The theorem admits an obvious generalization to any finite number of domains Ω_j. All the classes which arise in this case are again Faber invariant with respect to G.

For $\Omega_1 = G$, $\Omega_2 = \Delta$, $h_1(t) = h(t)^{\alpha}$, $h_2(t) = h(t)^{1-\alpha}$, $0 < \alpha < 1$, from Theorem 3 one obtains Shirokov's result [6].

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SPACES WITH "SMALL" ANNIHILATORS

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In this note one investigates the properties of subspaces G of $C(S)$, such that $G^{\perp}$ is "not a very large part" of the space $C(S)^*$. The fundamental result is: if $G^{\perp}$ is reflexive, then every operator from G^* into ℓ_2 is absolutely summable.

In this note we consider subspaces G of $C(S)$, whose annihilators $G^{\perp} = \{F \in C(S)^* : F(g) = 0, g \in G\}$ are "small" in some sense.

1. One of the remarkable results in the theory of p-absolutely summable operators is A. Grothendieck's theorem which states that every (linear and bounded) operator defined on the $\mathcal{L}_1$-space and taking values in ℓ_2 is 1-absolutely summable.† In [1], Lindenstrauss and Pelczyński have raised the question whether the converse theorem holds, i.e., whether the space X is necessarily a $\mathcal{L}_1$-space when the equality $\Pi_1(X, \ell_2) = B(X, \ell_2)$ holds.

The following theorem gives a negative answer to this question.

THEOREM 1. Let E be a reflexive subspace of the space $L_1(\mu)$. Then

$$\Pi_1(L_1(\mu)/E, \ell_2) = B(L_1(\mu)/E, \ell_2).$$

It is known that the quotient space $L_1(\mu)/E$ is not a $\mathcal{L}_1$-space when E is infinite-dimensional and reflexive.

†We adhere to the notations and to the terminology adopted in [1]. In particular, $\Pi_p(X, Y)$ is the collection of all p-absolutely summable operator from X into Y.

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We outline the proof of Theorem 1.

THEOREM 2. Let $G \subset C(S)$ and assume that the space $C(S)/G$ is reflexive. If X is such that $\Pi_2(Y,X) = \Pi_s(Y,X)$ for all Y and all s, $s \geq 2$, then $\Pi_2(G,X) = B(G,X)$.

In Theorem 2 one can take for X, e.g., the space $L_r(\lambda)$, $1 \leq r \leq 2$.

Theorem 1 follows from Theorem 2. Indeed, let π be the canonical mapping of $L_1(\mu)$ onto $L_1(\mu)/E$, $G = (L_1(\mu)/E)^* \subset L_\infty(\mu)$. From Theorem 2 it follows that every operator from G into ℓ_1 is 2-absolutely summable and thus it can be extended to an operator from $L_\infty(\mu)$ into ℓ_1. From here and from the fact that the space $L_1(\mu)/E$ is complemented in its second conjugate space, one obtains at once that every operator from c_0 into $L_1(\mu)/E$ can be "lifted" to an operator from c_0 into $L_1(\mu)$. But this means that for every unconditionally convergent series Σy_n in $L_1(\mu)/E$ there exists an unconditionally convergent series Σx_n in $L_1(\mu)$ such that $y_n = \pi(x_n)$. Now Theorem 1 can be easily derived from Grothendieck's theorem.

For the proof of Theorem 2 one requires two lemmas.

LEMMA 1. Let A, G, X be Banach spaces, $A \supset G$, $T \in B(G,X)$. Then there exist a space Z, an isometric embedding $j: X \to Z$, and an operator T_1 from A into Z such that the spaces A/G and Z/jX are isometric and $jT = T_1 i$; here i is the inclusion mapping of G into A.

Proof. We set $Z = (A \oplus X)_{\ell_1}/H$, where $H = \{(g, -Tg): g \in G\}$. T_1 and j are given by the formulas $T_1(a) = (a,0) + H$ and $j(x) = (0,x) + H$.

Let $2 < p \leq \infty$. We shall say that the space X possesses property (RM_p) (after Rosenthal and Maurey), if

$$\exists A \forall n \exists T_1 \in B(\ell_p^{(n)}, X) \quad \exists T_2 \in B(X, \ell_\infty^{(n)}):$$

$T_2 T_1$ is the identity embedding of $\ell_p^{(n)}$ in $\ell_\infty^{(n)}$,

$$\|T_2\| \cdot \|T_1\| \leq A.$$

LEMMA 2. If Y and X/Y do not possess property (RM_p), then X does not have property (RM_p).

For the proof of Theorem 2 one needs only the case $p = \infty$, and for this case the lemma is proved in [6].

Proof of Theorem 2. Let $T \in B(G,X)$. It is sufficient to prove that $T \in \Pi_q(G,X)$ for some $q > 2$. Applying Lemma 1 to the operator T (with $A = C(S)$), we obtain the space Z, the operator T_1, and the isometric embedding j. From the results of Rosenthal and Maurey [3, Chap. 8] it follows that neither X, nor $Z/jX = C(S)/G$, possesses property (RM_∞). By virtue of Lemma 2, the space Z does not have the property (RM_∞) either. Thus, $T_1 \in \Pi_q(C(S), Z)$ for some $q > 2$ ([3, Theorem 92] and [2, Proof of Proposition 2.1]). Since T is part of the operator T_1, we have $T \in \Pi_q(G,X)$).

Making use of the full extent of Lemma 2, one can obtain the following theorem.

THEOREM 3. If $C(S)/G$ does not have property (RM_p) and X does not have property (RM_r), then $\Pi_{max(p,r)}(G,X) = B(G,X)$.

2. THEOREM 4. Let $G_1 \subset C(S_1)$, $G_2 \subset C(S_2)$ such that both spaces $C(S_1)/G_1$ and $C(S_2)/G_2$ are reflexive. If $G_1^* \sim G_2^*$, then there exist spaces F_1 and F_2, $F_1 \subset C(S_1)^*$, $F_2 \subset C(S_2)^*$, such that: 1) the spaces $C(S_1)^*/F_1$ and $C(S_2)^*/F_2$ are finite-dimensional; 2) there exists an isomorphism φ, mapping F_1 and F_2, for which $\varphi(F_1 \cap G_1^\perp) = F_2 \cap G_2^\perp$.

COROLLARY. If E_1 and E_2 are reflexive subspaces in $L_1(\mu)$ and $L_1(\mu)/E_1 \sim L_1(\mu)/E_2$, then one of the subspaces E_1, E_2 is isomorphic to the product of the other one with a finite-dimensional space.

COROLLARY. There exists a continuum of pairwise nonisomorphic spaces X, not $\mathcal{L}_1$-spaces and such that $\Pi_1(X, \ell_2) = B(X, \ell_2)$.

Indeed, L_p can be embedded into L_1 if $p \in (1, 2]$.

3. The spaces G which occur in Theorem 2 possess a series of other properties which makes them similar to the spaces $C(S)$.

THEOREM 5. Assume that $C(S)/G$ is reflexive. Then: 1) G and all of its conjugates satisfy the Dunford–Pettis condition, while the spaces $G^*, G^{***}, \ldots, G^{(2n+1)}, \ldots$ are weakly sequentially complete; 2) the spaces $G, G^{**}, \ldots, G^{(2n)}, \ldots$ satisfy the Pelczyński condition.†

The second part of this theorem is obtained with the aid of Lemma 1, while the first part — with the aid of the following lemma which is easily derived from Rosenthal's theorem [4].

LEMMA 3. Let X be a Banach space, $Y \subset X$, and assume that Y does not have subspaces which are isomorphic to ℓ_1. Let φ be the canonical mapping of X onto X/Y, and let $\{x_n\}$ be a bounded sequence in X such that the sequence $\{\varphi(x_n)\}$ is weakly fundamental. Then $\{x_n\}$ contains a weakly fundamental subsequence.

4. It would be interesting to find out whether Theorem 2 remains valid if G is subjected to other conditions (e.g., if one requires that the space $G^\perp = (C(S)/G)^*$ be separable). I do not even know whether an analog of Theorem 2 holds for the classical space C_A (one can show that if the quotient space $C(S)/G$ is reflexive, then the spaces G^* and C_A^* are not isomorphic). Nevertheless, the following theorem holds.

THEOREM 6. A reflexive space can be embedded into C_A^* if and only if it can be embedded into some space $L_1(\mu)$. Moreover, if E is a reflexive subspace of the space C_A^*, then there exists an operator T from $B(C_A^*, C_A^*)$, such that $T|E$ is an isomorphism and such that on the space $T(E)$ there exists a cross section for the canonical mapping of $C(\mathbb{T})^*$ onto C_A^*.

Finally, for spaces with a separable annihilator, the following analog of the fundamental result of [5] holds.

THEOREM 7. Let S be a metric compactum, $G \subset C(S)$, and assume that the space $G^\perp$ is separable. If $T \in B(G, X)$ and the set $T^*(X^*)$ is not separable, then in G there exists a complemented subspace Y, isomorphic to $C([0,1])$, and such that T/Y is an isomorphism.

†The definitions mentioned in the conditions of Theorem 5 are described, e.g., in [7] (regarding [7], see also the footnote on p. 1103 of this issue).

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SPECTRUM OF OPERATORS IN IDEAL SPACES

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One considers "weighted translation" operators in ideal Banach spaces. It is proved that if the translation is aperiodic (the set of periodic points has measure zero), then the spectrum of such an operator is rotation-invariant. This result can be extended (under certain additional restrictions) to "weighted translation" operators acting in regular subspaces of ideal spaces, in particular, to operators in Hardy spaces.

In this note we prove the rotation-invariance of the spectrum of aperiodic operators of "weighted translation" in ideal* spaces and uniform B-algebras.†

THEOREM 1. Let (X,μ) be a space with a positive σ-finite measure, let E be the ideal space of measurable functions on X, and let φ be a measurable mapping of X onto X such that:

1) $\mu(e)=0 \Leftrightarrow \mu(\varphi(e))=0$;

2) the translation operator T_φ is defined and bounded in E. Assume that the set of φ-periodic points in X has measure zero and that $M\in L^\infty(X,\mu)$. Then the spectrum of the operator $T=MT_\varphi$ is rotation-invariant.

Proof. Assume that λ belongs to the continuous or to the point spectrum of $\sigma(T)$, and let $|\alpha|=1$. Let us show that $\alpha\lambda\in\sigma(T)$. For any measurable set F, $F\subset X$, we define the sets $\varphi_i(F), i=0,1,\ldots$, by the equalities $\varphi_0(F)=F$, $\varphi_i(F)=\varphi(\varphi_{i-1}(F))$. We fix a natural number N. By virtue of the aperiodicity of φ there exist (see [4]) sets $B_1,\ldots,B_N$ with the properties:

1) the sets $\varphi_j(B_i), i=1,\ldots,N, j=0,\ldots,N+2$, are pairwise disjoint;

*Regarding ideal spaces and the corresponding terminology, see [1, p. 91].

†For operators induced by ergodic transformations in L^2, the corresponding result has been obtained by Petersen [2], while for operators with unitary spectrum in uniform B-algebras — by the author [3].

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