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DIFFERENCE SCHEME FOR TWO-PHASE FLOW*

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Abstract: *A numerical method for two-phase flow with hydrodynamics behavior was considered. The nonconservative hyperbolic governing equations proposed by Saurel and Gallout were adopted. Dissipative effects were neglected but they could be included in the model without major difficulties. Based on the opinion proposed by Abgrall that "a two phase system, uniform in velocity and pressure at $t = 0$ will be uniform on the same variable during its temporal evolution", a simple accurate and fully Eulerian numerical method was presented for the simulation of multiphase compressible flows in hydrodynamic regime. The numerical method relies on Godunov-type scheme, with HLLC and Lax-Friedrichs type approximate Riemann solvers for the resolution of conservation equations, and nonconservative equation. Speed relaxation and pressure relaxation processes were introduced to account for the interaction between the phases. Test problem was presented in one space dimension which illustrated that our scheme is accurate, stable and oscillation free.*

Key words: compressible two-phase flow; relaxation process; approximate Riemann solver
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Introduction

The study of two-phase system in very important because a great number of two-phase flow phenomena exist in industrial processes and our daily life. By now two kinds of mathematical models have been developed. One is Euler-Lagrange model that treats fluid as continuum and particles as discrete phase, and tracks the particle trajectory in the Lagrange system. The other is two-fluid model in which both the fluid and discrete particles were thought as continuum and solve the equations in the Euler system. This paper focuses on the two-fluid model. Being known little about its flow mechanism and wave structure, numerical simulation to it is still in infancy and the main start point is to transfer the scheme for conservation equations to it. Some works have been

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done in this area. Toumi[1], based on the work of G. Dal Maso and P. Le Floch[2], tried to linearise the system and had solved it, but the choice of the integral path still be a question; L. Sainaulieu[3] used finite volume method to solve locally linearised two-fluid system, and from the numerical results, we can see that the scheme is inaccurate and has a low resolution for discontinuities; R. Saurel[4], on the work of Abgrall[5], used HLL scheme to discrete the conservative flux, but unphysical oscillation occurred near the discontinuities. In this paper, we use HLLC scheme to solve the equations proposed by D. A. Drew[6] and numerical tests are presented to test the accuracy and stability of the scheme.

1 Mathematical Model

The compressible two-phase flow model proposed by D. A. Drew[6], Saurel and Gallouet[7] was used here

$$\frac{\partial \alpha_g}{\partial t} + V_i \frac{\partial \alpha_g}{\partial x} = 0, \tag{1}$$

$$\frac{\partial (\alpha_g \rho_g)}{\partial t} + \frac{\partial (\alpha_g \rho_g u_g)}{\partial x} = \dot{m}, \tag{2}$$

$$\frac{\partial (\alpha_g \rho_g u_g)}{\partial t} + \frac{\partial (\alpha_g \rho_g u_g^2 + \alpha_g p_g)}{\partial x} = P_i \frac{\partial \alpha_g}{\partial x} + mV_i + F_d, \tag{3}$$

$$\frac{\partial (\alpha_g \rho_g E_g)}{\partial t} + \frac{\partial [u_g(\alpha_g \rho_g E_g + \alpha_g p_g)]}{\partial x} = P_i V_i \frac{\partial \alpha_g}{\partial x} + \dot{m} E_i + F_d V_i + Q_i, \tag{4}$$

$$\frac{\partial (\alpha_l \rho_l)}{\partial t} + \frac{\partial (\alpha_l \rho_l u_l)}{\partial x} = -\dot{m}, \tag{5}$$

$$\frac{\partial (\alpha_l \rho_l u_l)}{\partial t} + \frac{\partial (\alpha_l \rho_l u_l^2 + \alpha_l p_l)}{\partial x} = -P_i \frac{\partial \alpha_g}{\partial x} - \dot{m} V_i - F_d, \tag{6}$$

$$\frac{\partial (\alpha_l \rho_l E_l)}{\partial t} + \frac{\partial [u_l(\alpha_l \rho_l E_l + \alpha_l p_l)]}{\partial x} = -P_i V_i \frac{\partial \alpha_g}{\partial x} - \dot{m} E_i - F_d V_i - Q_i, \tag{7}$$

where subscripts g and l represent gas phase and liquid phase, respectively, $\alpha_k (k = l, g)$ is the phase volume fraction and satisfies relation $\alpha_g + \alpha_l = 1$, $\dot{m}$ is the mass transfer between phases, the drag force F_d is usually in form $F_d = \lambda(u_l - u_g)$ and λ is a positive parameter, P_i amd V_i are average pressure and velocity on the interface, and Q_i is connective heat exchange.

The estimate for P_i and V_i proposed by Saurel[4] was used here

$$P_i = \sum \alpha_k p_k \qquad (k = l, g), \tag{8}$$

$$V_i = \sum \alpha_k \rho_k u_k \Big/ \sum \alpha_k \rho_k \qquad (k = l, g). \tag{9}$$

From Eqs. (2) ~ (7) we can see that if we add the mass conservation, momentum conservation, and energy conservation equations, respectively; we will get the Euler equations for the mixture. Being no theoretical solution existed about two-phase flow equation, one commonly accepted method is to compare the mass-averaged numerical solution of the system with the thoeretical solution of the Euler equation of the mixture.

2 Difference Scheme

Assuming $\dot{m} = 0$ and $Q_i = 0$, Eqs. (2) ~ (7) can be written as follows, when F_d is

neglected,

$$\frac{\partial \boldsymbol{U}}{\partial t} + \frac{\partial \boldsymbol{F}(\boldsymbol{U})}{\partial x} = \boldsymbol{H}(\boldsymbol{U}) \frac{\partial \alpha_g}{\partial x}. \tag{10}$$

Equation (1) is still in nonconservative form

$$\frac{\partial \alpha_g}{\partial t} + V_i \frac{\partial \alpha_g}{\partial x} = 0, \tag{11}$$

where

$$\begin{aligned}
\boldsymbol{U} &= (\alpha_g\rho_g, \alpha_g\rho_g u_g, \alpha_g\rho_g E_g, \alpha_l\rho_l, \alpha_l\rho_l u_l, \alpha_l\rho_l E_l)^{\mathrm{T}},\\
\boldsymbol{F}(\boldsymbol{U}) &= (\alpha_g\rho_g u_g, \alpha_g\rho_g u_g^2 + \alpha_g p_g, u_g(\alpha_g\rho_g E_g + \alpha_g p_g),\\
&\qquad \alpha_l\rho_l u_l, \alpha_l\rho_l u_l^2 + \alpha_l p_l, u_l(\alpha_l\rho_l E_l + \alpha_l p_l))^{\mathrm{T}},\\
\boldsymbol{H}(\boldsymbol{U}) &= (0, P_i, P_i V_i, 0, -P_i, -P_i V_i)^{\mathrm{T}}.
\end{aligned}$$

In general, it is reasonable to assume that the two-phase system is in equilibrium due to that the characteristic time of the system is much longer than the relaxation time between the two phases. From this point of view, the whole solution procedure could be divided into three steps,

1) Solving Eq.(10) using differencing scheme;

2) Instantaneous velocity relaxation with interphase interaction;

3) Instantaneous pressure relaxation with interphase interaction.

Step 1 Solving Eq.(10) using differencing scheme

The criterion for building up the numerical scheme in this paper is proposed by Abgrall[5], it is stated as "a two phase system, uniform in velocity and pressure at $t = 0$ will be uniform on the same variable during its temporal evolution". The equation of the state used in this paper is

$$p = (\gamma - 1)\rho e - \gamma\pi. \tag{12}$$

To solve the system (10), we use following central difference scheme:

$$\boldsymbol{U}_j^{n+1} = \boldsymbol{U}_j^n - \lambda(\boldsymbol{F}_{j+1/2}^n - \boldsymbol{F}_{j-1/2}^n) + \Delta t \boldsymbol{H}(\boldsymbol{U}_j^n)\nabla, \tag{13}$$

where ∇ represents the discrete form of $\partial\alpha_g/\partial x$ which will be given later. Saurel[4] used the HLL scheme for term $\boldsymbol{F}_{j+0.5}^n$. Here we use HLLC scheme due to its higher resolution for contact discontinuities, i.e., we choose,

$$\boldsymbol{F}_{j+1/2}^{HLLC} = \begin{cases} \boldsymbol{F}_L & (S_L \geqslant 0),\\ \boldsymbol{F}_{*L} = \boldsymbol{F}_L + S_L(\boldsymbol{U}_{*L} - \boldsymbol{U}_L) & (S_L \leqslant 0 \leqslant S_*),\\ \boldsymbol{F}_{*R} = \boldsymbol{F}_R + S_R(\boldsymbol{U}_{*R} - \boldsymbol{U}_R) & (S_* \leqslant 0 \leqslant S_R),\\ \boldsymbol{F}_R & (S_R \leqslant 0), \end{cases} \tag{14}$$

where

$$\begin{aligned}
\boldsymbol{U} &= (\alpha_g^K\rho_g^K, \alpha_g^K\rho_g^K u_g^K, \alpha_g^K\rho_g^K E_g^K, \alpha_l^K\rho_l^K, \alpha_l^K\rho_l^K u_l^K, \alpha_l^K\rho_l^K E_l^K)^{\mathrm{T}},\\
\boldsymbol{F}_K(\boldsymbol{U}) &= (\alpha_g^K\rho_g^K u_g^K, \alpha_g^K\rho_g^K u_g^{K^2} + \alpha_g^K p_g^K, u_g^K(\alpha_g^K\rho_g^K E_g^K + \alpha_g^K p_g^K),\\
&\qquad \alpha_l^K\rho_l^K u_l^K, \alpha_l^K\rho_l^K u_l^{K^2} + \alpha_l^K p_l^K, u_l^K(\alpha_l^K\rho_l^K E_l^K + \alpha_l^K p_l^K))^{\mathrm{T}},
\end{aligned}$$

$$\boldsymbol{U}_{*K} = \left[\begin{array}{l} \dfrac{S_K\alpha_g^K\rho_g^K - \alpha_g^K\rho_g^K u_g^K}{S_K - S_*} \\ \dfrac{S_K\alpha_g^K\rho_g^K - \alpha_g^K\rho_g^K u_g^K}{S_K - S_*} S_* \\ \alpha_g^K\rho_g^K(S_K - u_g^K)[E_g^K + (S_* - u_g^K)\cdot \\ \qquad (S_* + \dfrac{p_g^K}{\rho_g^K(S_K - u_g^K)})]\Big/(S_k - S_*) \\ \dfrac{S_K\alpha_l^K\rho_l^K - \alpha_l^K\rho_l^K u_l^K}{S_K - S_*} \\ \dfrac{S_K\alpha_l^K\rho_l^K - \alpha_l^K\rho_l^K u_l^K}{S_K - S_*} S_* \\ \alpha_l^K\rho_l^K(S_K - u_l^K)[E_l^K + (S_* - u_l^K)\cdot \\ \qquad (S_* + \dfrac{p_l^K}{\rho_l^K(S_K - u_l^K)})]\Big/(S_k - S_*) \end{array}\right]_{K=L,R}.$$

Many estimates existed for shockwave speeds S_L and S_R (see Ref.[8]). The formula used here is

$$S_L = \min(u_g^L - c_g^L, u_l^L - c_l^L, u_g^R - c_g^R, u_l^R - c_l^R), \tag{15}$$

$$S_R = \max(u_g^L + c_g^L, u_l^L + c_l^L, u_g^R + c_g^R, u_l^R + c_l^R). \tag{16}$$

As for S_*, assuming pressure equilibrium on the two sides of interface, we get

$$S_* = \frac{P_m^R - p_m^L + \rho_m^L u_m^L(S_L - u_m^L) - \rho_m^R u_m^R(S_R - u_m^R)}{\rho_m^L(S_L - u_m^L) - \rho_m^R(S_R - u_m^R)}, \tag{17}$$

where

$$\begin{aligned} &p_m^K = \alpha_g^K p_g^K + \alpha_l^K p_l^K, \quad \rho_m^K = \alpha_g^K\rho_g^K + \alpha_l^K\rho_l^K, \\ &u_m^K = (\alpha_g^K\rho_g^K u_g^K + \alpha_l^K\rho_l^K u_l^K)/\rho_m^K \end{aligned} \qquad (K = L,R).$$

Supposing that Eq.(14) can be written as

$$\boldsymbol{F}_{j+1/2}^{HLLC} = \boldsymbol{F}_{j+1/2} + S_{j+1/2}(\boldsymbol{U}_{*j+1/2} - \boldsymbol{U}_{j+1/2})$$

and substituting it into Eq.(13), we get the difference scheme for the conservation terms of the system. Here we just do manipulation to the gas phase, the similar result is for the liquid phase.

For mass conservation equation

$$\begin{aligned} (\alpha_g\rho_g)_j^{n+1} = (\alpha_g\rho_g)_j^n - \lambda\{&(\alpha_g\rho_g u_g)_{j+1/2}^n + S_{j+1/2}^n[(\alpha_g\rho_g)_{*j+1/2}^n - (\alpha_g\rho_g)_{j+1/2}^n] - \\ &(\alpha_g\rho_g u_g)_{j-1/2}^n - S_{j-1/2}^n[(\alpha_g\rho_g)_{*j-1/2}^n - (\alpha_g\rho_g)_{j-1/2}^n]\}. \end{aligned} \tag{18}$$

For momentum conservation equation

$$\begin{aligned} (\alpha_g\rho_g u_g)_j^{n+1} = (\alpha_g\rho_g u_g)_j^n - \lambda\{&(\alpha_g\rho_g u_g^2 + \alpha_g p_g)_{j+1/2}^n + \\ &S_{j+1/2}^n[(\alpha_g\rho_g u_g)_{*j+1/2}^n - (\alpha_g\rho_g u_g)_{j+1/2}^n] - (\alpha_g\rho_g u_g^2 + \alpha_g p_g)_{j-1/2}^n - \end{aligned}$$

$$S^n_{j-1/2}[(\alpha_g\rho_g u_g)^n_{*j-1/2} - (\alpha_g\rho_g u_g)^n_{j-1/2}]\} + \Delta t P_i \nabla . \tag{19}$$

Note that $u^n_g = V^n_i = S^n_*$. If

$$\nabla = \frac{1}{\Delta x}(\alpha^n_{j+1/2} - \alpha^n_{j-1/2}), \tag{20}$$

we have $u^{n+1}_j = u^n_j$. So we have difference scheme for $\partial\alpha_g/\partial x$,

$$\frac{\partial\alpha_g}{\partial x} = \frac{1}{\Delta x}(\alpha^n_{j+1/2} - \alpha^n_{j-1/2}). \tag{21}$$

For energy conservation equation

$$\begin{aligned}(\alpha_g\rho_g E_g)^{n+1}_j = {} & (\alpha_g\rho_g E_g)^n_j - \lambda\{[u_g(\alpha_g\rho_g E_g + \alpha_g p_g)]^n_{j+1/2} + \\ & S^n_{j+1/2}[(\alpha_g\rho_g E_g)^n_{*j+1/2} - (\alpha_g\rho_g E_g)^n_{j+1/2}] - [u_g(\alpha_g\rho_g E_g + \alpha_g p_g)]^n_{j-1/2} - \\ & S^n_{j-1/2}[(\alpha_g\rho_g E_g)^n_{*j-1/2} - (\alpha_g\rho_g E_g)^n_{j-1/2}]\} + \Delta t P_i V_i \nabla .\end{aligned} \tag{22}$$

Making use of the relation $E_g = e_g + u^2_g/2$ and the equation of state, we can prove that only if

$$\begin{aligned}\alpha^{n+1}_j = {} & \alpha^n_j - \lambda\{u^n_j(\alpha^n_{j+1/2} - \alpha^n_{j-1/2}) + S^n_{j+1/2}(\alpha^n_{*j+1/2} - \alpha^n_{j+1/2}) - \\ & S^n_{j-1/2}(\alpha^n_{*j-1/2} - \alpha^n_{j-1/2})\},\end{aligned} \tag{23}$$

could we get $p^n_j = p^{n+1}_j$. And in this way, we get the numerical scheme for the scalar Eq.(11). Because we could not get $\alpha^n_{*j+1/2}$ directly, Roe average[9] was used for it

$$\alpha_{*j+1/2} = \frac{\sqrt{\alpha^L_g\rho^L_g}\alpha^L_g + \sqrt{\alpha^R_g\rho^R_g}\alpha^R_g}{\sqrt{\alpha^L_g\rho^L_g} + \sqrt{\alpha^R_g\rho^R_g}}. \tag{24}$$

It is easy to see in formula (23), that the classic discrete form $u^n_j(\alpha^n_{j+1} - \alpha^n_{j-1})$ for the convective term $V_i(\partial\alpha_g/\partial x)$ was adopted, and the other term $S^n_{j+1/2}(\alpha^n_{*j+1/2} - \alpha^n_{j+1/2}) - S^n_{j-1/2}(\alpha^n_{*j-1/2} - \alpha^n_{j-1/2})$, represents the viscosity of the scheme.

As a comparison, Lax-Friedrichs scheme was also used to discrete the conservation terms and we have

$$\boldsymbol{U}^{n+1}_j = \boldsymbol{U}^n_j - \lambda(\boldsymbol{F}^{LF}_{j+1/2} - \boldsymbol{F}^{LF}_{j-1/2}) + \Delta t\boldsymbol{H}(\boldsymbol{U}^n_j)\nabla , \tag{25}$$

where

$$\boldsymbol{F}^{LF}_{j+1/2} = \frac{1}{2}(\boldsymbol{F}^n_j + \boldsymbol{F}^n_{j+1}) + \frac{1}{2\lambda}(\boldsymbol{U}^n_j - \boldsymbol{U}^n_{j+1}), \tag{26}$$

$$\alpha^{n+1}_j = \alpha^n_j - \lambda\{u^n_j(\alpha^n_{j+1/2} - \alpha^n_{j-1/2})\} - \alpha^n_j + 0.5(\alpha^n_{j+1} + \alpha^n_{j-1}). \tag{27}$$

Step 2 Instantaneous velocity relaxation with interphase interaction

According to the momentum conservation of the whole system, we have

$$u = u_g = u_l = \frac{\alpha_g\rho_g u_{g0} + \alpha_l\rho_l u_{l0}}{\alpha_g\rho_g + \alpha_l\rho_l}, \tag{28}$$

and consequently there is a variation of energy in each phase

$$\frac{\partial\alpha_g\rho_g}{\partial t} = 0, \tag{29}$$

$$\frac{\partial \alpha_g \rho_g u_g}{\partial t} = \lambda(u_l - u_g), \tag{30}$$

$$\frac{\partial \alpha_g \rho_g E_g}{\partial t} = \lambda V_i(u_l - u_g), \tag{31}$$

and after some manipulation we have

$$e_g = e_{g0} + \frac{1}{2}(u_g - u_{g0})(V_i^n - u_{g0}), \tag{32}$$

then we should adjust the conservation variable U_0, where subscript "0" represents the result of the first step. The same result is similar for liquid phase.

Step 3 Instantaneous pressure relaxation with interphase interaction

Pressure relaxation will lead to variation of internal energy and volume fraction. Due to the fact that pressure relaxation is important to the resolution of interface, a reasonable estimate for μ is infinite especially when we know nothing about parameter μ. From equations

$$\frac{\partial \alpha_g}{\partial t} = \mu(p_g - p_l), \tag{33}$$

$$\frac{\partial \alpha_g \rho_g}{\partial t} = 0, \tag{34}$$

$$\frac{\partial \alpha_g \rho_g u_g}{\partial t} = 0, \tag{35}$$

$$\frac{\partial \alpha_g \rho_g E_g}{\partial t} = -\mu P_i(p_g - p_l), \tag{36}$$

we get

$$\frac{\partial \alpha_g \rho_g E_g}{\partial t} = -P_i \frac{\partial \alpha_g}{\partial t}. \tag{37}$$

Integrating with time, the following relation approximately holds,

$$e_g^* = e_g^0 - \bar{P}_i/(\alpha_g \rho_g)(\alpha_g^* - \alpha_g^0). \tag{38}$$

The similar result is for the liquid phase. For given the equation of the state, our aim is to find α_g satisfying pressure equilibrium condition

$$f(\alpha_g) = P_g(\rho_g, e_g) - P_l(\rho_l, e_l) = 0, \tag{39}$$

where $\bar{P}_i = (P_i + P_i^0)/2$, P_i^0 is computed from state $(\alpha_g^0, \rho_g^0, e_g^0)$ and P_i is from state (α_g, ρ_g, e_g). In this way the whole system is solved.

3 Numerical Test

Here the numerical test used in Ref.[4] is used and a comparison is made.

Consider a shock tube filled with high-pressure liquid on the left side and with air on right, that is,

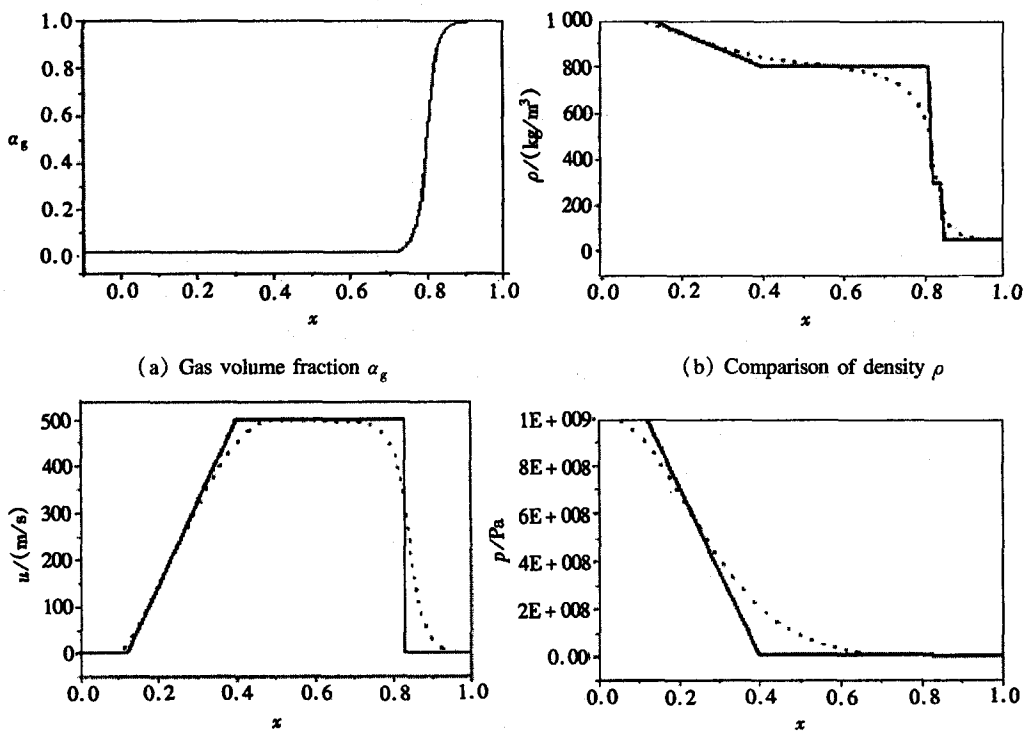
$W_L = (\varepsilon, 1E-6\ \text{kg/m}^3, 1E-6\ \text{m/s}, 1E-6\ \text{Pa}, 1\,000\ \text{kg/m}^3, 1E-6\ \text{m/s}, 1E-9\ \text{Pa})$,

$\gamma_L = 4.4$, $\pi_L = 6.1^8$,

$W_R = (1-\varepsilon, 50\ \text{kg/m}^3, 1E-6\ \text{m/s}, 1E+0.5\ \text{Pa}, 1E-6\ \text{kg/m}^3, 1E-6\ \text{m/s}, 1E-6\ \text{Pa})$,

$\gamma_R = 1.4, \pi_R = 0.0, \varepsilon = 1E-8$.

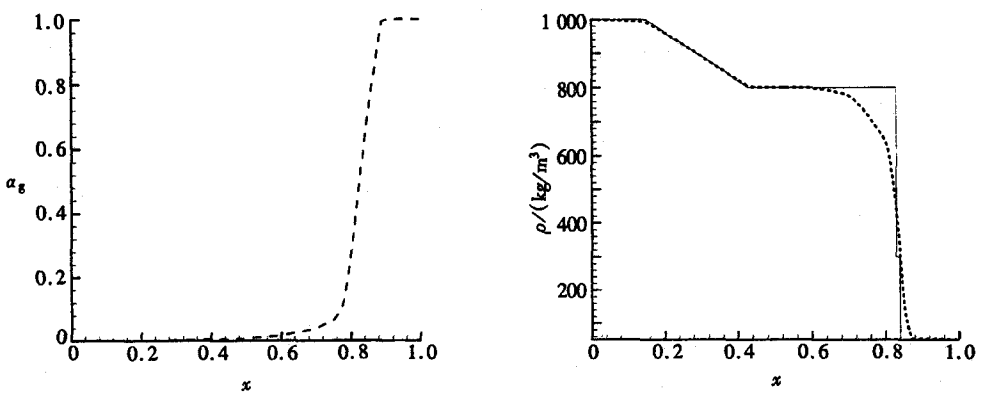
Numerical simulation was done at $t = 299\mu s$ with 100 and 1 000 computational cells, respectively, the result was shown in Fig.1 to Fig.5 (solid line represents the theoretical result and dotted line represents numerical result) and a comparison was done with that of Saurel[4].



(a) Gas volume fraction α_g (b) Comparison of density ρ

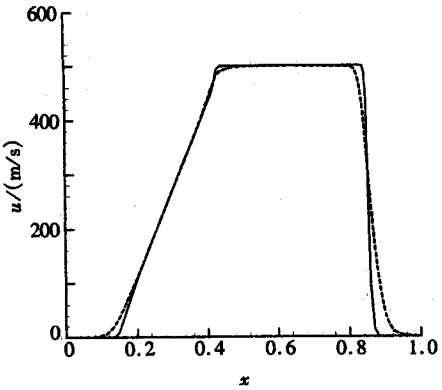
(c) Comparison of velocity u (d) Comparison of pressure p

Fig.1 Comparsion between exact and numerical solutions with 100 cells using Lax-Friedrichs scheme as the solver

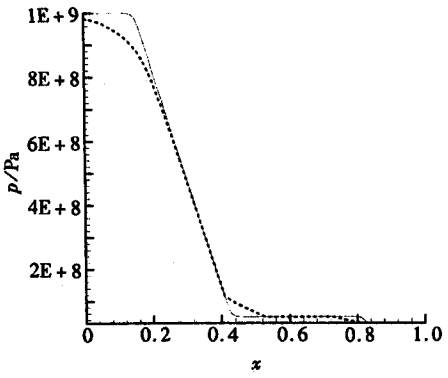


(a) Gas volume fraction α_g (b) Comparison of density ρ

Fig.2 Comparsion between exact and numerical solutions with 100 cells using HLL scheme as the solver

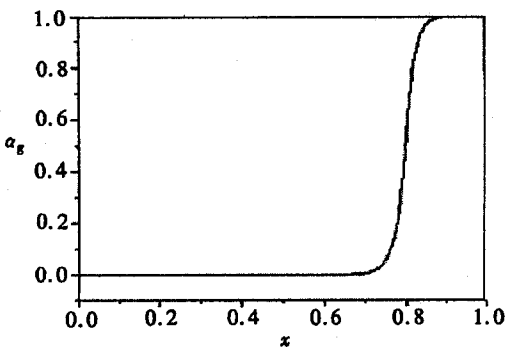


(c) Comparison of velocity u

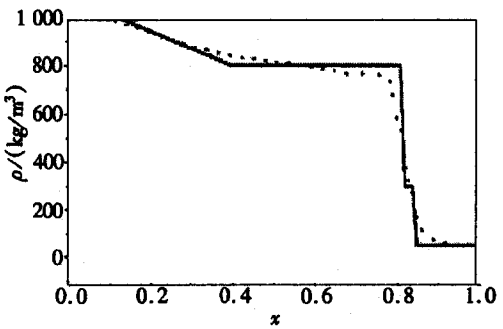


(d) Comparison of pressure p

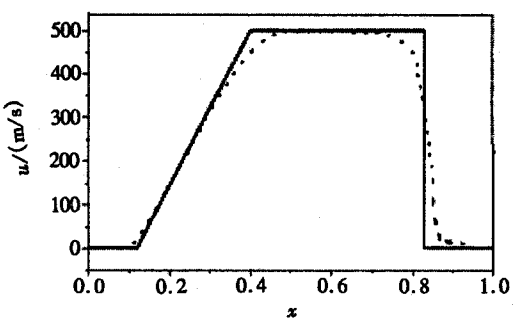
Fig.2 Comparsion between exact and numerical solutions with 100 cells using HLL scheme as the solver



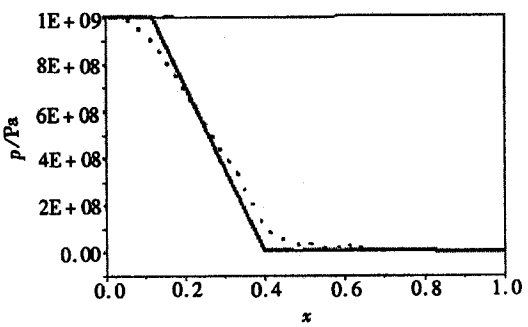
(a) Gas volume fraction α_g



(b) Comparison of density ρ



(c) Comparison of velocity u

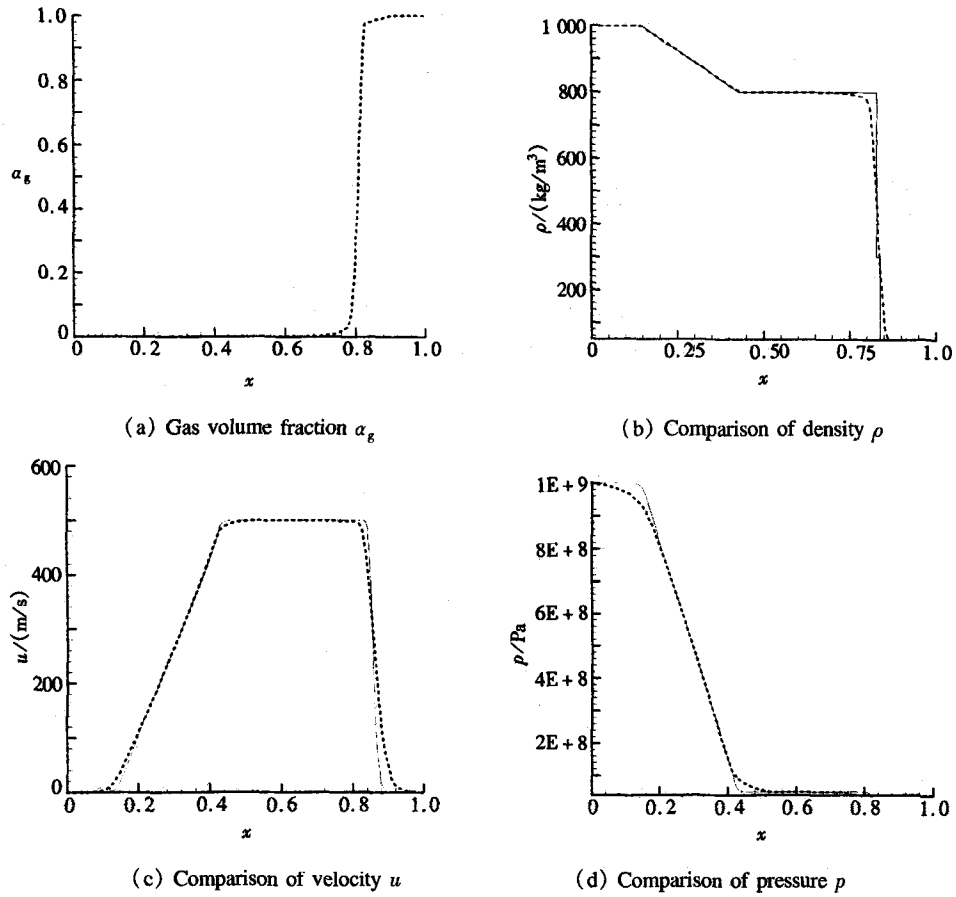


(d) Comparison of pressure p

Fig.3 Comparison between exact and numerical solutions with 100 cells using HLLC scheme as the solver

Figures 1, 2 and 3 are using HLLC scheme us the solver the numerical results with 100 cells using Lax-Friedrichs scheme, HLL scheme and HLLC scheme as the solver respectively. Figs.4 and 5 are the numerical results with 1 000 cells using HLL scheme and HLLC scheme as the solver

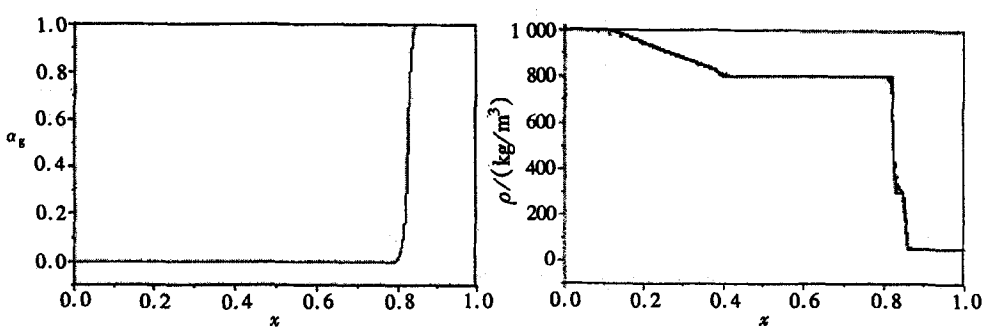
respectively. From the result, we can see all the schemes are stable and by comparison, the HLLC scheme has a higher resolution for shock waves and more stable than HLL scheme at discontinuities. And for contact discontinuities, the Lax-Friedrichs is smearing and HLLC scheme is sharp.



(a) Gas volume fraction α_g (b) Comparison of density ρ

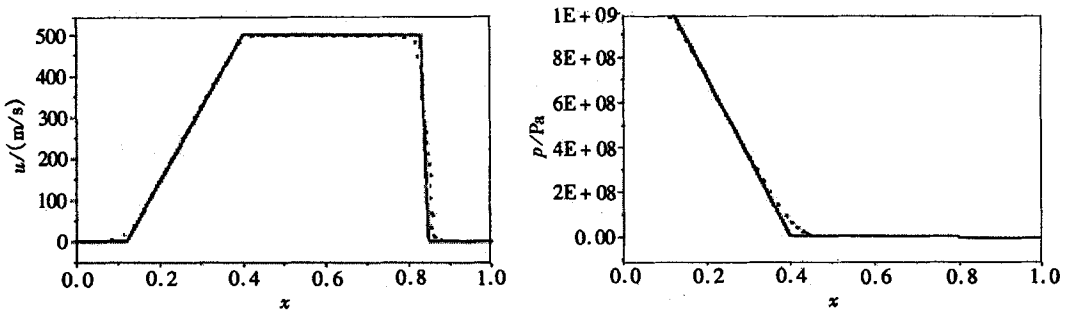
(c) Comparison of velocity u (d) Comparison of pressure p

Fig.4 Comparison between exact and numerical solution with 100 cells using HLL scheme as the solver



(a) Gas volume frection α_g (b) Comparison of density ρ

Fig.5 Comparison between exact and numerical solutions with 1 000 cells using HLLC scheme as the solver



(c) Comparison of velocity u (d) Comparison of pressure p

Fig.5 Comparison between exact and numerical solutions with 1 000 cells using HLLC scheme as the solver

4 Conclusion

Based on the work of Saurel[4], a new scheme was presented and numerical simulation was done. From the result we can see that the scheme is both stable and accurate and meanwhile has a higher resolution for shock waves. But the mathematical model used did not consider the mass and heat transfer between phases and from the computation procedure we can see that the scheme is expensive and most of the computation time is spent on the iteration for gas volume fraction. So it is necessary to pay more effort to simplify the pressure relaxation procedure and take into account the mass and heat transfer between phases.

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