vascular disease I. *J. Biomech.* **3**, 297–306.

Forrester, J. H. and Young, D. F. (1970*b*) Flow through a converging-diverging tube and its applications in occlusive vascular disease II. *Ibid.*, **3**, 307–317.

Fung, Y. C., Parrone, N. and Anliker, M. (1971) *Biomechanics—its foundation and objectives.* Prentice Hall, Englewood Cliffs, New Jersey.

Lee, J. S. and Fung, Y. C. (1970) Flow in a locally constricted tube at low Reynolds numbers. *J. Appl. Mech. Trans. ASME*, **37**, 9–16.

McDonald, D. (1974) *Blood flow in arteries.* Edward Arnold.

Mclachlan, N. W. (1955) *Bessel functions for engineers.* Clarendon Press, Oxford, 81.

Milnor, W. R. (1982) *Thermodynamics.* Williams & Williams, Baltimore.

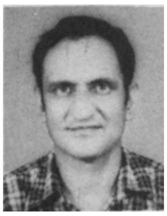
Padmanabhan, N. (1980) Mathematical model of arterial stenosis. *Med. & Biol. Eng. & Comput.*, **18**, 281–286.

YOUNG, D. F. (1968) Effect of a time-dependent stenosis on flow through a tube. *J. Eng. for Industry, Trans. ASME*, **90**, 248–254.

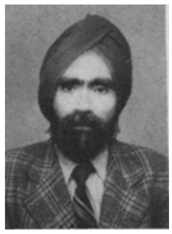
YOUNG, D. F. and TSAI, F. Y. (1973*a*) Flow characteristics in models of arterial stenosis: I steady flow. *J. Biomech.*, **6**, 395–410.

YOUNG, D. F. and TSAI, F. Y. (1973*b*) Flow characteristics in models of arterial stenosis: II unsteady flow. *Ibid.*, **6**, 547–559.

Authors' biographies



Dr Virender Kumar Sud graduated from Panjab University, India, then obtained his Master's degree in Applied Mathematics in 1964 from the University of Roorkee and his Ph.D. degree from the Indian Institute of Technology, Delhi in 1968. Dr Sud joined the Biophysics Department at the All-India Institute of Medical Sciences, New Delhi, in 1968, where he is currently working as Associate Professor. His areas of interest include mathematical applications in biology, especially cardiovascular flow. Dr Sud also worked at the US Aerospace Medical Research Laboratory, Ohio, for two years (1978–1980) as senior Postdoctoral Research Associate of the US NRC.



Dr G. S. Sekhon is a Professor in the Department of Applied Mechanics at the Indian Institute of Technology, Delhi. He obtained his B.Sc. (Mechanical Engineering) degree from Panjab Engineering College, Chandigarh, his MS (Production & Industrial Engineering) degree from the University of Roorkee and his Ph.D. from the Indian Institute of Technology, Delhi. His research interests include mechanics, solid mechanics and computational methods.