

Optimizing the horizontal structural diversity in uneven-aged northern hardwood stands

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Two mathematical programming formulations are presented which allow the determination of diameter distributions that maximize the diameter class diversity in uneven-aged northern hardwood stands. Distributions generated from these models were found to be comparable from a management standpoint and could be incorporated into existing linear programming models as alternative management scenarios. The models presented here provide an initial framework for quantitatively addressing the requirements of the US National Forest Management Act in matters of diversity in the planning process.

Keywords: diameter distributions; non-linear programming; Patil–Taillie diversity profiles

1. Introduction

The US National Forest Management Act (NFMA) final rule (*Federal Register* 47(190), 1982) requires that diversity be considered in formulating management alternatives in the national forest planning process. NFMA specifically states that ‘Forest planning shall provide for diversity of plant and animal communities and tree species consistent with the overall multiple-use objectives of the planning area’. In addition, it calls for the quantitative evaluation of diversity in both past and present conditions so that the impact of proposed management practices on diversity may be evaluated. The NFMA is vague as to how such quantifications of diversity are to be handled, however. Presumably, the drafters of NFMA saw this as an area open for future research.

In this paper, diversity is viewed as a community property; accordingly, the trophic, taxonomic or structural levels that comprise the community must be specifically delimited. Most often, it is unrealistic to identify the components of a community as all the living organisms within a region. Instead, the limited class of organisms for which diversity assessment is desired needs to be clearly determined; in reference to this class, Pielou (1974, p. 289) has coined the term *taxocene*, which may be applied with equal validity to taxonomic groups, structural components, or even communities within communities (e.g. forest stands within a landscape mosaic).

In making diversity comparisons, it is not uncommon to find that diversity has increased according to one index, but decreased according to a second index. This merely reflects the fact that diversity is a complex multidimensional property of a community. To view diversity through the lens of a single index is to project that multidimensional complexity on to a one-dimensional ordinal scale with distorted perception and possibly misleading conclusions. In view of the inadequacy of a single index, we quantify diversity by means of *diversity profiles*. A diversity profile is a curve

depicting the simultaneous values of a large collection of diversity indices. Thus, the profile portrays the views of diversity from many different vantage points simultaneously and in a single picture.

This paper takes up the issue of optimizing diversity in uneven-aged forest management. For concreteness, the discussion is in terms of diversity of the diameter-class distribution, but other forms of diversity could also be used. In uneven-aged forest management, there is the idealized notion of a 'target' stand: a forest, initially at the target stand, is allowed to grow and then is cut back to the target stand after a fixed number of years, known as the length of the cutting cycle (cc). The cycle then repeats itself with cuttings every cc years. The problem is to determine the target stand according to some optimality criterion. The growth equations also place limits on the permissible target stands, so this is a constrained optimization problem. Traditionally, the optimality criteria have been based on economic considerations. This paper brings diversity considerations into this framework through an optimality criterion with economic considerations incorporated in the form of constraints.

The optimal target stand in uneven-aged forest management is defined by: (1) the optimal species mix; (2) the optimal, sustainable residual diameter distribution (number of trees per acre by diameter class left after harvest) and associated maximum tree size; (3) the residual stocking level or basal area per acre (total cross-sectional area of all trees in the target stand); and (4) the optimal cutting cycle length. For simplicity, we use a growth model that incorporates all species in a stand into one set of growth equations, obviating the need to consider optimal species composition. In addition, we assume a cutting cycle length of 5 years. Thus, our model is reduced to one that defines the optimal diameter distribution for all species with sustainability, economic and stocking constraints. The model may be adapted to even-aged stand management, and many other types of constraints not considered here can be envisioned. For example, extensions to the model could incorporate wildlife concerns: tree diameter and height relationships could be used to incorporate some measure of foliage height diversity into the model which would allow the maximization of bird species diversity using relationships similar to those of MacArthur and MacArthur (1961). A solution to this problem would provide the manager with tree diameter and height distributions for management of multiple objectives.

2. Average species rarity and diversity

In this section we discuss the concept of community diversity as average species rarity, first proposed by Patil and Taillie (1979). Throughout this discussion we speak in terms of a conceptual community, C , which is composed of s species. However, it is important to realize that 'species' is simply a convenient label for the categories into which we aggregate individuals, and that the names or labels of the individuals themselves are of no consequence. In addition, in this discussion we define abundances in terms of numbers of individuals (by species). That is, in an s -species community, the absolute abundances are given as N_1, N_2, \dots, N_s , such that $\sum_{i=1}^s N_i = N$, the total number of individuals. Just as 'species' is used generically for some method of categorization, other measures of abundance could also be used; these include biomass, board foot or cubic foot volume, basal area, or any other mensurational quantity.

With the above thoughts in mind, we find that the absolute abundances and total number of individuals in a community are secondary quantities in diversity considerations; the apportionment, or relative distribution of individuals and the number of species are of primary interest. The relative abundance vector for a community is given by $\pi = (\pi_1, \dots, \pi_s)$, where $\pi_i = N_i/N$; therefore, $\sum_{i=1}^s \pi_i = 1$. The total number of species in the community, s , is called the *species richness*; the conceptual community may therefore be written as $C(s, \pi)$, or simply $C(\pi)$ since s is implied

in the dimension of π . Now consider a community such that all species have the same relative abundance; that is, $\pi_i = 1/s = \pi_E$ for all i , so that s alone determines the abundance vector. Such a community is denoted by $C_E(s)$ and is termed the *completely even community*.

Diversity is defined here as average community rarity. The rarity of species i is a quantitative measure associated with that species and is denoted by $R(\pi_i)$. Patil and Taillie (1979, 1982) discuss two types of rarity measures: *rank-type* and *dichotomous-type*. The dichotomous-type rarity measure is used here; it is given by

$$R_\beta(\pi_i) = \frac{(1 - \pi_i^\beta)}{\beta}, \quad -\infty < \beta < \infty. \quad (1)$$

Rarity is a species property while diversity is a property of the community. To determine community diversity, rarity is considered a measurable random variable and diversity is given as its expectation, $E[R(\pi_i)]$. Therefore, using the dichotomous rarity index, we find that the diversity for community $C(s, \pi)$ is

$$\Delta_\beta(\pi) = \sum_{i=1}^s \pi_i R_\beta(\pi_i) = \frac{(1 - \sum_{i=1}^s \pi_i^{\beta+1})}{\beta}, \quad \beta \geq -1. \quad (2)$$

The restriction on the parameter β is required in order that $\Delta_\beta(\pi)$ have certain desirable properties (see Patil and Taillie, 1979, 1982, for more discussion). Note that the normal limiting definition is used as $\beta = 0$ for both $R_\beta(\pi_i)$ and $\Delta_\beta(\pi)$.

The use of the Patil–Taillie diversity profile $\Delta_\beta(\pi)$ as the diversity measure has two important consequences. First, three of the common ecological diversity indices are special cases of $\Delta_\beta(\pi)$. When $\beta = -1$, $\Delta_{-1}(\pi)$ is the *species count*; at $\beta = 0$, $\Delta_0(\pi)$ is the *Shannon index*; and at $\beta = 1$, $\Delta_1(\pi)$ is the *Simpson index*. This ties the $\Delta_\beta(\pi)$ definition of diversity in with much of the ecological literature on diversity, both past and present.

The second important consequence of using $\Delta_\beta(\pi)$ is that if β is allowed to vary while π is held fixed, a plot of $\Delta_\beta(\pi)$ by β yields the Patil–Taillie diversity profile. Diversity profiles for several communities may be plotted simultaneously to provide visual diversity comparisons. If two profiles intersect, then the communities are not intrinsically comparable in terms of diversity, the results of the comparison being dependent upon the specific diversity measure employed. However, some conclusion about the comparative diversity may still be possible using the intersecting profiles. When β is small (close to -1), $\Delta_\beta(\pi)$ is more sensitive to rare species than when β is large (close to $+1$). This can easily be seen by noting that $\Delta_{-1}(\pi)$ is the species count index in which all species, irrespective of the associated community abundance vector, receive the same weight. However, Simpson's index, $\Delta_1(\pi)$, is insensitive to rare species. Therefore, β may be interpreted as a 'sensitivity' parameter. For example, in a mature, even-aged, mixed Appalachian hardwood forest, the forester may place great importance on the occasional 'high-value' species such as black walnut. In this case, the forester would emphasize the portion of the profile corresponding to small values of β in assessing forest diversity. The red-eyed vireo, however, who sees an unbroken canopy of choice mature oak habitat for nesting and foraging, would be more interested in measuring diversity at larger β since abundance of what it considers 'high-value' species is of primary importance.

3. Uneven-aged stand growth model

The model presented is a whole-stand, mixed-species, distance-independent growth model, for uneven-aged northern hardwoods. The original growth model developed by Ek (1974) used three

non-linear regression equations to predict the number of ingrowth, upgrowth, and mortality trees over 5 year growth periods. This set of equations subsequently was modified by Adams and Ek (1974) for use in their study, and has been used by numerous other investigators in optimization studies. Their growth model is expressed in the following set of equations:

$$\begin{aligned}
 I(t) &= 7.07933 \left(\frac{\text{BPA}}{\text{TPA}} \right)^{-1.40072} \\
 U_i(t) &= 0.00330 N_i(t-1)^{0.88218} S Y_{1i}^{0.48383} \exp(-0.00286 \text{BPA}), \quad i = 1, \dots, s \\
 M_i(t) &= 0.04109 N_i(t-1), \quad i = 1, \dots, s.
 \end{aligned} \tag{3}$$

Here

$I(t)$ = ingrowth: the number of trees per acre growing into the 6-inch diameter class during the growth period;

$U_i(t)$ = upgrowth: the number of trees per acre growing from diameter class i to $i + 1$ during the growth period;

$M_i(t)$ = mortality: the number of trees dying in diameter class i during the growth period;

$N_i(t-1)$ = number of trees in diameter class i at the *beginning* of the growth period;

$N_i(t)$ = number of trees in diameter class i at the end of the growth period. These are calculated according to the following three equations:

$$N_i(t) = N_i(t-1) + I(t) - U_i(t) - M_i(t), \quad \text{for } i = 1,$$

$$N_i(t) = N_i(t-1) + U_{i-1}(t) - U_i(t) - M_i(t), \quad \text{for } i = 2, \dots, s \quad \text{and}$$

$$N_{s+1}(t) = U_s(t), \quad \text{for the last diameter class.} \tag{4}$$

s = number of diameter classes; $s = 9$ here;

Y_{1i} = 2-inch diameter class midpoint diameters at breast height (DBH = 4.5 feet), beginning at 6 inches (i.e. 6, 8, ..., 22);

$Y_{2i} = \pi Y_{1i}^2 / (2 \times 12)^2 = 0.005454 Y_{1i}^2$ = tree basal (cross-sectional) areas in square feet corresponding to the Y_{1i} (tree cross-sections are generally assumed circular for basal area calculations).

S = site index: a measure of site quality; $S = 55$ here;

$\text{BPA} = \sum_{i=1}^s Y_{2i} N_i(t-1)$ = basal area per acre in square feet;

$\text{TPA} = \sum_{i=1}^s N_i(t-1)$ = number of trees per acre.

This non-linear growth model predicts the number of trees growing into the smallest diameter class (ingrowth), the number of trees growing up one class (upgrowth) for each diameter class, and the number of trees dying in each diameter class over the 5-year growth period. Equation set (4) provides the linkage between this growth model and the two optimization models presented in the next section.

4. Maximizing diversity

Earlier, it was noted that for any given vector of relative abundances π a diversity profile could be generated by allowing β to vary in (2). In this section, $\Delta_\beta(\pi)$ is viewed in the opposite sense: β is held fixed and π is allowed to vary subject to the constraints that $\beta \geq -1$ and $\sum_{i=1}^s \pi_i = 1$. When this is done, a *diversity surface* is generated for the specified β .

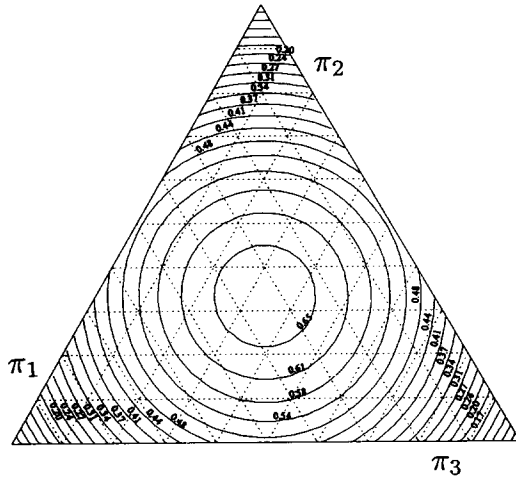


Figure 1. Δ_β diversity contours for three-species communities ($\beta = 1$).

Figure 1 presents a triangular chart of a three-species ($s = 3$) community with each of the three axes of the chart scaled such that $0 \leq \pi_i \leq 1, i = 1, \dots, 3$. This type of chart is useful for envisioning the diversity surface since it automatically incorporates the constraint $\sum_{i=1}^s \pi_i = 1$. The contours plotted on the interior of the chart represent constant values of the $\Delta_\beta(\pi)$ diversity surface when $\beta = 1$ (Simpson's index). Any corner point on the chart represents a single-species community, edges are two-species communities, and interior points are three-species communities. The chart clearly shows for $\beta = 1$ that the $\Delta_\beta(\pi)$ diversity surface reaches its maximum at the center – the completely even community.

A similar chart is shown in Fig. 2 for the $\Delta_\beta(\pi)$ surface for $\beta = 2$. Note the slight difference in the shape of the diversity surface level curves when compared with Fig. 1. In Fig. 2, the curves are less circular and are beginning to become somewhat triangular in shape. Indeed, if the $\Delta_\beta(\pi)$ surface is

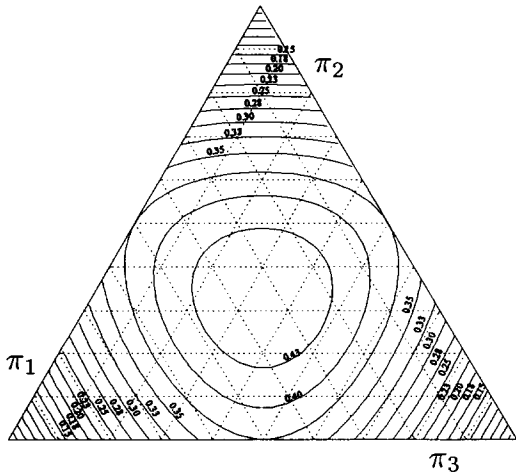


Figure 2. Δ_β diversity contours for three-species communities ($\beta = 2$).

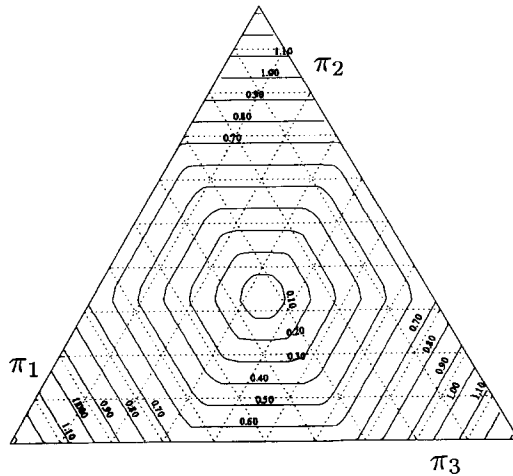


Figure 3. Model (6) contours for three-species communities. (Note that the surface is actually piecewise linear.)

plotted as $\beta \rightarrow \infty$, the level curves become more and more triangular. The maximum, again, is clearly seen to occur at the completely even community when $\beta = 2$.

Moving in the other direction, the diversity surface becomes flatter as β approaches -1 . In the limit of $\beta = -1$, the surface is constant on the interior of the triangle, so conventional contours cannot be drawn.

That the completely even community maximizes diversity for a given number of species s is well known (Patil and Taillie, 1979, 1982; Pielou, 1974; Solomon, 1979). In general, the problem may be formulated for $\Delta_\beta(\boldsymbol{\pi})$ as

$$\max_{\{\boldsymbol{\pi}\}} \Delta_\beta(\boldsymbol{\pi}) \text{ subject to } \sum_{i=1}^s \pi_i = 1. \quad (5)$$

It is straightforward to show that the solution to (5) is $C_E(s)$. This is an interesting finding because it allows the introduction of an alternative objective function into model (5); maximizing (5) is equivalent to the following problem:

$$\min_{\{\boldsymbol{\pi}\}} \sum_{i=1}^s |\pi_E - \pi_i| \text{ subject to } \sum_{i=1}^s \pi_i = 1. \quad (6)$$

The *unevenness* measure $\sum_{i=1}^s |\pi_E - \pi_i|$ in (6) is known as the Pietra index of income inequality in economics (Arnold, 1987). The diversity surface for this formulation is presented in Fig. 3. Note the difference in the shape of the level curves in this surface when compared to Figs 1 and 2; the level curves for (6) are hexagonal. This surface also is minimized at the completely even community, implying that diversity is at its maximum.

5. Maximizing diameter class diversity

Consider the following question for an uneven-aged, northern hardwood stand: given certain

stocking, economic, and biological growth constraints, what is the diameter distribution that maximizes diameter class diversity? Based on the results of the last section, the intuitive answer to this question is the diameter distribution that is most nearly even. However, the result will not be a completely even diameter distribution if the constraints used in the model impose any true restriction on the diversity surface.

Under the current scenario, notice that ‘species’ has now become synonymous with *diameter class*; therefore, s is now the number of diameter classes which is held constant. Relative abundances composing π are determined in terms of the number of trees per acre.

A mathematical programming model structure first introduced by Adams and Ek (1974) is adopted here. Two general model formulations are presented and solved as non-linear programs. The concepts discussed in the previous section may be extended to $s > 3$ in these two models to maximize diameter class diversity. The first formulation in equation set (7) maximizes the diameter-class diversity using the objective function from (6) and is termed Model I.

$$\begin{aligned} \min_{\{\pi\}} \sum_{i=1}^s |\pi_E - \pi_i| \quad \text{subject to:} \\ \sum_{i=1}^s \pi_i = 1; \\ N_i(t) - N_i(t-1) \geq 0, \quad i = 1, \dots, s+1; \\ 0 < \pi_i \leq 1; \\ \text{BPA} = \text{PSL}; \\ \text{LEV} = \text{ESL}. \end{aligned} \tag{7}$$

Here

s = number of diameter classes (‘species’); $s = 9$ in this study. The diameter class width used was 2 inches, with a minimum diameter class of 6 inches;

$N_i(t-1)$ = number of trees in diameter class i in the optimal stand at the *beginning* of the 5-year growth period;

$N_i(t)$ = number of trees in diameter class i in the optimal stand at the *end* of the 5-year growth period given the growth dynamics predicted by the growth model;

BPA = total basal area per acre in the optimal stand (taken at the midpoint of the 2-inch diameter classes);

LEV = $\text{VG}/((1 + \text{arr})^{cc} - 1) - \text{VGS}$ = land expectation value for the optimal stand;

VGS = $\sum_{i=1}^s V_i N_i(t-1)$ = value of initial growing stock;

VG = $\sum_{i=1}^{s+1} V_i N_i(t) - \text{VGS}$ = 5-year value growth;

cc = cutting cycle (time between harvests) length in years ($cc = 5$ in this study);

V_i = individual tree value in dollars for the i th diameter class. The individual tree values used in the computation of LEV are Martin’s (1982) fair site (site index 55) values (see Table 1);

arr = alternative rate of return (3% in this study);

PSL = some physical stocking level of basal area per acre;

ESL = some economic stocking level in present value dollars per acre.

Model II maximizes $\Delta_{\beta}(\pi)$ with the added constraint that β must be fixed, the rest of the model is

the same as Model I. The complete formulation is given as follows:

$$\begin{aligned}
 & \max_{\{\pi\}} \Delta_{\beta}(\pi) \quad \text{subject to:} \\
 & \sum_{i=1}^s \pi_i = 1; \\
 & N_i(t) - N_i(t-1) \geq 0, \quad i = 1, \dots, s+1; \\
 & 0 < \pi_i \leq 1; \\
 & \text{BPA} = \text{PSL}; \\
 & \text{LEV} = \text{ESL}; \\
 & \beta = b,
 \end{aligned} \tag{8}$$

where β may be fixed at any value b such that $-1 \leq b < \infty$.

The growth dynamics for both formulations are modelled with the set of non-linear whole-stand diameter-class growth equations given in (3); therefore, the diameter distribution recovered in the optimal stand is for the entire community composed of all species – no individual species distributions are available.

The physical stocking constraint on basal area and the economic constraint on LEV keep the solutions from both models feasible from a biological perspective. If both of these are set simply to be positive, Model I will lead to a degenerate solution; Model II may find a feasible solution with non-zero π , however, the stand basal area and trees per acre effectively will be zero. Therefore, the growth constraints alone do little to determine a biologically reasonable solution. In addition, if only one of these two constraints is used and if BPA is restricted between lower and upper bounds (e.g. $\text{BPA}_L \leq \text{BPA} \leq \text{BPA}_U$), both models always seem to converge to a solution at the lower bound.

6. Model results

Models I and II were optimized using the generalized reduced gradient program GRG2 (Lasdon and Waren, 1986). Solutions were found at several economic and physical stocking combinations; all solutions presented satisfied the Kuhn–Tucker stationarity conditions.

Table 1 presents solutions to Model I with LEV constrained only to be positive, but with stand basal area set at several stocking levels. Note that the diameter distributions in Table 1 are not completely even; this is a consequence of the constraints on growth and basal area which are all binding in both model formulations. A plot of the $\Delta_{\beta}(\pi)$ profiles for these three communities is shown in Fig. 4. The distribution at 60 ft² is the most diverse community according to the diversity ordering of the $\Delta_{\beta}(\pi)$ profiles. In addition, the unevenness criterion correctly orders each community with respect to diversity in this example.

The most striking aspect of the distributions in Table 1 is that LEV is zero for all solutions. The reason for this is that LEV and evenness work against each other in these formulations. In order to even out a distribution (i.e. maximize diversity), as many trees as possible are put into the larger diameter classes. This happens in accordance with satisfying the growth and BPA constraints until LEV reaches its lower bound of zero. However, trees in the sawtimber size classes (≥ 12 inches) contribute substantially more to holding costs in the calculation of LEV; therefore, few trees are needed in these classes to drive LEV to zero. Thus, the positive constraint on LEV is a mechanism

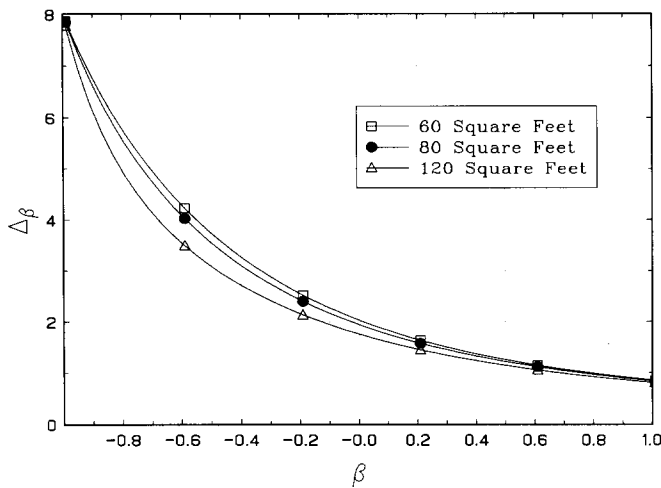


Figure 4. Δ_β profiles of maximum diversity diameter distributions for Model I at different basal area per acre stocking levels.

that works against evening out the distribution at optimality. Indeed, if LEV were left unconstrained, it would be driven negative in the optimal solution, resulting in a more diverse community than the 60-ft² solution in Table 1. Such a result may be reasonable if financial considerations are of no concern to the landowner.

Maximizing Model II is not as straightforward as minimizing Model I. The reason for this was pointed out earlier: inasmuch as different diversity surfaces are generated at each β , $\Delta_\beta(\pi)$ should be optimized several times, each at a different level of β to allow for comparison of the resulting

Table 1. Maximum diversity diameter distributions for Model I at different basal area stocking levels.

Diameter class (inches)	Basal area per acre			Value per tree ^a
	60 ft ²	80 ft ²	120 ft ²	
Trees per acre				
6	18.44	30.03	64.63	0.11
8	13.66	21.90	45.62	0.30
10	10.67	16.86	34.16	0.54
12	8.63	13.47	25.02	3.83
14	7.16	11.05	20.08	6.15
16	6.05	9.25	16.35	8.61
18	5.20	7.87	4.80	11.23
20	4.52	3.01	0.11	14.66
22	1.47	0.56	0.04	17.79
Total TPA	75.80	114.00	210.81	
LEV \$/acre	0.00	0.00	0.00	
Unevenness ^b	0.47	0.55	0.72	

^a The tree values used are from Martin's (1982) fair site guides; values for 24 inch and 26 inch trees used by Martin were \$21.19 and \$24.97 respectively.

^b Unevenness is defined as $\sum_{i=1}^s |\pi_E - \pi_i|$.

Table 2. Maximum diversity diameter distributions with 80 ft² of basal area per acre for Model II at different β .

Diameter class (inches)	β			
	0.01	1.0	10.0	100.0
	Trees per acre			
6	30.53	29.99	30.03	34.78
8	22.26	21.87	21.90	25.29
10	17.13	16.84	16.86	18.51
12	13.69	13.45	13.47	10.29
14	11.22	11.04	11.05	8.47
16	7.66	9.24	9.25	7.11
18	6.53	7.86	7.87	6.06
20	3.03	2.66	3.01	5.24
22	2.01	0.89	0.56	1.91
Total TPA	114.06	113.84	114.00	117.66
LEV \$/acre	0.00	0.00	0.00	0.00
Unevenness	0.58	0.55	0.55	0.67

distributions. Table 2 presents the results of this process for four levels of β at 80 ft² of basal area per acre. All of the constraints (with the exception of that on β) have remained the same in each of these solutions; therefore, the solution space has not changed – the only difference contributing to the slightly different results is the shape of the diversity surface at each β . This phenomenon can be envisioned readily by imagining one or two simple linear constraints in Figs 1, 2, and 3. Note that depending upon the arrangement of the constraints in these figures, the optimal solution may be slightly different in each case. This same reasoning applies to the results in Table 2.

The results in Table 2 show a range of only four trees per acre difference between the resulting stands; therefore, from both a biological and practical perspective, there is no difference between the resulting distributions at different β . Theoretically, the value of the unevenness statistic might be used to judge which of these distributions is, in fact, the most diverse at the 80-ft² level. However, the unevenness criterion is only a one-dimensional statistic and has not been shown to order communities consistently for a given s as have the $\Delta_\beta(\pi)$ profiles. Indeed, a plot of the four $\Delta_\beta(\pi)$ profiles (not shown) reveals that they are not intrinsically comparable since the profiles cross. In this case, it seems reasonable to pick the community at $\beta = 10$ since it has the smallest unevenness and is the solution to Model I (see Table 1). Given the practical considerations, nothing is compromised in this decision.

One further cautionary note is in order when optimizing Model II. If β is set equal to -1 , the diversity surface generated by $\Delta_\beta(\pi)$ is a constant at $s - 1$ for *all* π , as noted earlier. In this case, the only factors restricting the solution are the constraints, and any point which satisfies the constraints within the feasible solution space may be chosen as a solution. Thus, Model II should never be optimized at $\beta = -1$ as solution vectors having little relation to the results of Model I may result. In addition, if β is left unconstrained in Model II, the same result occurs since $\beta \rightarrow -1$ in this case.

The results of adding a LEV constraint different from zero to Model I are shown in Table 3. The first distribution constrains LEV to be \$100 per acre while allowing BPA to go free. The other two distributions constrain both LEV and BPA. Comparing the 60-ft² and 80-ft² distributions in Tables 1 and 3 clearly shows that the effect of the LEV constraint is to add more trees to the smaller

Table 3. Maximum diversity diameter distributions for Model I at different basal area stocking levels with LEV constrained to \$100.

Diameter class (inches)	Basal area per acre		
	50.7 ^a ft ²	60 ft ²	80 ft ²
	Trees per acre		
6	26.78	25.56	42.38
8	19.79	18.84	30.72
10	15.42	14.64	23.53
12	11.34	11.80	18.71
14	9.40	9.75	15.28
16	4.98	8.22	4.64
18	2.25	5.06	3.62
20	0.07	0.15	1.93
22	0.03	0.00	0.00
Total TPA	90.06	94.02	140.81
LEV \$/acre	100.00	60.00	80.00
Unevenness	0.74	0.62	0.75

^a The BPA constraint was free in this distribution.

diameter classes while removing trees from the larger sawtimber classes. This causes a decrease in the holding costs, allowing LEV to increase over the distributions in Table 1. It also decreases the unevenness statistic and, therefore, the diversity, as expected. This illustrates the interplay between evenness and LEV alluded to earlier.

7. Concluding remarks

The models presented provide a framework for quantitatively considering diversity as part of natural resource mathematical programming models. Diversity here is considered the objective to be maximized in both model formulations. However, Gove *et al.* (1994) consider how diversity may be reinterpreted into constraint form if some other objective was desired. The limiting factor in these formulations is the non-linearity of the $\Delta_{\beta}(\pi)$ function and growth equations, requiring solution techniques which necessarily fall within the realm of non-linear programming. Biologically and mathematically such non-linearity makes sense, however, it excludes the explicit use of such functions in large linear programming models such as those used in national forest planning. Diameter distributions produced by solving Model I could, however, be incorporated into linear programming models in the form of alternative management scenarios.

The solutions of Models I and II suggest that both models will give approximately the same answers. However, because of the nature of the diversity surfaces generated in these models and the uncertainty of diversity ordering based on a single index like unevenness alone, it is recommended that both models be solved, as was done in the previous section. In addition, Patil–Taillie $\Delta_{\beta}(\pi)$ profiles should *always* be plotted when comparing models for diversity ordering. Other profiles are available and may also be useful. For example, in comparing the distributions of Table 2, the $\Delta_{\beta}(\pi)$ profiles plot very close to each other, and it is difficult to determine if and where they cross. In this case the Patil–Taillie right *tail-sum* profiles of the relative abundance vector were extremely helpful (see Patil and Taillie, 1979).

In each of the tables presented, the absolute abundance vectors are given because this measure is necessary for management. It is therefore possible to incorrectly interpret the results of these tables by trying to judge evenness based merely on the diameter distributions alone, while not taking into account the respective total trees per acre. In addition, the 'species richness' was held constant for each distribution to facilitate comparison. If new distributions were generated with different numbers of diameter classes, this would also enter into the subsequent evaluation of diversity ordering. It should be remembered that the relative abundances and species richness are the keys for evaluating diversity.

Models I and II were kept relatively simple in order to introduce the concept of maximizing diversity and related diversity ordering. For example, π is treated as a deterministic vector in both models. Actually, because of the stochastic nature of the underlying growth equations, π is a random vector with unknown sampling distribution. Gove and Fairweather (1992) provide a method for evaluating the precision of the optimal diameter distribution by using a Weibull distribution function to characterize the diameter distribution, thus reducing the dimensionality of the problem to three parameters. They bootstrapped the original plot data yielding a multivariate distribution of the Weibull parameters for the optimal stand diameter distribution. This multivariate distribution was transformed and subsequently modeled with the multivariate normal distribution to enable confidence limits to be placed on the optimal diameter distribution. Similar analyses could be done with the models presented here to quantify the stochastic nature of these models.

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