# Deformation Measurement by Phase-shifting Digital Holography

by Y. Morimoto, T. Nomura, M. Fujigaki, S. Yoneyama and I. Takahashi

ABSTRACT—The out-of-plane displacement of a cantilever is measured by phase-shifting digital holography. From four phase-shifted holograms of a cantilever recorded by a CCD image sensor, a complex amplitude at each pixel of the CCD plane is obtained using the conventional phase-shifting method. The complex amplitude of a cantilever is reconstructed from the complex amplitude of the CCD plane using the Fresnel diffraction integral. The phase difference distribution on the cantilever before and after deformation, i.e., the out-of-plane displacement distribution, is calculated. In order to decrease the effect of speckle noise, a new method using divided holograms is proposed. The theory and experimental results are shown.

KEY WORDS—Phase-shifting digital holography, holographic interferometry, displacement measurement, out-of-plane displacement, speckle noise reduction

## Introduction

Holographic interferometry is useful to measure the small displacement of materials and structures because of its high sensitivity, full-field and non-contact characteristics.[1,2] One of the authors[3] has previously analyzed stress wave propagation in a laminated composite material using holographic interferometry. In the displacement measurement by holographic interferometry, two holograms, before and after the deformation of a specimen, are recorded on a film using the double exposure method.[1–3] Two object images, before and after deformation, are simultaneously reconstructed from the holograms on the film by illuminating the reference laser beam on the holograms. The two reconstructed images make a fringe pattern, which is the superposition of the complex amplitudes of the two reconstructed images, before and after the deformation, and represents the displacement distribution of the specimen. However, the development process of a film is troublesome and time-consuming. Therefore holographic interferometry is seldom used in industry. If the analysis without the film development process is performed, the range of applicability to displacement measurement on materials and structures would increase. Recently, digital holography has been developed.[4,5] In digital holography, a hologram is recorded on a CCD sensor instead of film and the reconstructed image can be obtained from the hologram using a computer. Since digital holography has the feature that digital information of both the amplitude and the phase of the reconstructed fringe pattern can be analyzed in a short time by a computer, it has been studied actively in various fields. Yamaguchi and co-workers[6–10], in particular, have developed phase-shifting digital holography. Phase-shifting digital holography can extract only the first-order diffraction image without the superposition of the zeroth and minus first-order diffraction images, by calculating the amplitude and phase values on the reconstructed object from the Fresnel diffraction integral of the amplitude and phase values of a hologram.

Electronic speckle pattern interferometry (ESPI) is considered as a type of holography. Hung et al.[11] analyzed displacement distributions using a type of digital holography without the Fresnel diffraction integral. Hung's ESPI method utilizes a lens for focusing the image of the object on a CCD image sensor. Since the image of the object is focused on the CCD image plane in a camera, the Fresnel diffraction integral of the hologram is not required. Although the method is simple and easy, it can be used to analyze only the area of the object focused on the CCD image sensor. However, since a hologram has essentially three-dimensional information, the information at any point of the object surface can be analyzed by calculating the Fresnel diffraction integral of the hologram.

A holographic image of a diffused object has essentially speckle noise. Where the brightness change at a speckle with low brightness amplitude is small, the phase distribution is scattered and the accuracy of the phase values is not so high. In a conventional analysis, a spatial filter is used to decrease the effect of the speckle. In such cases, although the distribution becomes smooth, the accuracy at each point on the object is poor and it is especially bad at the edge of the object.

In this paper, we propose the out-of plane deformation measurement method, using a combination of techniques: phase-shifting digital holography and double exposure holographic interferometry. The fringe pattern of double exposure holographic interferometry is obtained as the superposition of the complex amplitudes before and after the deformation of an object. In phase-shifting digital holography, it is obtained as the difference between the phases of the complex

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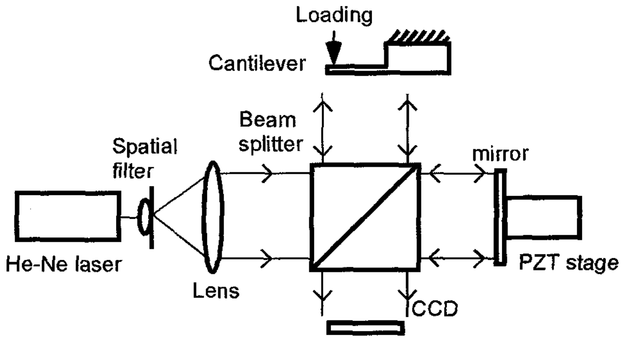



Fig. 1—Schematic diagram of optical setup for phase-shifting digital holography

amplitude of the reconstructed images before and after the deformation of the object. The phase difference distribution corresponds to the fringe pattern obtained by conventional double exposure holographic interferometry. After all, the phase difference between the complex amplitudes of the reconstructed holographic object before and after deformation provides a displacement distribution with high resolution.

Furthermore, a method to make it possible to determine the displacement at any point on an object is proposed without any focusing lens for the hologram. The displacement distribution is calculated by the Fresnel diffraction integral of the holograms recorded by a CCD image sensor.

In order to decrease the effect of speckle noise, a new method is also proposed, using the phenomenon by which the intensity of a speckle is altered by changing the hologram size and the observation direction. The theory and an example applied to the deflection distribution of a cantilever are shown.

## Theoretical Background of Proposed Method

### *Principle of Phase-shifting Digital Holography*

The optical setup of the proposed method is shown schematically in Fig. 1. A collimated light beam from a laser is divided into an object wave and a reference wave by a beam splitter. The phase of the reference wave is shifted by a PZT stage. The complex amplitudes of the object wave and the reference wave are expressed, respectively, at the pixel coordinates $(x, y)$ on the CCD plane as follows:[6]

$$A_o(x, y) = a_o(x, y) \exp\{i\phi_o(x, y)\}, \quad (1)$$

$$A_r(x, y) = a_r(x, y) \exp[i\{\phi_r(x, y) + \alpha\}]. \quad (2)$$

Here, $A(x, y)$, $a(x, y)$, and $\phi(x, y)$ denote the complex amplitude, the amplitude and the phase, respectively. The subscripts $o$ and $r$ represent the object wave and the reference wave, respectively. $\alpha$ denotes the phase-shift value of the reference wave. The value $\alpha$ is set as 0, $\pi/2$, $\pi$ and $\pi/2$ in this phase-shifting digital holography. The phase-shifted four digital holograms are recorded on the CCD plane in a CCD camera without any focusing lens. The intensity of the recorded digital holograms on the CCD plane is expressed as

$$I_\alpha(x, y) = a_o(x, y) a_r(x, y) \cos\{\phi_o(x, y) - \phi_r(x, y) - \alpha\}, \quad (3)$$

where $I_\alpha(x, y)$ denotes the intensity of the $\alpha$-phase-shifted digital hologram. Using the phase-shifting method, the amplitude $a_0(x, y)$ and the phase $\phi_0(x, y)$ of the object wave at the CCD plane can be expressed as follows:

$$a_o(x, y) = \frac{1}{4}\sqrt{\{I_{3\pi/2}(x, y) - I_{\pi/2}(x, y)\}^2 + \{I_0(x, y) - I_\pi(x, y)\}^2}, \quad (4)$$

$$\tan\phi_o(x, y) = \frac{I_{3\pi/2}(x, y) - I_{\pi/2}(x, y)}{I_0(x, y) - I_\pi(x, y)}. \quad (5)$$

By calculating the Fresnel diffraction integral from the complex amplitude $g(x, y)$ on the CCD plane, the complex amplitude of the reconstructed image at the position $(X, Y)$ on the reconstructed object surface, being at the distance $R$ from the CCD plane, is expressed as

$$u(X, Y) = \exp\left[\frac{ik(X^2 + Y^2)}{2R}\right] F\left\{\exp\left[\frac{ik(x^2 + y^2)}{2R}\right] g(x, y)\right\}, \quad (6)$$

where $u(X, Y)$, $k$, and $F$ denote the complex amplitude on the reconstructed plane, the wavenumber, and the operator of Fourier transform, respectively. Equation (6) is called the Fresnel diffraction integral. By calculating the intensities of these complex amplitudes on the reconstructed object surface, a holographic reconstructed image is obtained.

### *Principle of Holographic Interferometry*

The concept of holographic interferometry is schematically explained using Fig. 2. The complex amplitudes of the object waves before and after deformation, respectively, are expressed as

$$A_o(X, Y) = a_o(X, Y) \exp\{i\phi_o(X, Y)\}, \quad (7)$$

$$A'_o(X, Y) = a'_o(X, Y) \exp\{i\phi'_o(X, Y)\}, \quad (8)$$

where the prime denotes after deformation. If the deformation is small, the amplitudes before and after deformation are unchanged but the phases before and after deformation change. In this form of holographic interferometry, the relationship between the amplitudes and the phases before and after deformation are expressed as

$$a'_o(X, Y) = a_o(X, Y) \quad (9)$$

$$\phi'_o(X, Y) = \phi_o(X, Y) + \Delta\phi(X, Y), \quad (10)$$

where $\Delta\phi(X, Y)$ is the phase difference before and after deformation. The out-of-plane displacement $w(X, Y)$ is given by

$$w(X, Y) = \frac{\lambda}{4\pi}\Delta\phi(X, Y), \quad (11)$$

where $\lambda$ is the wavelength of the He–Ne laser.

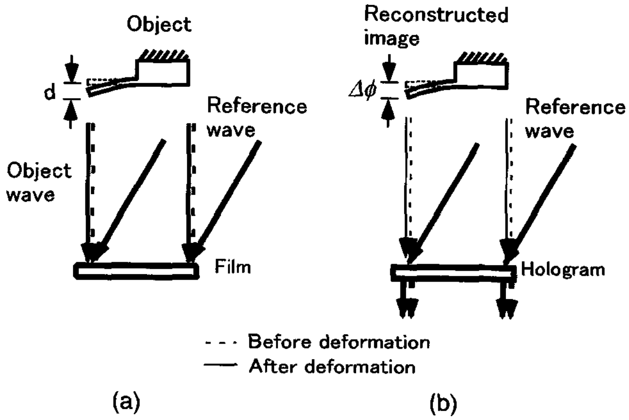



Fig. 2—Schematic explanation of holographic interferometry: (a) recording process; (b) reconstructing process

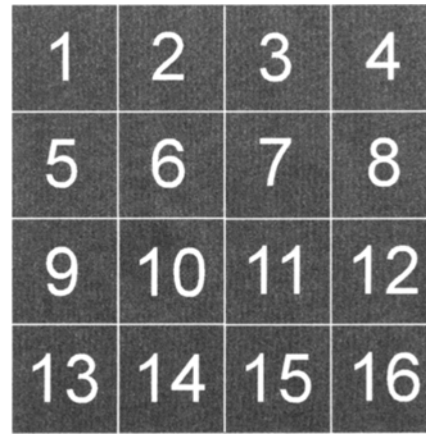



Fig. 3—Original digital hologram with 960 × 960 pixels divided into small holograms with 240 × 240 pixels and numbers applied to make a full-size hologram

### *Speckle Pattern Formation*

In holographic interferometry, a speckle pattern appears on a reconstructed image. The amplitude of the brightness change at each reconstructed point during phase-shifting depends on the brightness change of the speckle. When some random noise is included in the hologram, the analyzed phase value is not very reliable at the reconstructed point where the brightness change during phase-shifting is small. That is, the analyzed phase value with a larger intensity at a reconstructed point during phase-shifting is more reliable. Although the formation of a speckle pattern of a reconstructed image is complicated, the intensity of a speckle at a point of the reconstructed image is a function of the hologram position and the hologram size. Any subdivision of a hologram records the entire information of the recorded three-dimensional object. The speckle pattern obtained from a divided hologram is different from the speckle pattern obtained from any other divided hologram. On the other hand, the phase difference is the same, if the displacement is only out-of-plane along the direction normal to the CCD plane and the object is small compared with the distance $R$. However, some random noise is included in the phase difference. The phase difference at a reconstructed point obtained from the hologram that has larger intensity is more reliable, i.e., it has a higher signal-to-noise ratio.

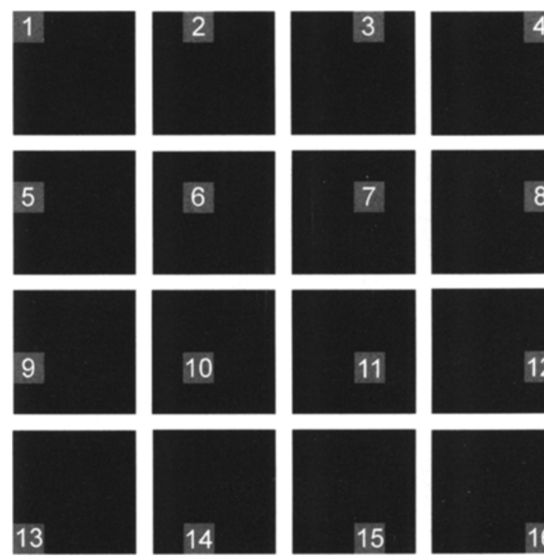
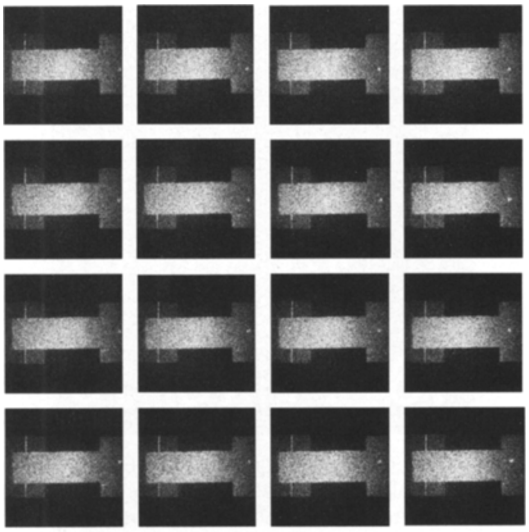



Fig. 4—Divided small holograms pasted at original positions on full-size holograms with null data

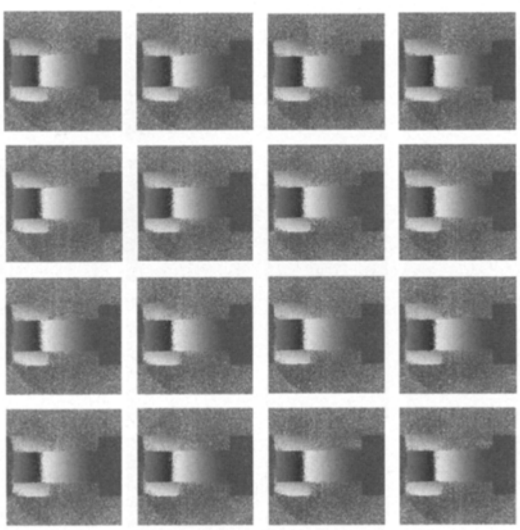
### *Extraction of Displacement Information*

The procedure for displacement determination is provided as follows. The example shown in this section will be used in the experiment shown in the next section. The original full size digital hologram with 960 × 960 pixels is shown in Fig. 3. The hologram is divided into16 small holograms with 240 × 240 pixels. Each divided hologram is numbered as shown in Fig. 3. The divided small holograms are pasted at the same original positions on full-size holograms with null data, respectively, as shown in Fig. 4. The reconstructed image is calculated from each divided full-size hologram as shown in Figs. 5 (a) and (b) by using eq (6).

Figure 6 shows the magnified image with 20 × 20 pixels near the center parts of some images in Fig. 5 (a). The intensity obtained from a divided full-size hologram at the same reconstructed point is different from the intensity at the same reconstructed point obtained from the other divided full-size holograms; that is, the speckles move. However, as shown in Fig. 7, the speckles do move minimally at the same reconstructed point obtained from the same divided full-size holograms before and after deformation. That is, whichever set of divided holograms before and after deformation is used, the phase differences before and after deformation at the same reconstructed point should be the same if there is no noise. If there is some noise, among the phase differences obtained from each set of divided full-size holograms, the phase difference with the maximum amplitude at a reconstructed point is the most reliable. Therefore, after calculating the average intensities obtained from before and after deformation for all the divided full-size holograms, the phase difference value with the maximum average intensity is adopted at each reconstructed point.

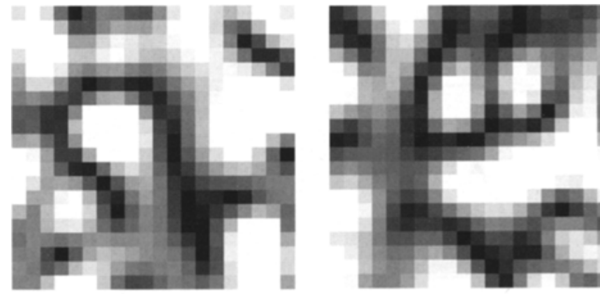


(a)

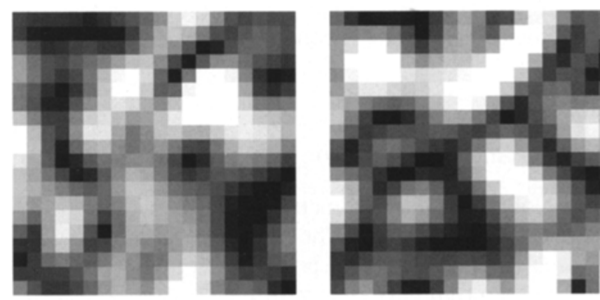


(b)

Fig. 5—Reconstructed images calculated from each divided full-size hologram: (a) reconstructed intensity distribution; (b) phase difference distribution



(a) (b)



(c) (d)

Fig. 6—Magnified images with 20 × 20 pixels near center parts of reconstructed images in Fig. 5(a): (a) from part 1; (b) from part 4; (c) from part 13; (d) from part 16

(a) (b)

Fig. 7—Magnified images (from part 1) with 20 × 20 pixels near center parts of reconstructed images before and after deformation: (a) before deformation; (b) after deformation

## Experimental Results

The experimental optical setup shown in Fig. 1 is used in this study. The light source is an He–Ne laser. The power is 30 mW and the wavelength is 632.8 nm. The pixel size of the CCD sensor is $4.65 \times 4.65\ \mu m^2$. A captured image is sampled by 1280(V) × 960(H) pixels and the image is digitized with 8 bits. The image of 960(V) × 960(H) pixels near the center of the recorded image is used for the analysis in this study.

A cantilever beam shown in Fig. 8 is analyzed. The cantilever is cut out of a thick stainless steel plate. The cantilever size is 10 mm wide, 30 mm long, and 1 mm thick. The loading point is 25 mm from the fixed end. The displacement at the loading point is given by a micrometer with a wedge. To improve the reflection from the specimen, lusterless white lacquer is sprayed on the surface of the cantilever. The distance $R$ from the CCD to the cantilever is 300 mm. The phase of the reference wave is shifted every $\pi/2$ using the mirror controlled by a PZT stage. Then, four phase-shifted digital holograms for one cycle of phase-shifting are recorded on the memory of a personal computer.

First, the analysis of a full-size hologram with 960 × 960 pixels is shown; then the analysis of the divided holograms is provided as mentioned above. The intensity distribution of the hologram obtained before deformation is shown in Fig. 3. The complex amplitude of the brightness change at each pixel

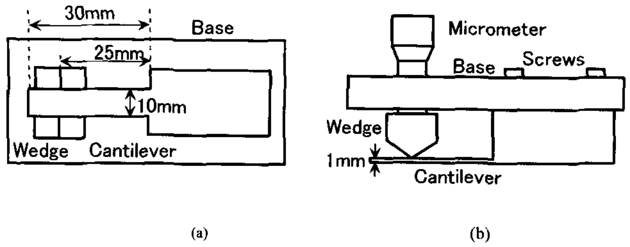



Fig. 8—Specimen (cantilever) and loading device (micrometer with wedge): (a) front view; (b) upper view

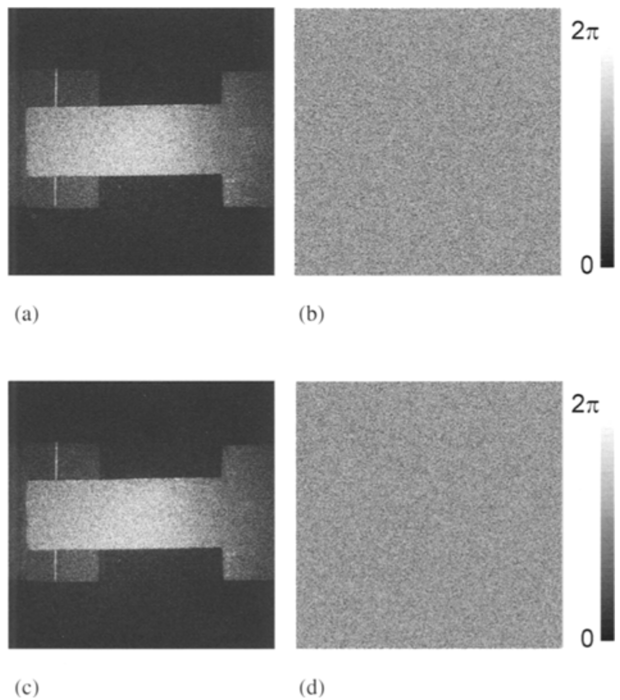



Fig. 9—Reconstructed images obtained from full-size holograms: (a) reconstructed image before deformation; (b) phase distribution before deformation; (c) reconstructed image after deformation; (d) phase distribution after deformation

on the hologram is calculated using the phase-shifting method expressed in eqs (4) and (5). The reconstructed complex amplitude of the object waves at a reconstructed object surface is calculated from the complex amplitudes of the holograms using the Fresnel diffraction integral expressed in eq (6). After the cantilever is deformed, the reconstructed complex amplitude of the object waves is obtained similarly. Figure 9 shows the reconstructed images and the phase distributions obtained from the holograms with 960 × 960 pixels before and after deformation. Figures 10(a) and (b) show the fringe pattern obtained from the real part of the sum of the complex amplitudes and the phase difference distribution, respectively, before and after deformation.

The phase difference distribution along the centerline A of the cantilever shown in Fig. 10(b) is plotted in Fig. 11(a). The displacement at each point on the reconstructed object is regarded as one-dimensional out-of plane deformation. The theoretical curve of the cantilever is shown in Fig. 11(b). The theoretical curve is obtained from a cantilever beam theory with boundary conditions at the fixed point and the loading

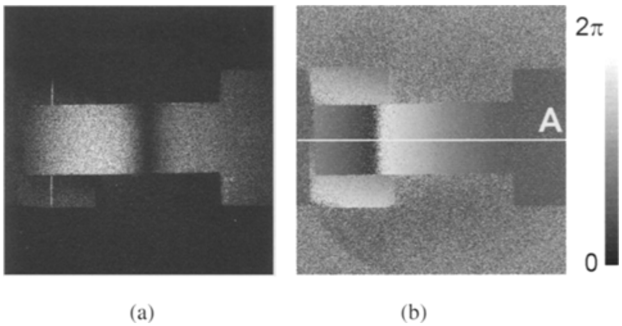



Fig. 10—Reconstructed fringe pattern and phase difference distribution obtained from full-size holograms shown in Figs. 9(b) and (d): (a) fringe pattern; (b) phase difference distribution

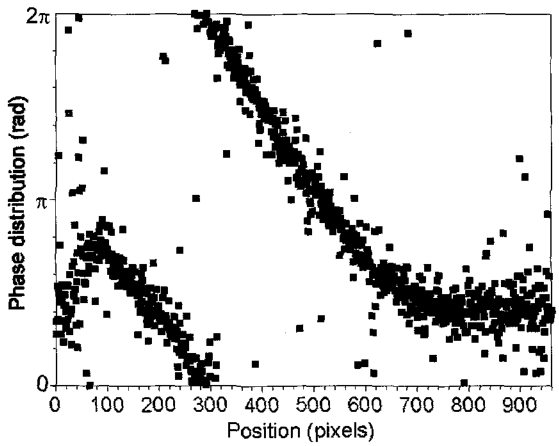
point. The displacement data at the points are obtained from the averages of 10 data values near the points, respectively.

The standard deviation of the displacement distribution near the center (between $X = 350$ and 600 pixels when the analyzed area is expressed by 960 × 960 pixels obtained from the phase difference between Figs. 11(a) and (b) is 23.48 nm. The phase difference values are scattered because the brightness values with small intensities have small signal-to-noise ratios.

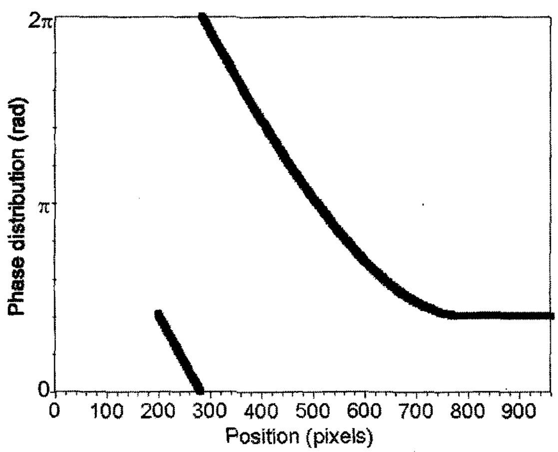
Let us show an example of the proposed method using the divided full-size holograms. The hologram shown in Fig. 3 is divided into 16 small holograms. The full-size divided holograms are made, as shown in Fig. 4, by adding null data to each corresponding divided small hologram. The intensity and the phase values on the surface points of the cantilever are calculated using the Fresnel diffraction integral expressed in eq (6). The distributions are shown in Figs. 5(a) and (b), respectively. At a pixel point of the reconstructed image, 16 values of the intensity and 16 values of the phase difference are calculated from 16 sets of the full-size divided holograms before and after deformation, respectively. The phase difference with the maximum average intensity among the 16 values is adopted at each pixel point. The resultant distribution of the phase difference is shown in Fig. 12. The phase difference distribution along the centerline B of the cantilever is shown in Fig. 13. The theoretical curve of the cantilever is obtained by the same method as shown in Fig. 11(b). The standard deviation of the displacement distribution near the center (between $X = 350$ and 600 pixels) obtained from the phase difference between Fig. 13, and the theoretical curve is 8.27 nm. The standard deviation obtained from the divided holograms decreases into 35% of that obtained from the full-size hologram. The effect of speckle becomes considerably smaller in spite of the use of only the data at one reconstructed point without any spatial filter. This proposed method is reliable and accurate.

## Conclusions

We have proposed a displacement measurement method using phase-shifting digital holography and the Fresnel diffraction integral of the hologram. The out-of-plane displacement of a cantilever was measured by the proposed method. In order to decrease the effect of speckle noise, we propose a new method using divided holograms. By dividing a hologram into 16 pieces and calculating the phase differences of all the divided holograms, the phase difference




(a)




(b)

Fig. 11—Phase difference distribution along centerline (480th line): (a) phase difference distribution obtained from full-size holograms; (b) theoretical curve of cantilever

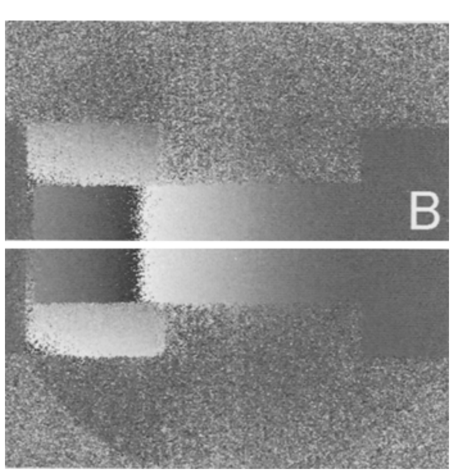



Fig. 12—Phase difference distribution obtained from phase value with maximum intensity at each point

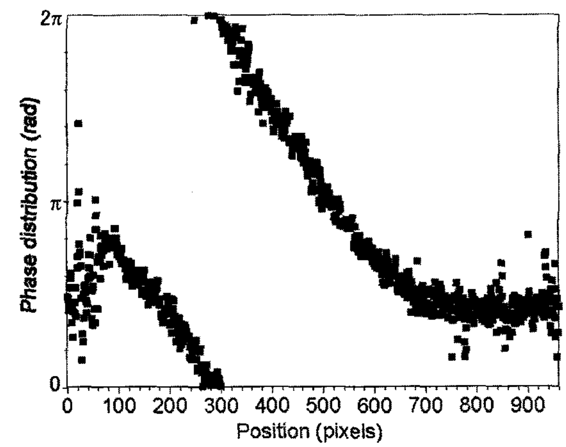



Fig. 13—Phase difference distribution along centerline B (480th line) of Fig. 12

with the maximum intensity among the results of 16 divided holograms provides a more reliable and more accurate result. Although ESPI and digital holography using a focusing lens can analyze only the focused points, this proposed method can analyze a displacement distribution at any point of an object. This method will be a powerful tool for the stress and strain analysis of actual structures.

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