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SYMPLECTIC SOLUTION SYSTEM FOR REISSNER PLATE BENDING *

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Abstract: *Based on the Hellinger-Reissner variatonal principle for Reissner plate bending and introducing dual variables, Hamiltonian dual equations for Reissner plate bending were presented. Therefore Hamiltonian solution system can also be applied to Reissner plate bending problem, and the transformation from Euclidian space to symplectic space and from Lagrangian system to Hamiltonian system was realized. So in the symplectic space which consists of the original variables and their dual variables, the problem can be solved via effective mathematical physics methods such as the method of separation of variables and eigenfunction-vector expansion. All the eigensolutions and Jordan canonical form eigensolutions for zero eigenvalue of the Hamiltonian operator matrix are solved in detail, and their physical meanings are showed clearly. The adjoint symplectic orthonormal relation of the eigenfunction vectors for zero eigenvalue are formed. It is showed that the all eigensolutions for zero eigenvalue are basic solutions of the Saint-Venant problem and they form a perfect symplectic subspace for zero eigenvalue. And the eigensolutions for nonzero eigenvalue are covered by the Saint-Venant theorem. The symplectic solution method is not the same as the classical semi-inverse method and breaks through the limit of the traditional semi-inverse solution. The symplectic solution method will have vast application.*

Key words: Reissner plate; Hamiltonian system; symplectic geometry; separation of variable

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Introduction

The problem of middle-thickness plate is one of important parts in theory of plate and shell, and the analytical solving process is always a difficult problem. The traditional method of elasticity is mainly the semi-inverse method, which can only be used to solve some special

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problems. The traditional semi-inverse method belongs to the Lagrangian system of one kind of variable and lacks rational analysis so that the solving range is limited very largely. The Ref. [1] establishes a new systematic methodology for theory of elasticity and acquires some achievements[1~3]. And the systematic methodology can be applied to many branches of applied mechanics[4]. The Refs. [5 ~ 7] set up the analogue theory between plane problem and Kirchhoff plate bending, and present another series of basic equations and solving method for the classical theory of Kirchhoff plate bending. Thereby it breaks through the limitations of the classical solution for Kirchhoff plate bending, extends the range of the analytical solutions, and establishes a new way of analyzing and solving plate bending problem. Based on the above, the Reissner plate bending problem is discussed in this paper. First, based on the Hellinger-Reissner variational principle for Reissner plate bending and introducing dual variables of the original variables, the state variables in the symplectic space are built, and Hamiltonian dual equations for Reissner plate bending are presented, therefore Hamiltonian solution system can be applied to Reissner plate bending problem and Reissner plate bending can be solved in the symplectic space via the method of separation of variables and eigenfunction vectors expansion. Secondly by rational analyzing all the eigenvectors and their Jordan canonical form eigenvectors for zero eigenvalue of the Hamiltonian operator matrix can be obtained directly, which have special physical meanings and are the basic solutions of Saint-Venant problem.

1 Variational Principle of Reissner Plate Bending

The plate discussed in this paper is in a rectangular domain $-a < x < a$, $0 < y < l$, and $a \ll l$, where the positive directions of the internal forces of Reissner plate bending is illustrated as in Fig. 1; and the corresponding generalized displacements include the deflection w and the slopes ψ_x and ψ_y.

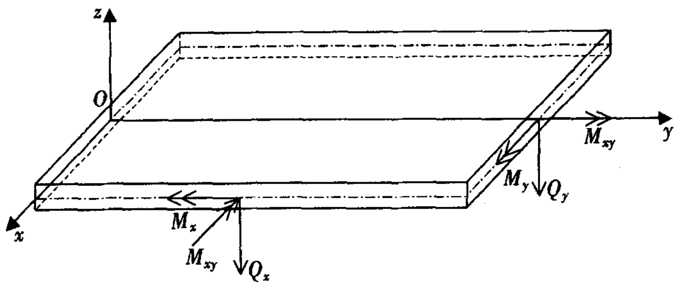

Fig. 1 Internal force positive sign for Reissner plate bending

The control equations of Reissner plate bending can be derived from the Hellinger-Reissner variational principle

$$\delta \Pi_2 = 0, \tag{1}$$

where

$$\Pi_2 = \int_0^l \int_{-a}^{a} \Big[M_x \frac{\partial \psi_x}{\partial x} + M_y \frac{\partial \psi_y}{\partial y} - M_{xy}\Big(\frac{\partial \psi_x}{\partial y} + \frac{\partial \psi_y}{\partial x}\Big) - Q_x\Big(\frac{\partial w}{\partial x} - \psi_x\Big) - Q_y\Big(\frac{\partial w}{\partial y} - \psi_y\Big) - U - q_w w - m_x \psi_x - m_y \psi_y \Big] \mathrm{d}x\mathrm{d}y, \tag{2}$$

and the complementary energy function is defined as

$$U = \frac{1}{2D(1-\nu^2)}[M_x^2 + M_y^2 - 2\nu M_x M_y + 2(1+\nu)M_{xy}^2] + \frac{1}{2C}(Q_x^2 + Q_y^2). \quad (3)$$

Coordinate y here is employed to simulate the time variable in the Hamiltonian system, and a symbol '·' in the following derivation will be used denoting the differential with respect to y, i.e. $(\dot{\ }) = \partial/\partial y$. By making stationary for Eq.(1) with respect to M_x and Q_x, M_x and Q_x can be expressed as follows:

$$M_x = D(1-\nu^2)\frac{\partial \psi_x}{\partial x} + \nu M_y, \quad Q_x = C\left(\psi_x - \frac{\partial w}{\partial x}\right). \quad (4)$$

Substituting Eqs.(4) into Eq.(1), the Hamiltonian mixed energy variational principle can be given as

$$\delta\left\{\int_0^l\int_{-a}^a [\boldsymbol{p}^{\mathrm{T}}\dot{\boldsymbol{q}} - H - m_x q_1 - m_y q_2 - q_w q_3]\,\mathrm{d}x\mathrm{d}y\right\} = 0, \quad (5)$$

where Hamiltonian density function is defined as

$$H = \frac{1}{2D}p_2^2 + \frac{1}{D(1-\nu)}p_1^2 + \frac{1}{2C}p_3^2 - \frac{D(1-\nu^2)}{2}\left(\frac{\partial q_1}{\partial x}\right)^2 -$$
$$\frac{C}{2}\left(\frac{\partial q_3}{\partial x} - q_1\right)^2 - \nu p_2\frac{\partial q_1}{\partial x} - p_1\frac{\partial q_2}{\partial x} + p_3 q_2, \quad (6)$$

and for convenience, let

$$\boldsymbol{q} = \{q_1, q_2, q_3\}^{\mathrm{T}} = \{\psi_x, \psi_y, w\}^{\mathrm{T}}, \ \boldsymbol{p} = \{p_1, p_2, p_3\}^{\mathrm{T}} = \{-M_{xy}, M_y, -Q_y\}^{\mathrm{T}}. \quad (7)$$

2 Hamiltonian Dual Equation

Now the state vector $\boldsymbol{v}$ which consists of the original variables $\boldsymbol{q}$ and its dual variables $\boldsymbol{p}$ are introduced, $\boldsymbol{v} = \{\boldsymbol{q}^{\mathrm{T}}, \boldsymbol{p}^{\mathrm{T}}\}^{\mathrm{T}}$, it forms a symplectic geometry space according to following symplectic inner product:

$$\langle \boldsymbol{v}^{(1)}, \boldsymbol{v}^{(2)}\rangle \overset{\mathrm{def}}{=} \int_{-a}^a (\boldsymbol{v}^{(1)})^{\mathrm{T}}\boldsymbol{J}\boldsymbol{v}^{(2)}\mathrm{d}x =$$
$$\int_{-a}^a (q_1^{(1)}p_1^{(2)} + q_2^{(1)}p_2^{(2)} + q_3^{(1)}p_3^{(2)} - p_1^{(1)}q_1^{(2)} - p_2^{(1)}q_2^{(2)} - p_3^{(1)}q_3^{(2)})\mathrm{d}x, \quad (8)$$

where the operator matrix $\boldsymbol{J}$ is the unit symplectic matrix

$$\boldsymbol{J} = \begin{bmatrix} \boldsymbol{0} & \boldsymbol{I}_3 \\ -\boldsymbol{I}_3 & \boldsymbol{0} \end{bmatrix}, \quad \boldsymbol{I}_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}. \quad (9)$$

The stationary requirement of Eq.(5) can yield a group of dual equations as follows:

$$\dot{\boldsymbol{v}} = \boldsymbol{H}\boldsymbol{v} + \boldsymbol{h}_p, \quad (10)$$

where the Hamiltonian operator matrix $\boldsymbol{H}$ is defined as

$$\boldsymbol{H} = \begin{bmatrix} 0 & -\partial/\partial x & 0 & 2/D(1-\nu) & 0 & 0 \\ -\nu\partial/\partial x & 0 & 0 & 0 & 1/D & 0 \\ 0 & 1 & 0 & 0 & 0 & 1/C \\ C - D(1-\nu^2)\partial^2/\partial x^2 & 0 & -C\partial/\partial x & 0 & -\nu\partial/\partial x & 0 \\ 0 & 0 & 0 & -\partial/\partial x & 0 & -1 \\ C\partial/\partial x & 0 & -C\partial^2/\partial x^2 & 0 & 0 & 0 \end{bmatrix}, \quad (11)$$

the nonhomogeneous part corresponding to body forces is given as

$$\boldsymbol{h}_p = \{0 \quad 0 \quad 0 \quad -m_x \quad -m_y \quad -q_w\}^{\mathrm{T}}. \tag{12}$$

Solving the nonhomogeneous linear partial differential Eq.(10), we should first solve the corresponding homogeneous equation

$$\dot{\boldsymbol{v}} = \boldsymbol{H}\boldsymbol{v}, \tag{13}$$

where two sides are assumed as satisfying free boundary conditions

$$D(1-\nu^2)\frac{\partial q_1}{\partial x} + \nu p_2 = 0, \quad p_1 = 0, \quad q_1 - \frac{\partial q_3}{\partial x} = 0 \qquad \text{when } x = \pm a. \tag{14}$$

Integrating by parts, we give as

$$\begin{aligned}\langle \boldsymbol{v}^{(1)}, \boldsymbol{H}\boldsymbol{v}^{(2)}\rangle &= \langle \boldsymbol{v}^{(2)}, \boldsymbol{H}\boldsymbol{v}^{(1)}\rangle + \Big\{p_1^{(1)}q_2^{(2)} - p_1^{(2)}q_2^{(1)} - \\ &q_1^{(1)}\Big[D(1-\nu^2)\frac{\partial q_1^{(2)}}{\partial x} + \nu p_2^{(2)}\Big] - Cq_3^{(2)}\Big[q_1^{(1)} - \frac{\partial q_3^{(1)}}{\partial x}\Big] + \\ &q_1^{(2)}\Big[D(1-\nu^2)\frac{\partial q_1^{(1)}}{\partial x} + \nu p_2^{(1)}\Big] + Cq_3^{(1)}\Big[q_1^{(2)} - \frac{\partial q_3^{(2)}}{\partial x}\Big]\Big\}\Big|_{-a}^{+a},\end{aligned} \tag{15}$$

that is, if $\boldsymbol{v}^{(1)}$ and $\boldsymbol{v}^{(2)}$ satisfy free boundary conditions (14) on two sides, the following identical equation can be given as

$$\langle \boldsymbol{v}^{(1)}, \boldsymbol{H}\boldsymbol{v}^{(2)}\rangle \equiv \langle \boldsymbol{v}^{(2)}, \boldsymbol{H}\boldsymbol{v}^{(1)}\rangle. \tag{16}$$

So $\boldsymbol{H}$ is Hamilton operator matrix in the symplectic geometrical space. It is the same to any other boundary conditions such as fixed and simply supported boundary conditions.

3 Symplectic Eigen Problem

To the homogeneous Eq.(13) the usual method of separation of variables can be used to solve it. Let

$$\boldsymbol{v}(x,y) = \xi(y)\boldsymbol{\Psi}(x), \tag{17}$$

and substituting the above into Eq.(13), we can get

$$\xi(y) = \exp(\mu y) \tag{18}$$

and eigen-equation

$$\boldsymbol{H}\boldsymbol{\Psi}(x) = \mu\boldsymbol{\Psi}(x). \tag{19}$$

Where μ is the eigenvalue to be determined; $\boldsymbol{\Psi}(x)$ is the eigenfunction vector and should satisfy the corresponding homogeneous boundary conditions on two sides, namely Eqs.(14).

The above proved that $\boldsymbol{H}$ is Hamiltonian operator matrix in the symplectic geometrical space, thus the eigen problem has special characteristics[1,2]:

1) If μ is an eigenvalue of Hamiltonian operator matrix $\boldsymbol{H}$, then $-\mu$ must also be its eigenvalue.

2) The eigenfunction vectors have the adjoint symplectic orthonormality relation.

Let $\boldsymbol{\Psi}_i$ and $\boldsymbol{\Psi}_j$ be the eigenfunction vectors corresponding to eigenvalues μ_i and μ_j respectively, when $\mu_i + \mu_j \neq 0$, $\boldsymbol{\Psi}_i$ and $\boldsymbol{\Psi}_j$ must be the symplectic orthogonality, i.e.

$$\langle \boldsymbol{\Psi}_i, \boldsymbol{\Psi}_j\rangle = \int_{-a}^{a} \boldsymbol{\Psi}_i^{\mathrm{T}}\boldsymbol{J}\boldsymbol{\Psi}_j \mathrm{d}x = 0. \tag{20}$$

The symplectic adjoint eigenfunction vectors with $\boldsymbol{\Psi}_i$ must be the eigenfunction vector (or Jordan canonical eigenfunction vector) corresponding to eigenvalue $-\mu_i$.

Based on the adjoint symplectic orthonormality relationship, any state vector $\boldsymbol{v}$ can be expanded by the eigenfunction vectors

$$\boldsymbol{v}(x,y) = \sum_{i=1}^{\infty}[a_i \exp(\mu_i y)\boldsymbol{\Psi}_i + b_i \exp(-\mu_i y)\boldsymbol{\Psi}_{-i}], \tag{21}$$

where a_i and b_i are the coefficients to be determined, $\boldsymbol{\Psi}_i$ and $\boldsymbol{\Psi}_{-i}$ $(i = 1,2,\cdots)$ are eigenfunction vector. Of course, if finite item is selected, its approximate solution can be obtained. Applying the adjoint symplectic orthonormality relationship of the eigenfunction vectors, the coefficients and the solution of Eq.(13) are obtained.

4 The Eigensolutions for Zero Eigenvalue

Eigenvalue zero is a very special eigenvalue in Hamiltonian eigen problems, this eigensolution has also the special important meanings in elasticity. To the rectangular Reissner plate bending problem, because free boundary condition of two sides is selected, eigenvalue zero must be multiple eigenvalue.

Now, we seek these eigenfunction vector for eigenvalue zero, namely solve the differential equation

$$\boldsymbol{H\Psi}(x) = \boldsymbol{0}. \tag{22}$$

Solving Eq.(22) with boundary (14), the basic eigensolutions corresponding to eigenvalue zero can be given as

$$\boldsymbol{v}_0^{(1)} = \boldsymbol{\Psi}_0^{(1)} = \{0 \quad 0 \quad 1 \quad 0 \quad 0 \quad 0\}^{\mathrm{T}} \tag{23}$$

and

$$\boldsymbol{v}_0^{(2)} = \boldsymbol{\Psi}_0^{(2)} = \{1 \quad 0 \quad x \quad 0 \quad 0 \quad 0\}^{\mathrm{T}}. \tag{24}$$

$\boldsymbol{\Psi}_0^{(1)}$ and $\boldsymbol{\Psi}_0^{(2)}$ are the basic eigen vectors, so they are just the solutions of the original (13), $\boldsymbol{v}_0^{(1)}$ and $\boldsymbol{v}_0^{(2)}$. Obviously the physical meaning of $\boldsymbol{v}_0^{(1)}$ and $\boldsymbol{v}_0^{(2)}$ are rigid body translation and rigid body rotation in the plane xOz respectively. Because eigenvalue zero is multiple eigenroot, and $\boldsymbol{\Psi}_0^{(1)}$ is the symplectic orthonormal with $\boldsymbol{\Psi}_0^{(2)}$, namely

$$\langle \boldsymbol{\Psi}_0^{(1)}, \boldsymbol{\Psi}_0^{(2)} \rangle = \int_{-a}^{a} (\boldsymbol{\Psi}_0^{(1)})^{\mathrm{T}} \boldsymbol{J} \boldsymbol{\Psi}_0^{(2)} \mathrm{d}x = 0, \tag{25}$$

Jordan canonical eigenfunction vector must exit, and $\boldsymbol{\Psi}_0^{(1)}$ and $\boldsymbol{\Psi}_0^{(2)}$ are both the heads of two Jordan chains. According to mathematics physics methods. Jordan canonical eigenfunction vectors for eigenvalue zero satisfy

$$\boldsymbol{H\Psi}_k^{(j)} = \boldsymbol{\Psi}_{k-1}^{(j)}, \tag{26}$$

where the subscript $k(=1,2,3)$ denotes that it is the k- th order Jordan canonical eigensolution, the superscript $j(=1,2)$ denotes that it is in the j- th Jordan canonical chain. Of course, Jordan canonical eigenfunction vectors satisfy not only Eq.(26), but also homogeneous boundary conditions (14).

By solving, the first-order Jordan canonical eigensolutions can be solved as

$$\boldsymbol{\Psi}_1^{(1)} = \{0 \quad 1 \quad 0 \quad 0 \quad 0 \quad 0\}^{\mathrm{T}} \tag{27}$$

and

$$\boldsymbol{\Psi}_1^{(2)} = \left\{0,\ x - \frac{2\mathrm{sh}(mx)}{m\mathrm{ch}(ma)},\ 0,\ \frac{2C}{m^2}\left[1 - \frac{\mathrm{ch}(mx)}{\mathrm{ch}(ma)}\right],\ 0,\ \frac{2C\mathrm{sh}(mx)}{m\mathrm{ch}(ma)}\right\}^{\mathrm{T}}, \tag{28}$$

where

$$m = \sqrt{2C/D(1-\nu)}, \tag{29}$$

the two Jordan canonical eigensolutions are not the solutions of the original (13), but they can constitute the solutions of the original (13) respectively as follows:

$$\boldsymbol{v}_1^{(1)} = \boldsymbol{\Psi}_1^{(1)} + y\boldsymbol{\Psi}_0^{(1)} = \{0,\ 1,\ y,\ 0,\ 0,\ 0\}^{\mathrm{T}} \tag{30}$$

and

$$\boldsymbol{v}_1^{(2)} = \boldsymbol{\Psi}_1^{(2)} + y\boldsymbol{\Psi}_0^{(2)} = \left\{y,\ x - \frac{2\mathrm{sh}(mx)}{m\mathrm{ch}(ma)},\ xy,\ \frac{2C}{m^2}\left[1 - \frac{\mathrm{ch}(mx)}{\mathrm{ch}(ma)}\right],\ 0,\ \frac{2C\mathrm{sh}(mx)}{m\mathrm{ch}(ma)}\right\}^{\mathrm{T}}. \tag{31}$$

Obviously the physical meaning of $\boldsymbol{v}_1^{(1)}$ is rigid body rotation in the plane yOz, and the physical meaning of $\boldsymbol{v}_1^{(2)}$ is pure torsion. Because $\boldsymbol{\Psi}_1^{(2)}$ and $\boldsymbol{\Psi}_0^{(1)}$ have the symplectic orthonormality relation, simultaneously $\boldsymbol{\Psi}_1^{(2)}$ and $\boldsymbol{\Psi}_0^{(2)}$ have the symplectic adjoint relation

$$\langle \boldsymbol{\Psi}_0^{(2)}, \boldsymbol{\Psi}_1^{(2)} \rangle = \int_{-a}^{a} (\boldsymbol{\Psi}_0^{(2)})^{\mathrm{T}} \boldsymbol{J} \boldsymbol{\Psi}_1^{(2)} \mathrm{d}x = \frac{8C}{m^3}[am - \mathrm{th}(ma)] \neq 0. \tag{32}$$

So the second Jordan chain only includes two eigensolutions $\boldsymbol{\Psi}_0^{(2)}$ and $\boldsymbol{\Psi}_1^{(2)}$. On the other hand, because $\boldsymbol{\Psi}_1^{(1)}$ is symplectic orthonormality with $\boldsymbol{\Psi}_0^{(1)}$ and $\boldsymbol{\Psi}_0^{(2)}$, there must be the second order Jordan canonical eigensolution in the first Jordan chain. By solving, it can be given as

$$\boldsymbol{\Psi}_2^{(1)} = \left\{-\nu x,\ 0,\ -\frac{1}{2}\nu x^2 + d,\ 0,\ D(1-\nu^2),\ 0\right\}^{\mathrm{T}}, \tag{33}$$

and the corresponding solutions of the original (13) are

$$\boldsymbol{v}_2^{(1)} = \boldsymbol{\Psi}_2^{(1)} + y\boldsymbol{\Psi}_1^{(1)} + \frac{y^2}{2}\boldsymbol{\Psi}_1^{(0)} = \left\{-\nu x,\ y,\ \frac{1}{2}(y^2 - \nu x^2) + d,\ 0,\ D(1-\nu^2),\ 0\right\}^{\mathrm{T}}. \tag{34}$$

Obviously the physical meaning of $\boldsymbol{v}_2^{(1)}$ is pure bending solution in the plane yOz. Because $\boldsymbol{\Psi}_2^{(1)}$ is symplectic orthonormality with $\boldsymbol{\Psi}_0^{(1)}$ and $\boldsymbol{\Psi}_0^{(2)}$, so there must be the third order Jordan canonical eigensolution in the first Jordan chain. By solving, it can be given as

$$\boldsymbol{\Psi}_3^{(1)} = \left\{0,\ \frac{2\nu a\mathrm{ch}(mx)}{m\mathrm{sh}(ma)} - \frac{1}{2}\nu x^2 + \frac{2}{m^2} + d,\ 0,\ \frac{2C\nu}{m^2}\left[\frac{a\mathrm{sh}(mx)}{\mathrm{sh}(ma)} - x\right],\ 0,\ -D(1-\nu)\left[1 + \frac{\nu ma\mathrm{ch}(mx)}{m\mathrm{sh}(ma)}\right]\right\}^{\mathrm{T}}, \tag{35}$$

and the corresponding solution of the original (13) is

$$\boldsymbol{v}_3^{(1)} = \boldsymbol{\Psi}_3^{(1)} + y\boldsymbol{\Psi}_2^{(1)} + \frac{y^2}{2}\boldsymbol{\Psi}_1^{(1)} + \frac{y^3}{6}\boldsymbol{\Psi}_0^{(1)} = \left\{\begin{matrix} -\nu xy \\ \frac{2\nu a\mathrm{ch}(mx)}{m\mathrm{sh}(ma)} - \frac{1}{2}\nu x^2 + \frac{2}{m^2} + \frac{1}{2}y^2 + d \\ -\frac{1}{2}\nu x^2 y + \frac{1}{6}y^3 + \mathrm{d}y \\ D\nu(1-\nu)\left[\frac{a\mathrm{sh}(mx)}{\mathrm{sh}(ma)} - x\right] \\ D(1-\nu^2)y \\ -D(1-\nu)\left[1 + \frac{\nu ma\mathrm{ch}(mx)}{\mathrm{sh}(ma)}\right] \end{matrix}\right\}. \tag{36}$$

The physical meaning of $\boldsymbol{v}_3^{(1)}$ is the constant shear stress bending solution. Because $\boldsymbol{\Psi}_3^{(1)}$ and $\boldsymbol{\Psi}_0^{(2)}$ have the symplectic orthonormality relation, simultaneously $\boldsymbol{\Psi}_3^{(1)}$ and $\boldsymbol{\Psi}_0^{(1)}$ have the symplectic adjoint relation, namely

$$\langle \boldsymbol{\Psi}_3^{(1)}, \boldsymbol{\Psi}_0^{(1)} \rangle = \int_{-a}^{a} (\boldsymbol{\Psi}_3^{(1)})^{\mathrm{T}} \boldsymbol{J} \boldsymbol{\Psi}_0^{(1)} \mathrm{d}x = 2Da(1 - \nu^2) \neq 0, \tag{37}$$

so the first Jordan chain also ends. Thus we obtain all the eigensolutions of eigenvalue zero for Reissner plate, every eigensolution has its special physical meaning. Obviously the eigensolutions $\boldsymbol{v}_0^{(1)}, \boldsymbol{v}_1^{(1)}, \boldsymbol{v}_2^{(1)}$ and $\boldsymbol{v}_3^{(1)}$ in the first chain are the symmetric deformation about axis y, and similarly the eigensolutions $\boldsymbol{v}_0^{(2)}$ and $\boldsymbol{v}_1^{(2)}$ in the second chain are the antisymmetric deformation about axis y.

The adjoint symplectic orthonormality relation of the six eigenfunction vectors are shown in Table 1, where 0 represents that they are certainly orthonormal, ※ represents the symplectic adjoint relation, and d represents that the two eigenfunction vectors can be made symplectic orthonormal by choosing appropriate constant d as

$$d = \frac{m^2 a^2 \nu(1 + 3\nu) - 6(1 + 2\nu) - 6ma\nu^2 \mathrm{th}(ma)}{6m^2(1 + \nu)}, \tag{38}$$

So far, we have achieved the adjoint symplectic orthonormality relation of the eigenfunction vector for eigenvalue zero, i.e. they constituted one adjoint symplectic orthonormality vector group.

The six eigensolutions for eigenvalue zero are all basic solutions of the Saint-Venant problem, which can form a perfect symplectic subspace for eigenvalue zero and take effects in the total domain. Eq.(13) have infinite eigensolutions for nonzero eigenvalue, which are covered by the Saint-Venant theorem, damped and take effects in the local domain. With the length limited, it isn't discussed here.

Table 1 Adjoint symplectic orthonormality relation among the six eigenfunction vectors for eigenvalue zero

	$\boldsymbol{\Psi}_0^{(1)}$	$\boldsymbol{\Psi}_1^{(1)}$	$\boldsymbol{\Psi}_2^{(1)}$	$\boldsymbol{\Psi}_3^{(1)}$	$\boldsymbol{\Psi}_0^{(2)}$	$\boldsymbol{\Psi}_1^{(2)}$
$\boldsymbol{\Psi}_0^{(1)}$	0	0	0	※	0	0
$\boldsymbol{\Psi}_1^{(1)}$		0	※	0	0	0
$\boldsymbol{\Psi}_2^{(1)}$			0	d	0	0
$\boldsymbol{\Psi}_3^{(1)}$				0	0	0
$\boldsymbol{\Psi}_0^{(2)}$					0	※
$\boldsymbol{\Psi}_1^{(2)}$						0

5 Concluding Remarks

This paper shows that Reissner plate bending problem can be applied in Hamiltonian system, and the corresponding symplectic solution system is presented. The symplectic solution system solve directly all basic solutions of the Saint-Venant solutions via entirely rational analysis, which constitute a perfect symplectic subspace. The significance is that it not only present a new methodology to solve the analytical solution for Reissner plate bending problem, but also introduces the unified methodology into the analysis of couple stress plane elasticity according the analogy between Reissner plate bending and couple stress plane elasticity[8], in this aspect there are many tasks to do.

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