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An Experimental Study of Voting Rules and Polls in Three-Candidate Elections[1]

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Abstract: We report the results of elections conducted in a laboratory setting, modelled on a three-candidate example due to Borda. By paying subjects conditionally on election outcomes, we create electorates with (publicly) known preferences. We compare the results of experiments with and without non-binding pre-election polls under plurality rule, approval voting, and Borda rule. We also refer to a theory of voting "equilibria," which makes sharp predictions concerning individual voter behavior and election outcomes. We find that Condorcet losers occasionally win regardless of the voting rule or presence of polls. Duverger's law (which asserts the predominance of two candidates) appears to hold under plurality rule, but close three-way races often arise under approval voting and Borda rule. Voters appear to poll and vote strategically. In elections, voters usually cast votes that are consistent with some strategic equilibrium. By the end of an election series, most votes are consistent with a single equilibrium, although that equilibrium varies by experimental group and voting rule.

I Introduction

In 1770, Jean-Charles de Borda raised objection to the then-generally-held opinion that "in an election by ballot the plurality of voices indicates the will of the electors." He argued that this opinion, "true in the case where the election is conducted between two candidates only, may lead to error in all other cases." He provided an example in which two candidates, both preferred to a third by a majority of the electorate, might split the votes of that majority, permitting the third candidate to receive a plurality of votes and win the election. He used this example as the basis for his proposal of "election by order of merit," the now-well-known "Borda rule."[2]

Borda's arguments were based on the tacit assumption that each voter would vote "sincerely," i.e., would cast a plurality vote for his or her most-preferred

[1] We thank the Dispute Resolution Research Center at Northwestern University for funding this research. We also thank Roy Gardner, Ronald Harstad, Roberta Herzberg Reinhard Selten and several anonymous referees for helpful comments and Maria Deniston and Scott Schur for research assistance.

[2] See De Grazia [1953] for a translation of and comments on Borda's work.

candidate, and would report his or her preference ranking accurately under Borda's procedure. Assumptions of sincere voting are common in the literature. (For example, see Black [1958].) Yet, as early as 1776, John Adams spoke from a different perspective: "Reason, justice, and equality never had weight enough on the face of the earth to govern the councils of men. It is interest alone that does it, and it is interest alone which can be trusted..."[3] This perspective foreshadowed recent models of strategic voter behavior.[4]

At times, history has borne out Borda's concern. In the 1912 U.S. presidential election, Roosevelt and Taft (former and incumbent Republican presidents) split a majority of the popular vote, allowing Wilson to win. In the 1970 New York race for a U.S. Senate seat, the Democratic and Liberal-Republican nominees split the liberal vote and the Conservative candidate was elected with only 41% of the vote. (It was this latter election which first motivated the proposal of the "approval voting" system.) Similar historical examples are rife.[5]

On the other hand, perhaps the only statement accorded the status of "Law" in political science is that of Duverger: "The simple-majority single-ballot system favours the two-party system." (Duverger [1967, p. 217]) This statement can be justified only by the assumption that voters sometimes vote strategically, i.e., focus their attention upon two "serious" candidates and then vote for the more-preferred of those two, even when they strictly prefer other candidates. History, in the large, bears out Duverger's Law. As Duverger himself points out, "An almost complete correlation is observable between the simple-majority single-ballot system and the two-party system: dualist countries use the simple-majority system and simple-majority vote countries are dualist." (Duverger [1967, p. 217])

Can the apparently-conflicting historical evidence be consistent with rational voter behavior? Our purpose in this paper is to give an affirmative answer to this question.

We discuss herein a theory of "voting equilibria" (developed in more detail in Myerson and Weber [1993]), which provides for Borda's example two different types of equilibria when rational voters participate in a plurality-rule election. In some equilibria, the majority vote is split and the minority candidate is elected, realizing Borda's fears. In other equilibria, a "Duverger effect" leads to the election of one of the majority-supported candidates, with the other majority-supported candidate finishing last. If the election is conducted using approval voting or Borda rule, the equilibria for Borda's example differ. Under approval voting, there are two types of equilibria, in both of which voters vote sincerely, and in neither of which the minority candidate expects a clear victory. Under Borda rule, the unique equilibrium predicts a close three-way race, with a substantial amount of strategic voting by individuals. (The prediction accords well

[3] De Grazia [1953] comments on this quote found in Burnett [1941].

[4] For example, see Farquharson [1969], Niemi and Frank [1982 and 1985], Felsenthal [1990], and Myerson and Weber [1993]. See Rietz [1993] for a summary of these models.

[5] For some examples, see Riker (1982).

with Adams' view and weakens Borda's arguments advocating use of this proposed procedure.)

In order to test the predictions of the theory, as well as to determine conditions under which one or another of the equilibria arises, we conducted a series of laboratory-based voting experiments. We held sequences of elections, with candidates in fixed positions, in order to provide an opportunity for convergence to equilibrium and to study the effect of the outcomes of previous events (elections or polls) on individual voter behavior. (The repetition can also be viewed as modelling a sequence of real elections involving similar candidates from a fixed list of parties.) To compare among the three electoral systems, we conducted three separate sets of elections using plurality rule, approval voting, and Border rule procedures. A reporting of the experimental results forms the main portion of this paper

II Previous Research

There is a long tradition of experimental research studying elections (see McKelvey and Ordeshook [1987]). Some of this research has examined plurality rule in three-candidate elections (Plott [1977]). Some of it has looked at the impact of polls (Plott [1977] and McKelvey and Ordeshook [1985a and 1985b]). In general, it has focused on environments with incomplete information. Either the candidates do not know the voters' preferences or the voters do not know each candidate's position and, hence, the value of a candidate's election to them. In this paper, we consider a much simpler, complete-information setting, in which candidate positions are specified exogenously and voter preferences over the candidates are commonly known.

In terms of the information available to the voters, our voting experiments are similar to those conducted by Felsenthal, Rapoport and Maoz [1988], and Rapoport, Felsenthal and Maoz [1991], who tested several previously-proposed classes of "equilibrium" models that assume bloc voting. (The tested models make the questionable assumption that voters with the same preferences vote in the same deterministic manner, and frequently fail to yield predictions.[6]) Each bloc of voters was represented by a single experimental subject, and the subject's ballot was scaled according to the bloc size. Similar to the results reported here, in Forsythe, Myerson, Rietz, and Weber [1993] and in Myerson, Rietz and Weber [1994], these earlier experiments yielded a high degree of strategic voting: Voters often voted for a less-preferred candidate when they believed that their most-preferred candidate was unlikely to win. This evidence calls into serious question the commonly adopted assumption of sincere voting in multi-candidate elections.

[6] See Felsenthal [1990] and Rietz [1993] for descriptions the block voting models tested. They are developed in Farquharson [1969], Niemi [1984], Niemi and Frank [1982 and 1985], and Felsenthal [1990]. Rietz [1993] contains a summary.

In contrast to Felsenthal, Rapoport and Maoz [1988], and Rapoport, Felsenthal and Maoz [1991], we allow voters with identical preferences to vote independently. We feel it is important to expand the scope of experimental research to test models that do not assume bloc voting because mathematical models are, at best, approximations of reality. A standard and fundamental test which must be applied to any proposed method of analysis is that the results of the analysis not change drastically when the model is changed slightly. (More precisely, predictions should vary upper-hemicontinuously in the parameters of the model.) Bloc voting models fail this test: Slight changes in the preferences of some members of a "bloc" can yield gross changes in the predicted behavior of individuals (by breaking the bloc into pieces). Our equilibrium theory, which does not force bloc voting, passes this test. We find that voters in blocs defined by identical preferences often cast different vote vectors. Recent laboratory studies also find that this behavior is pervasive.[7] Thus, both theoretical arguments and the experimental evidence calls into serious question the commonly adopted assumption of bloc voting in multi-candidate elections. For these reasons, we choose here to have classes of several voters with identical induced preferences in a setting with commonly-known demographic data. While blocs of voters with (induced) common preferences exist, these voters are not required to vote as blocs. We are unaware of any other experiments that compare alternative voting mechanisms in such a framework.

III The Experiment

The experiment consisted of six sessions, with twenty-eight subjects in each session. For each of the three voting rules, there was one session involving only elections, and a second which also involved pre-election polls. We identify sessions by the voting rule used ("P" for plurality voting, "A" for approval voting and "B" for Borda rule) and the presence of lack of polls (with "WOP" indicating no polls and "WP" indicating polls). For example, the Plurality voting session WithOut Polls is designated 'PWOP," the Borda rule session With Polls is designated "BWP," etc.

In each session we initially assigned subjects to two independent voting groups. Each of these groups participated in eight elections (independent of each other). Then all subjects were randomly re-assigned to two new, independent voting groups for a second series of eight elections each. Finally, subjects were again randomly reassigned to two more independent voting groups for a third series of eight elections each. Thus, there were six treatments with a single cohort of

[7] Besides the results reported here, see also: Forsythe, Myerson, Rietz and Weber (1993); Myerson, Rietz and Weber (1994) and Gerber, Morton and Rietz (1994). All show that voters within a like preference bloc often cast different votes.

subjects participating in each treatment. Within each cohort, there were six largely independent voting groups. Within each group, there were eight elections.

We conducted all sessions at the University of Iowa.[8] Subjects were recruited for a three hour session from a large, volunteer subject pool recruited directly from M.B.A. and undergraduate classes. The sessions with polls typically lasted just under three hours. The sessions without polls were significantly shorter. Subjects earned an average of $24.09 with an earnings standard deviation of $2.59.

Upon arrival, subjects were seated in a large classroom and given copies of the instructions for the session. (This appendix is available on the World-Wide-Web at http://www.econlab.arizona.edu) The instructions were read aloud and questions were answered in public in order to make all instructional information common knowledge.

Each subject was given a voter identification number and assigned to an initial voting group consisting of 14 of the 28 subjects. At all times, there were two distinct voting groups in the room. Each voting group was divided into voters of three "types," differing by their payoffs conditional on the winning candidate. Subjects knew that the composition of each voting group would remain unchanged for eight voting periods. This allowed voters to form expectations and develop voting strategies based on a group's common history. After eight periods, voters were randomly re-assigned to new groups and new types (and new payoff schedules were used, with randomly rearranged and relabeled rows and columns). This allowed us to observe several different groups in each session while minimizing any repeated-game effects that might carry over from one group to the next. In each of the six sessions, we conducted three series of eight elections each. Thus, a total of six independent voting groups were formed per session, and $6 \cdot 6 \cdot 8 = 288$ elections were conducted (together with 144 polls). Each subject participated in three voting groups sequentially and in a total of twenty-four elections, yielding a total of $6 \cdot 28 \cdot 24 = 4032$ voter responses in elections and $3 \cdot 28 \cdot 24 = 2016$ voter responses to polls.

At the beginning of each session, each voter's folder contained the payoff schedules for each of the three voting groups in which he or she would participate. Each group (of the 36 formed in the course of the experiment) used a payoff schedule equivalent to the payoff schedule given in Figure 1.[9] For each voting group, rows and columns of this payoff schedule were randomly shuffled and relabeled as discussed above.[10] Within a group, each individual payoff schedule

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[8] We also used Northwestern University students for two sessions similar to the approval voting session reported here. The results are similar, but are not reported here because of the possibility of subject pool effects. All data are available from the authors.

[9] Voter types are designated by their most preferred candidate here. They were designated only by number in the actual payoff tables.

[10] Given the structure of payoffs, subjects presumably could identify which candidate was the same as the Blue candidate in this payoff schedule and which voter type was the same as Voter Type 3(B). However, they should not be able to identify the other two candidates or voter types from one group to the next. Throughout this paper, we will refer to this payoff schedule. The actual voter types and responses have been transformed so they match this schedule for reporting purposes.

Voter Type	Payoff Schedule Group: Election Winner			Total Number of Each type
	Orange	Green	Blue	
1 "O"	$1.60	$1.20	$0.30	4
2 "G"	$1.20	$1.60	$0.30	4
3 "B"	$0.60	$0.60	$1.90	6

Fig. 1. "Symmetric" payoff schedule

was identical except for a box placed around that individual's voter type. In this way, each voter knew his or her own payoffs, the payoffs to the other voter types in the group, and the number of voters of each type. However, voters did not know the specific assignment of types to others in the room. Furthermore, since poll and election responses were collected from both groups simultaneously and outcomes for both groups were posted publicly, the voters did not know the specific identities of others in their groups.

The actual voter types and responses have been transformed so they match this schedule for reporting purposes. We will call the candidates corresponding to Orange and Green the "majority" candidates, and the candidate corresponding to Blue the "minority" candidate. Similarly, we will call voters who prefer the Orange or Green candidates "majority voters" and voters who prefer the Blue candidate "minority voters." Of the two candidates corresponding to Orange and Green in the payoff matrix, the first to appear (from left to right) in the payoff schedule and on the poll and election ballots is reported as Orange.

Notice that, under the payoff schedule used, the Blue candidate is a Condorcet loser (i.e., a majority of the voters prefers Orange to Blue, and a majority also prefers Green to Blue). However, if all voters vote sincerely, Blue will win plurality-rule elections. While the majority of the electorate prefers either Orange or Green to Blue, they are evenly split. *A priori*, neither candidate appears to be more likely to win. We modeled this payoff structure on the preferences that Borda feared would lead to the "wrong" outcome (a Condorcet loser winning), while not giving majority voters any obvious coordination devices such as asymmetric payoffs or voter numbers.

A Implementing Voting Rules

Each voter's folder contained a set of election ballots. Voters were told that they could choose to abstain in any election (or poll), by turning in blank ballots. If they did vote, then they were required to vote according to a specific rule.

The wording from the instructions for implementing each voting rule was as follows:

Plurality: "If you not abstain, you may vote for at most one candidate. To do this, place a check next to the candidate for whom you are voting."

Approval: "If you do not abstain, you may cast one vote each for as many candidates as you wish. To do this, place a check next to each candidate for whom you are voting."

Borda Rule: "If you do not abstain, you must give two votes to one candidate and one vote to one of the other candidates. To do this, write "2" next to the candidate to whom you are giving two votes and write "1" next to the candidate to whom you are giving one vote."

In practice, the admissible vote vectors under plurality rule were (1,0,0), (0,1,0), (0,0,1), and (0,0,0). Under approval voting, the vectors (1,1,1), (1,1,0), (1,0,1), and (0,1,1) were also admissible. Finally, under Borda rule, the vectors (2,1,0), (2,0,1), (1,2,0), (0,2,1), (1,0,2), (0,1,2), and (0,0,0) were admissible.

If a tie occurred between two or more candidates, we selected the winner randomly. To do this, we placed colored balls corresponding to the names of the tied candidates in a box and asked one of the subjects to draw a ball from the box. The candidate whose name was the same as the color of the selected ball was declared the winner.

B Implementing Polling Rules

In sessions with polls, the instructions informed the subjects that polls were non-binding. Subjects were told that they could vote in the election even if they abstained from the poll, and that their vote need not match their poll response. In these sessions, each voter's folder contained a set of polling forms. Before each period's election, we asked each voter to submit a polling form. (Voters could abstain by submitting blank polling forms.) Polls were conducted according to the same voting rule as the election. Before conducting the election, we announced the total number of poll-votes for each candidate and recorded these totals on the blackboard in the front of the room.

C Voting Equilibria

A voter can affect the election outcome only if two or more candidates receive vote totals which are nearly equal and exceed the vote totals of all other candidates. How the voter perceives the relative likelihood of various "close races" should play a role in ballot choice. Following Myerson and Weber [1993], we assume the following: First, near-ties between two candidates are perceived to be much more likely than between three or more candidates. Second, the probability that a particular ballot moves one candidate past another is perceived to be proportional to the difference in votes cast on the ballot for the two candidates. And third, voters seek to maximize their expected utility gain from the outcome of the election. Let $K = \{1, 2, \ldots, k\}$ be the set of candidates. Then a voter who assigns utility u_i to the election of candidate i and perceives the likelihood of a near-tie between candidates i and j to be p_{ij}, will cast the vote vector $(v_1, \ldots, v_k)$ which maximizes:

$$\sum_{i \in K} \sum_{j \neq i} p_{ij}(u_i - u_j)(v_i - v_j) = \sum_{i \in K} v_i \sum_{j \neq i} p_{ij}(u_i - u_j). \tag{1}$$

At a voting *equilibrium*, the voters' perceptions of the relative chances of various close races support voting behavior which leads to an outcome justifying the original perceptions. Table I describes the voting equilibria that arise from our payoffs under each voting rule. Equilibria are defined by optimal vote vectors for each voter type and by the conditional probability of each pair of candidates being tied for victory, given that a tie occurs. (For each candidate ranking which can arise in equilibrium, we write "$\approx$" to represent a "close" race and "$\gg$" to represent a "strict ranking." For example, $B \gg O \approx G$ denotes the candidate ranking at an equilibrium in which Blue has the highest expected vote total, followed by Orange and Green, who have lower, but equal, expected vote totals.[11] While voters place positive probabilities on all two-way close races in an $O \approx B \approx G$ equilibria, these probabilities are not equal in any of the equilibria here. In each $O \approx B \approx G$ here, voters perceive two-way close races for the lead between Blue and a majority candidate (Orange or Green) as far less likely than a close race between the majority candidates (Orange and Green). Thus, though we call these equilibria close, three-way races, the candidates do not all have equal chances of winning. (To understand the unequal probabilities, consider the $O \approx B \approx G$ equilibrium under approval voting. At equilibrium, the Type O voters must be indifferent between voting for both Orange and Green, and voting for Orange alone. In a large electorate, this can only be the case if $-4p_{OG} + 9p_{GB} = 0$. A similar requirement for Type G voters implies that $-4p_{OG} + 9p_{OB} = 0$. Together with $p_{OG} + p_{OB} + p_{GB} = 1$, this yield $p_{OG} = 9/17$.[12])

Under plurality rule, the symmetric payoff matrix results in three possible equilibria. If all voters vote sincerely, Blue wins the election followed by Orange and Green, who are in a close race for second. We denote this equilibrium by $B \gg O \approx G$. Since neither Orange nor Green appears to have any advantage as

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[11] In the actual rankings listed in the data and statistical supplement, "$\gg$" implies that the candidates differed by two or more votes, "$>$" implies that the candidates differed by one vote, and "$=$" implies that the candidates were tied. In the former case, no single voter could change the winner by switching between un-dominated vote vectors. In the latter two cases, a single voter could change the outcome.

[12] Strictly speaking, we are assuming risk neutrality to derive these probabilities.

Table I. Equilibrium pivot probabilities for each candidate pair and consistent individual vote vectors (Probability and vote vectors are in the order orange, green and blue)

Voting Rule	Voter Type	Vote Vectors Consistent with the Equilibrium: $O \approx G \approx B$	$O \gg B \gg G$	$G \gg B \gg O$	$B \gg O \approx G$
	O	–	(1, 0, 0)*	(0, 1, 0)**	(1, 0, 0)*
	G	–	(1, 0, 0)**	(0, 1, 0)*	(0, 1, 0)*
	B	–	(0, 0, 1)*	(0, 0, 1)*	(0, 0, 1)*
Plurality Rule	Supporting Conditional Tie Probabilities:	–	$p_{OG} = 0$ $p_{OB} = 1$ $p_{GB} = 0$	$p_{OG} = 0$ $p_{OB} = 0$ $p_{GB} = 1$	$p_{OG} = 0$ $p_{OB} = a$ $p_{GB} = 1 - a$ $a \in [9/22, 13/22]$.
	Expected Vote Totals		(8, 0, 6)	(0, 8, 6)	(4, 4, 6)
	O	(1, 0, 0)* & (1, 1, 0)*	(1, 0, 0)*	(1, 1, 0)*	–
	G	(0, 1, 0)* & (1, 1, 0)*	(1, 1, 0)*	(0, 1, 0)*	–
	B	(0, 0, 1)*	(0, 0, 1)*	(0, 0, 1)*	–
Approval Voting	Supporting Conditional Tie Probabilities:	$p_{OG} = 9/17$ $p_{OB} = 4/17$ $p_{GB} = 4/17$	$p_{OG} = 0$ $p_{OB} = 1$ $p_{GB} = 0$	$p_{OG} = 0$ $p_{OB} = 0$ $p_{GB} = 1$	–
	Expected Vote Totals	(6, 6, 6)	(8, 4, 6)	(4, 8, 6)	
	O	(2, 1, 0)* & (2, 0, 1)**	–	–	–
	G	(1, 2, 0)* & (0, 2, 1)**	–	–	–
	B	(1, 0, 2)* & (0, 1, 2)*	–	–	–
Borda Rule	Supporting Conditional Tie Probabilities:	$p_{OG} = 49/57$ $p_{OB} = 4/57$ $p_{GB} = 4/57$	– –	– –	– –
	Expected Vote Totals	(14, 14, 14)			

* The vote vector is sincere in that it weakly ranks the candidates the same as the voters' preferences.
** The vote vector is not sincere, but it is strategic since it is the voter's best response given the equilibrium

a challenger to Blue, no one has any incentive to change their vote. The expected vote totals under this equilibrium are 4, 4, and 6 for Orange, Green, and Blue respectively. In this equilibrium, we expect the Condorcet loser (Blue) to win. The other two equilibria are "coordinated" in the sense that Type O and G voters form a coalition and all vote for either Orange or Green. These equilibria are $O \gg B \gg G$ and $G \gg B \gg O$. Here, the "strategic" Type O and G voters (Type O voters who vote for Green and vice versa) have no incentive to change. Voting for their favorite candidate will be perceived (at equilibrium) to increase the chance that Blue wins much more than the chance that their favored candidate wins. The vectors of expected vote totals for these equilibria are (8, 0, 6) and (0, 8, 6), respectively.

According to (1), under approval voting a voter should be willing to vote for candidate i only if

$$\sum_{j \neq i} p_{ij}(u_i - u_j) \geqq 0. \tag{2}$$

This voter response pattern gives three possible equilibria: $O \gg B \gg G$, $G \gg B \gg O$, and $O \approx B \approx G$. The first two equilibria result when either Type O or G voters cast approval votes for their second-favorite candidate, making this candidate the winner. They do not have an incentive to withdraw this vote because this would increase the chances that Blue would win. We expect the leading candidate to win with expected vote totals of (8, 4, 6) and (4, 8, 6), respectively. The third equilibrium results when Type O and G voters are indifferent between casting 0 and 1 votes for their second favorite candidate. This results in a close three-way race with conditional tie probabilities of $p_{OG} = 9/17$, $p_{OB} = 4/17$, and $p_{GB} = 4/17$.

Borda rule yields a unique equilibrium: $O \approx B \approx G$. If Blue were perceived to be no threat, Type O and G voters would "dump" their 1-votes on Blue—and Blue would win! Hence, at equilibrium Blue *must* be perceived to have some chance of winning. Type B voters "dump" their 1-votes evenly between Orange and Green. Conditional tie probabilities of $p_{OG} = 49/57$, $p_{OB} = 4/57$, and $p_{GB} = 4/57$. Support this equilibrium, making all voter types indifferent between casting their 1-vote for their second-favorite or least-favorite candidate.[13]

IV Results

Recall that the actual payoff tables used by subjects were randomly scrambled versions of Figure 1. For reporting purposes, we standardize the candidate names. Regardless of their actual names, we will call the two majority preferred candidates "Orange" and "Green" with "Orange" being the first listed such

[13] Again, stricly speaking, we are assuming risk neutrality.

candidate. We will call the minority preferred candidate "Blue." Forsythe, Myerson, Rietz and Weber [1993] found that there could be ballot position effects and this reporting convention controls for such effects. Finally, we will call voters who prefer Orange or Green the "majority" voters and voters who prefer Blue the "minority" voters.

A Results for Elections Not Preceded by Polls

For each voting rule, six randomly composed groups of subjects (from a single cohort) each participated in a sequence of eight elections. Table II gives the ordinal ranking of the candidates by vote totals in each election. The symbol "=" indicates that two candidates received equal vote totals, ">" indicates a difference of exactly one vote and "≫" a difference of two or more votes.

Under plurality rule, the first three voting groups established patterns of behavior consistent with both Duverger's Law and the asymmetric voting equilibria: One of the majority candidates became "focal" in an early round and won in all subsequent rounds. Voting group "D" contained two particularly stubborn supporters of the trailing majority candidate. In voting group "E," several supporters of the miniority candidate recognized that change could only come from them and they voted for the trailing majority candidate in round 5. This encouraged several supporters of the trailing majority candidate, who had been voting for the leader, to "come home" (to their chagrin) in rounds 6 and 7. In voting group "F," a supporter of the minority candidate tried the same tactic, with less success.

Under approval voting and Borda rule, the minority candidate never emerged as a clear winner (a winner who wins by two or more votes). The minority candidate won five elections by a single vote under approval voting and four elections by a single vote under Borda rule. Note that the election outcome was more varied from round to round (within each voting group) under approval voting and Borda rule than it was under plurality rule. This accords well with the predictions of close races associated with the symmetric voting equilibrium under approval voting and the unique voting equilibrium under Borda rule.

The mean first-, second-, and third-place vote totals (as percentages of all votes cast) were 52%, 38%, and 10% for plurality rule. This accords well with Duverger's Law. The comparable percentages were 38%, 33%, and 29% for approval voting, and 39%, 33%, and 28% for Borda rule. We interpret these percentages as confirmation of close three-way races. In a later section we will combine this data with data from the sessions with polls and show that these differences are significant.

The "gaming" that took place in three of the plurality-rule voting groups suggested a second set of sessions, with polls preceding all elections. With such sessions, we hoped to disentangle direct payoff-relevant behavior (in elections) from strategic signalling (in polls).

Table II. Election outcomes for sessions without polls

Plurality Rule

Election Number	Voting Group A	Voting Group B	Voting Group C	Voting Group D	Voting Group E	Voting Group F
1	G≫O=B	B≫G>O	B>G≫O	B≫G>O	B≫O=G	O≫B>G
2	G=B≫O	B≫G≫O	G≫B≫O	B>G≫O	B>G≫O	O>B≫G
3	G>B≫O	B>O=G	G≫B≫O	G=B≫O	G≫B≫O	O≫B≫G
4	G>B≫O	O>B≫G	G≫B≫O	G=B≫O	G≫B≫O	O≫B≫G
5	G≫B≫O	O≫B≫G	G>B≫O	G=B≫O	G≫O≫B	O≫B≫G
6	G≫B≫O	O≫B≫G	G≫B≫O	G=B≫O	G=B>O	O≫G=B
7	G≫B≫O	O≫B≫G	G>B≫O	G=B≫O	G=B≫O	O≫G=B
8	G≫B≫O	O≫B≫G	G≫B≫O	G=B≫O	G≫B≫O	O=G>B

Approval Rule

Election Number	Voting Group A	Voting Group B	Voting Group C	Voting Group D	Voting Group E	Voting Group F
1	G≫B>G	O=G≫B	O=G>B	G>O>B	O≫G>B	O=G>B
2	B>G>O	G>B>O	G=B≫O	O=B>G	O≫G=B	B>O=G
3	G>B>O	G=B>O	O=G>B	O>G=B	B>O=G	O=G>B
4	O=G=B	O=G≫B	G>B>O	G≫O=B	B>O=G	O≫G=B
5	G=B≫O	G>O≫B	G>B>O	G>O>B	O>G=B	O=G=B
6	G≫O=B	G≫B>O	B>O=G	G>B>O	O>B>G	O>G=B
7	G≫B>O	G≫O=B	G>B>O	G>B>O	O=B>G	O>G=B
8	O≫G=B	G≫O=B	G=B>O	G>B≫O	O=B>G	O≫G=B

Borda Rule

Election Number	Voting Group A	Voting Group B	Voting Group C	Voting Group D	Voting Group E	Voting Group F
1	G≫O≫B	O≫G>B	O=G≫B	O≫G=B	G>O>B	O≫G>B
2	G≫O≫B	O≫G≫B	O>B>G	O≫G=B	G≫O=B	G≫O≫B
3	G≫O≫B	G>B≫O	G≫O≫B	O=G=B	G≫O≫B	O>B>G
4	O=G≫B	B>O>G	G≫O>B	O=G≫B	G>B>O	O>B>G
5	O≫G≫B	G>B>O	G>O>B	G>O>B	G≫O≫B	G>O>B
6	B>O>G	B>G>O	O≫G>B	G>O>B	B>O>G	O≫G>B
7	G>O≫B	G≫O≫B	G≫O≫B	O≫G=B	O>G>B	O≫G≫B
8	G>O>B	O=G≫B	G≫O=B	O=G=B	G≫O=B	G≫B≫O

B Results for Elections Preceded by Polls

For each voting rule, six randomly composed groups of subjects (from a single cohort) each participated in eight consecutive poll-election rounds. Table III gives the ranking of the poll and election vote totals for each candidate in each poll and election. In each column, poll results are listed to the left and election

Table III. Poll and election outcomes for sessions with polls

Plurality Rule

Round Number	Voting Group A Polls	Voting Group A Elections	Voting Group B Polls	Voting Group B Elections	Voting Group C Polls	Voting Group C Elections	Voting Group D Polls	Voting Group D Elections	Voting Group E Polls	Voting Group E Elections	Voting Group F Polls	Voting Group F Elections
1	B ≫ O = G	B ≫ O = G	B ≫ O = G	B ≫ O ≫ G	B ≫ O > G	O > B ≫ G	B > G ≫ O	G ≫ B ≫ O	O = B ≫ G	O > B ≫ G	G > O = B	B > G ≫ O
2	G > B ≫ O	G ≫ B ≫ O	B ≫ G > O	G = B ≫ O	G > O = B	G ≫ B ≫ O	B > O = G	B > G ≫ O	G > B > O	B > G ≫ O	O > B > G	O > B ≫ G
3	O ≫ G > B	O ≫ B ≫ G	G ≫ B ≫ O	G ≫ B ≫ O	O = B ≫ G	O > B ≫ G	G > B ≫ O	G > B ≫ O	O ≫ B > G	O ≫ B ≫ G	G > B ≫ O	G ≫ B ≫ O
4	O ≫ G = B	O = B ≫ G	G ≫ B ≫ O	G ≫ B ≫ O	G > B > O	G ≫ B ≫ O	G > O ≫ B	B ≫ O = G	O > B ≫ G	O = B ≫ G	O ≫ B > G	O ≫ B ≫ G
5	O = G > B	G = B > O	G ≫ O ≫ B	G ≫ O = B	O ≫ B ≫ G	O > B ≫ G	O ≫ B > G	O > B ≫ G	G ≫ B > O	G ≫ B ≫ O	G ≫ B > O	G ≫ B ≫ O
6	G ≫ O > B	G ≫ B ≫ O	G ≫ B > O	G > B ≫ O	G > B ≫ O	G > B ≫ O	O ≫ B > G	O > B ≫ G	O > G = B	O > B ≫ G	O ≫ B > G	O ≫ B ≫ G
7	G ≫ B ≫ O	G ≫ B ≫ O	G ≫ B ≫ O	G = B ≫ O	O > B ≫ G	O > B ≫ G	G = B > O	G = B ≫ O	B > O > G	O = B ≫ G	G > O > B	G ≫ B ≫ O
8	O ≫ G > B	O > B ≫ G	G ≫ B ≫ O	G ≫ B ≫ O	G ≫ B > O	G > B ≫ O	O > B ≫ G	O > B ≫ G	O > B > G	O ≫ B ≫ G	O > G = B	O > B ≫ G

Approval Voting

Round Number	Voting Group A Polls	Voting Group A Elections	Voting Group B Polls	Voting Group B Elections	Voting Group C Polls	Voting Group C Elections	Voting Group D Polls	Voting Group D Elections	Voting Group E Polls	Voting Group E Elections	Voting Group F Polls	Voting Group F Elections
1	O > G ≫ B	G > O ≫ B	G ≫ O = B	G ≫ B ≫ O	O > G > B	G > B > O	B > G ≫ O	G = B > O	O = G = B	O ≫ G = B	G ≫ O ≫ B	O = G > B
2	G > O ≫ B	G ≫ O > B	G ≫ O > B	G ≫ B ≫ O	O > B > G	G = B > O	O = G = B	O = B > G	O ≫ G > B	O > G = B	G ≫ O ≫ B	G = B > O
3	G ≫ B > O	G ≫ B ≫ O	G > B ≫ O	G ≫ B ≫ O	O ≫ B > G	O ≫ B > G	O ≫ B > G	O ≫ G = B	O > G ≫ B	O > G > B	G ≫ O ≫ B	G ≫ B ≫ O
4	G ≫ O = B	G = B ≫ O	G = B ≫ O	G ≫ B ≫ O	O > B > G	O = B > G	O > B ≫ G	O ≫ B ≫ G	O ≫ G ≫ B	O = G ≫ B	G > O ≫ B	G = B > O
5	G ≫ O = B	G = B ≫ O	G > O ≫ B	B > G > O	B ≫ O ≫ G	O ≫ B ≫ G	B > O ≫ G	O ≫ B > G	O > G ≫ B	G > O = B	O ≫ G > B	G > B > O
6	G ≫ O = B	G = B ≫ O	G ≫ O ≫ B	G ≫ B > O	B ≫ O ≫ G	O > B > G	B > O > G	O ≫ G = B	O ≫ G ≫ B	O ≫ G = B	G ≫ O ≫ B	G ≫ B > O
7	G = B ≫ O	G = B > O	G ≫ B > O	G ≫ B > O	O ≫ B > G	O ≫ B > G	O ≫ B ≫ G	O ≫ B ≫ G	O ≫ G = B	O > B > G	O = G ≫ B	G ≫ B ≫ O
8	G > O = B	B > O = G	G ≫ O > B	G ≫ B ≫ O	O ≫ B > G	O ≫ G = B	O > B ≫ G	O ≫ B ≫ G	O ≫ G > B	O ≫ G = B	G > O = B	G > B > O

Borda Rule

Round Number	Voting Group A Polls	Voting Group A Elections	Voting Group B Polls	Voting Group B Elections	Voting Group C Polls	Voting Group C Elections	Voting Group D Polls	Voting Group D Elections	Voting Group E Polls	Voting Group E Elections	Voting Group F Polls	Voting Group F Elections
1	G ≫ O ≫ B	G ≫ O ≫ B	G ≫ B ≫ O	G ≫ B ≫ O	G ≫ B ≫ O	O ≫ G > B	G = B ≫ O	O ≫ G ≫ B	O = G = B	O ≫ B ≫ G	O ≫ G > B	O = G = B
2	G ≫ O ≫ B	G > O ≫ B	O ≫ B ≫ G	G > O > B	B ≫ O ≫ G	G ≫ B ≫ O	G ≫ O ≫ B	O = G ≫ B	G ≫ O > B	B > O > G	O ≫ G ≫ B	O = G ≫ B
3	G ≫ O ≫ B	G > O > B	O = B ≫ G	O = G = B	B ≫ G ≫ O	O ≫ G > B	B ≫ O ≫ G	O = G ≫ B	G ≫ O ≫ B	O ≫ G > B	O ≫ G = B	B > O > G
4	G ≫ O ≫ B	B > G > O	O ≫ G = B	G ≫ O = B	B ≫ O ≫ G	G > B > O	B ≫ G = O	O > B > G	G > B > O	O = G = B	G ≫ O = B	G > O > B
5	G > O ≫ B	O > G ≫ B	O ≫ B ≫ G	O ≫ G = B	O = G = B	O = G ≫ B	O > B ≫ G	G ≫ O > B	G ≫ O ≫ B	G ≫ O = B	G > O ≫ B	O > G > B
6	G ≫ O ≫ B	G ≫ O > B	O > B > G	O = G = B	B ≫ O > G	O ≫ G ≫ B	O > B ≫ G	G ≫ O = B	O > G ≫ B	G > B > O	O = B ≫ G	G ≫ O ≫ B
7	O > B > G	G > O > B	O > G ≫ B	O ≫ G = B	B ≫ O > G	O = G ≫ B	O = G ≫ B	O > G > B	G ≫ O ≫ B	O > B > G	B > O ≫ G	G > O > B
8	O ≫ G ≫ B	O > G > B	O ≫ G ≫ B	O ≫ B > G	B ≫ G ≫ O	B > G > O	G > B > O	O ≫ G = B	G ≫ O ≫ B	G ≫ B ≫ O	O ≫ B ≫ G	G ≫ O = B

results are listed to the right. (Note that "=" indicates equal vote totals, ">" indicates a difference of exactly one vote and "≫" a difference of two or more votes.)

Consider first the results under plurality rule. In 44 instances, one of the majority candidates drew strictly more votes than the other in the poll. In only 3 of these instances did that majority candidate fail to at least tie for victory in the subsequent election. Furthermore, in every instance, a majority candidate either finished clearly in last place in the election or was tied for last place. The minority candidate was a clear victor in only 3 elections and won 3 elections by a single vote.[14] All 6 of these elections were in early rounds (5 in the first or second round) and 3 of the 6 were elections following a tie between the majority candidates in the preceding poll.

These results accord well with Duverger's Law, which predicts that a three-candidate race will degenerate to a serious race between only two of the candidates. They also accord well with the asymmetric voting equilibria of the electoral situation studied here, which predict that the serious race will be between one of the majority candidates and the minority candidate. Both Duverger's Law and the asymmetric voting equilibria predict "no" votes for the third-place finisher. Indeed, the mean vote totals (across all 48 elections, expressed as percentages of the total votes cast) for the candidates in the elections were 51% for the candidate in first place, 39% for the candidate in second, and only 10% for the third-place finisher.

In only 4 instances were the majority candidates tied in a poll. In 3 of those 4 instances, the minority candidate won the following election. In the remaining instance, the minority candidate tied for victory. This suggests that, in the absence of a coordinating signal, the election outcome failed to satisfy Duverger's Law and instead corresponded to the symmetric voting equilibrium in which the minority candidate is the predicted victor.

Polls served as relatively good predictors of election winners under plurality rule. In only 5 instances did an election fail to yield a candidate at least tied for victory who was also at least tied for the lead in the preceding poll (and 4 of those instances occurred in the first two rounds).

Repeated game effects seem to be present in the results corresponding to the third and sixth voting groups. In both cases, the voting groups quickly established a pattern of coordinated alternation between elections of the two majority candidates.

Next, consider the results under approval voting. The minority candidate was never the clear victor and won by a single vote in only 2 elections. This candidate was the clear loser in 2 elections and lost by a single vote 3 times. The races were much closer than under plurality rule. The first-, second-, and third-place finishers in the elections drew mean vote totals of 41%, 33%, and 26% of all votes cast.

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[14] Again, we define a clear winner as one who wins by 2 or more votes.

The symmetric voting equilibrium predicts close vote totals, and fewer times that the minority candidate is in contention for victory than is either of the other candidates. Indeed, the minority candidate achieved only 14.6 victories (out of 48 chances, counting two-way ties as 1/2-victories and three-way ties as 1/3-victories).

Polls were poor predictors of election winners under approval voting. Of the 43 instances in which a single candidate led in the poll, only 23 times did that candidate win the following election outright. In 10 instances, the election yielded a (single) victor who trailed some other candidate in the preceding poll. There were 5 polls led by the minority candidate. The following elections yielded 4 losses and one tie for victory for that candidate. Obviously, poll results had an impact on voter behavior and the election outcome. However, that impact tended to invalidate the results of the polls.

Finally, consider the Borda rule results. The minority candidate was never a clear winner and won only 4 elections by a single vote. This candidate was the clear loser in 11 elections, lost by a single vote in 13 and tied for last in 11. Yet again, as predicted by the unique voting equilibrium, the races were closer than under plurality rule, with mean vote totals, listed in order of election ranking, of 37%, 33%, and 30% of all votes cast.

Of the 42 polls which yielded a leader (by 1 or more votes), only 18 were followed by elections in which the poll leader either won or tied for victory. The minority candidate was the poll leader nine times, yet only won or tied for victory in a single subsequent election.

Generally, it appears that the focal information conveyed by polls under plurality rule was information differentiating the two majority candidates, while polls under approval voting and Borda rule served primarily to determine the level of threat posed by the minority candidate, as perceived by the supporters of the majority candidates. Again, in a later section, we will show the significance of this effect.

C Test for Duverger and Condorcet Effects

Duverger's law predicts that the third place candidate will receive a much smaller percentage of the vote than the other candidates under plurality voting. This arises because of the need to form winning coalitions under this rule. It makes no such prediction for approval voting or Borda rule. Instead, we observe a tendency for close, three-way races under these rules, with third place vote percentages much closer to second (and first) place percentages. Thus, Duverger's Law implies a large spread between the second and third place vote percentages under plurality voting. Similarly, three-way close races imply a small spread between the second and third place vote percentages. Here, we present Kruskal-Wallis tests[15] to show the significance of this effect. All tests have the null hypothesis that

[15] See Conover (1980) for a description of this test and its properties.

Table IV. Kruskal-Wallis tests for duverger law effects vs close three-way races

Data Used (Observations/Rule)	Voting Rule	Average 2nd to 3rd Place Spread (%)	Rank Sum	χ^2-Stat. 2 d.o.f. (Prob > χ^2)
All Elections	Plurality	24.14	19411.50	70.673*
(96/Rule)	Approval	5.96	11819.50	0.0001
	Borda Rule	4.92	10385.00	
Last Elections in Each	Plurality	30.95	336.50	
Voting Group	Approval	7.55	204.50	17.136*
(12/Rule)	Borda Rule	2.78	125.00	0.0002
Last Elections in Each	Plurality	32.14	40.00	
Session (Cohort)	Approval	7.92	26.00	7.538*
(4/Rule)	Borda Rule	2.38	12.00	0.0231

* Reject null of identical distributions of spreads across voting rules at the 95% level of confidence.

the distributions of second to third place percentage spreads are identical across all three voting rules. We present three different tests. The first test uses all data. To avoid effects of learning and correlation within a voting group, we present a second test that uses data only from the last period of each voting group. To avoid effects of learning during a session (across groups), we present a third test that uses data only from the last election in the last voting groups (E and F) of each session. The results appear in Table IV. In all cases, the voting rule has a significant effect on spreads with plurality voting having the highest ranks, approval voting second and Borda rule third. This fits well with voters arriving at an asymmetric equilibrium under plurality voting, shuffling between the three equilibria under approval voting and finding the unique, close three-way race equilibrium under Borda rule.

Both approval voting and Borda rule were proposed as alternatives to plurality voting to avoid Condorcet losers winning in elections with exactly the type of electorate structures as we use in our experiments. Recall that, in our electorates, Blue is a Condorcet loser (i.e., Blue would lose two way races against either Orange or Green). In one equilibrium (out of three) under plurality rule, the Condorcet loser should win. In one equilibrium (out of three) under approval voting, the Condorcet loser has a small chance of winning. In the only equilibrium under Borda rule, the Condorcet loser has an even smaller chance of winning. Thus, predictions about Condorcet losers' winning frequencies depend on whether and, in the case of plurality and approval voting, which equilibria are selected by voters.

Table V shows how often the Condorcet loser won overall under each treatment. Generally, majority voters were able to coordinate sufficiently to avoid having the Condorcet loser win many elections. Nevertheless, the Condorcet loser won more often under plurality voting than the other two voting rules.

Table V. Effects of treatments on Condorcet losers' winning frequencies

	Winning Frequency of the Condorcet Loser (Blue)*	
Voting	Without Polls	With Polls
Plurality	0.2604	0.1979
Approval	0.1910	0.1458
Borda	0.0972	0.1111
Kruskal-Wallis Equality of Populations Rank Test Across Voting Rules ($\chi^2(2)$):		4.571 Prob$>\chi^2$:0.1017
Mann-Whitney Two-Sample U Test (Plurality vs Other Voting Rules):		1.85 Prob$>\|z\|$:0.0641

*Counted as follows: Outright Win = 1, Two-Way Tie for First = 1/2, Three Way Tie = 1/3.

According to a Kruskal-Wallis test, this effect is barely significant across the three types of voting rules at approximately the 90% level of confidence. If we simply ask whether plurality voting leads to Condorcet losers winning more often than the other rules, we find significance according to a Mann-Whitney, two-sample test.[16] Note also that polls decrease the frequency of Condorcet losers winning under plurality voting and approval voting. However, the frequency increases slightly with polls under Borda rule. While suggestive, we lack significance due to the small number of sessions.

D Results on Individual Voting Behavior

We classify voting behavior into four categories. A *sincere* ballot is one on which the votes cast for the candidates vary monotonically with the voter's utilities (more precisely, with equi-probable pivot probabilities). A *strategic* ballot is one consistent with *some* set of pivot probabilities. An *equilibrium-consistent* ballot is consistent with pivot probabilities supporting an equilibrium. A *dominated* ballot is one which is inconsistent and, therefore, for any set of beliefs, has another ballot which is "better." Both sincere and equilibrium-consistent are subsets of strategic. For all voter types, Table VI shows the vote vectors that are sincere, strategic, equilibrium consistent, and dominated. (Vectors show the number of votes for the Orange, Green, and Blue candidates in order.)

We begin by examining the specific ballots cast by the majority voters in the polls and elections. Table VII gives the fraction of votes cast by majority voters that are sincere, strategic, equilibrium consistent, and dominated. To aggregate across the two (essentially identical) types of majority voters, we transform the

[16] See Conover (1980) for a description of this test and its properties.

Table VI. Vote vector* classifications

Voter Type (Preference Ordering)	Vector Type	Voting Rule: Plurality	Approval	Borda Rule
	Sincere	(1,0,0)	(1,0,0),(1,1,0)	(2,1,0)
Type O	Strategic	(1,0,0),(0,1,0)	(1,0,0),(1,1,0)	(2,1,0),(2,0,1),(1,2,0)
(Orange≻ Green≻	Equilibrium Consistent	(1,0,0),(0,1,0)	(1,0,0),(1,1,0)	(2,1,0),(2,0,1)
Blue)	Dominated	(0,0,1),(0,0,0)	(1,0,1),(0,1,0),(0,0,1), (0,1,1),(1,1,1),(0,0,0)	(0,2,1),(1,0,2),(0,1,2), (0,0,0)
	Sincere	(0,1,0)	(0,1,0),(1,1,0)	(1,2,0)
Type G	Strategic	(1,0,0),(0,1,0)	(0,1,0),(1,1,0)	(2,1,0),(0,2,1),(1,2,0)
(Green≻ Orange≻	Equilibrium Consistent	(1,0,0),(0,1,0)	(0,1,0),(1,1,0)	(0,2,1),(1,2,1)
Blue)	Dominated	(0,0,1),(0,0,0)	(1,0,1),(1,0,0),(0,0,1) (0,1,1),(1,1,1),(0,0,0)	(2,0,1),(1,0,2),(0,1,2), (0,0,0)
	Sincere	(0,0,1)	(0,0,1)	(1,0,2),(0,1,2)
Type B	Strategic	(0,0,1)	(0,0,1)	(1,0,2),(0,1,2)
(Blue≻ Orange≈	Equilibrium Consistent	(0,0,1)	(0,0,1)	(1,0,2),(0,1,2)
Green)	Dominated	(1,0,0),(0,1,0), (0,0,0)	(1,0,0),(0,1,0),(1,0,1), (0,1,1),(1,1,0),(1,1,1)	(2,1,0),(2,0,1),(0,2,1), (1,2,0),(0,0,0)

* Vote vectors give the number of votes cast for the Orange, Green and Blue candidates, respectively.

vote vectors so that they give votes for the voter's most favored candidate (x), second most favored candidate (y), and least favored candidate (z) in the order $[x, y, z]$. In each case, the first-listed vote vector is "naively sincere" and the subsequently-listed vote vectors correspond to strategic actions. Under the "other" category are lumped all vote vectors which fail to be optimal with respect to any voter perceptions for the stage game. Recall that the voters were actually playing repeated games with the same sets of participants. Therefore, a ballot in the "other" category is not necessarily irrational if its casting might influence the result of subsequent elections. Still, relatively few such ballots were cast by supporters of the majority candidates.

A substantial amount of "gaming" occurred on polls under both approval voting and Borda Rule. This is not surprising. Under approval voting, a supporter of one of the majority candidates can hope to frighten voters supporting the other majority candidate into double-voting by raising the standing of the minority candidate in the preceding poll. Under Borda rule, a similar, but perhaps stronger, motivation exists. And we see, in fact, more than half of the poll ballots under Borda rule and a nearly a quarter of the poll ballots under approval

Table VII. Percentages of ballots cast by majority voters (N = 384 per cell)
(Ballots give votes for the voter's most favored candidate (x), second most favored candidate (y) and least favored candidate (z) in the order $[x, y, z]$)

Session	Vote Vector $[x, y, z]$	Percentage of Poll Ballots		Percentage of Election Ballots	
	[1,0,0]:*			60.2%	
PWOP	[0,1,0]:**	N.A.		36.5%	
	Other:****				3.4%
	[1,0,0]:*	62.2%		62.2%	
PWP	[0,1,0]:**	31.0%		36.7%	
	Other:****		6.8%		1.0%
	[1,0,0]:*			40.6	
AWOP	[1,1,0]:*	N.A.		46.4%	
	Other:****				13.0%
	[1,0,0]:*	50.5%		56.5%	
AWP	[1,1,0]:*	26.3%		36.5%	
	Other:****		23.2%		7.0%
	[2,1,0]:*			71.9%	
	[2,0,1]:**			19.3%	
BWOP	[1,2,0]:***	N.A.		6.5%	
	Other:****				2.3%
	[2,1,0]:*	32.0%		75.8%	
	[2,0,1]:**	4.9%		13.5%	
BWP	[1,2,0]:***	5.5%		7.8%	
	Other:****		57.6%		2.9%

* Sincere, Equilibrium Consistent and Strategic Vector if Cast in a Single Election.
** Equilibrium Consistent and Strategic Vector if Cast in a Single Election.
*** Strategic Vector if Cast in a Single Election.
**** Dominated Vector if Cast in a Single Election.

voting fell into the "other" class. On the other hand, the incentive under plurality rule is to signal, rather than frighten. In fact, we see few "other" ballots (i.e., votes for the minority candidate) being cast in the plurality-rule polls.

Table VIII gives the fraction of votes cast by minority voters that are sincere, strategic, equilibrium consistent, and dominated. As in Table VII, we transform the vote vectors so that they give votes in descending order of the voter's preference. (Note that the majority candidates are interchangeable for minority voters.) The frequencies of "other" vote vectors for minority voters are uniformly larger than for majority voters in elections. In contrast, when responding to polls, minority voters' vectors frequently fell into the "other" category, presumably in attempts to cloud the polls' signals to majority voters.

Table VIII. Percentages of ballots cast by minority voters (N = 288 per cell)
(Ballots give votes for the voter's most favored candidate (x), second most favored candidate (y) and least favored candidate (z) in the order $[x, y, z]$)

Session	Vote Vector $[x, y, z]$	Percentage of Poll Ballots		Percentage of Election Ballots	
PWOP	[1,0,0]:*	N.A.		87.5%	
	Other:**				12.5%
PWP	[1,0,0]:*	68.1%		94.8%	
	Other:**		31.9%		5.2%
AWOP	[1,0,0]:*	N.A.		78.1%	
	Other:**				21.9%
AWP	[1,0,0]:*	50.7%		80.6%	
	Other:**		49.3%		19.4%
BWOP	[2,1,0]:*	N.A.		81.3%	
	Other:**				18.8%
BWP	[2,1,0]:*	27.8%		95.1%	
	Other:**		72.2%		4.9%

* Sincere, Equilibrium Consistent and Strategic Vector if Cast in a Single Election.
** Dominated Vector if Cast in a Single Election.

To formally address whether voters behave as predicted by strategic equilibria, we use Selten's [1991] measure for predictive success. First, we partition the space of admissible vote vectors into those vectors explained and not explained by each type of voter behavior (sincere, equilibrium consistent, strategic and dominated). We define the area, a, predicted by each behavior type as the fraction of admissible vote vectors explained by the type of behavior. (For example, one out of four admissible vote vectors is sincere for minority voters under plurality voting, giving an area of 0.25.) Next, we define the hit rate, r, as the fraction of actual votes cast that conform to each type of behavior. Finally, we define the measure of predictive success for each behavior type according to:

$$m = r - a \tag{3}$$

We compare the measure of predictive success across types of voter behavior as suggested by Selten [1991] for predictive behavior models.[17] We also ask whether voters were significantly more likely to follow any particular behavioral pattern than predicted by random behavior. Under the (null) assumption that all admissible vote vectors were equally likely, we simply calculate the likelihood of observing a larger or smaller portion of votes within each behavior type than actually observed. We reject the null that voters were voting randomly if the likelihood of observing more (or less) than the actual portion of votes was less than 0.95. Finally, treating each session as an observation, we run a Wilcoxon Sign-Rank Test[18] and a Mann-Whitney Test to determine whether the predictive power of equilibrium consistent behavior significantly exceeded the predictive power of naively sincere behavior.

Table IX shows the measures of predictive success for majority voters while Table X shows these measures for minority voters. These tables also show whether the fraction of votes of any particular type are significantly larger or smaller than the fraction predicted by random voting. Note that, under all voting rules naively sincere, strategic and equilibrium consistent vote vectors are identical for minority voters. Thus, from Table X we only conclude that strategic voting explains minority voter behavior much better than dominated voting. Minority voters are significantly more likely to vote in a strategic manner than predicted by random behavior. They are significantly less likely to vote in a dominated manner than predicted by random behavior. Similarly, dominated voting has little predictive power for majority voters. Again, voters are significantly less likely to vote in a dominated manner and more likely to vote in naively sincere, equilibrium consistent and strategic manners than predicted by random behavior. With one exception (Borda Rule, with polls), Table IX also shows a substantial increase in the predictive power of equilibrium consistent behavior over simple naive voting. Note also, that under Borda Rule (the only voting rule for which there is a difference) the predictive power of equilibrium consistent behavior exceeds that of simple strategic behavior as well. Finally, Table XI shows Wilcoxon Signed-Rank and Mann-Whitney tests indicating that, for majority voters, equilibrium consistent behavior has significantly more predictive power than naively sincere behavior. Thus, overall, we conclude that voters do not vote in a dominated manner, nor do they simply vote naively for their favored candidates. Instead, voters cast votes strategically and in equilibrium consistent manners.

We now ask whether the results of previous polls or elections affect a voter's choice in an election. We consider here the effect of the immediately-previous "event"—a poll, if polls were conducted, and the election in the preceding round otherwise—on the choice of ballot by supporters of majority candidates. Table XII shows the effect of the previous event on majority voters in plurality voting elections. Table XIII shows the same effect in approval voting and Borda

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[17] See Selten [1991] for a discussion of the properties of this measure.

[18] See Conover [1980] for a description of this test and its properties.

Table IX. Measure of predictive success for majority voters

Session	Vote Vector Classification	Fraction of Actual Votes Cast (r)	Area Predicted (a)	Measure of Predictive Success ($m=r-a$)
	Naively Sincere	0.6016*	0.2500	0.3516
PWOP	Equilibrium Consistent	0.9691*	0.5000	0.4661
	Strategic	0.9691*	0.5000	0.4661
	Dominated	0.0339**	0.5000	−0.4661
	Naively Sincere	0.6224*	0.2500	0.3724
PWP	Equilibrium Consistent	0.9896*	0.5000	0.4896
	Strategic	0.9896*	0.5000	0.4896
	Dominated	0.0104**	0.5000	−0.4896
	Naively Sincere	0.4063*	0.1250	0.2773
AWOP	Equilibrium Consistent	0.8698*	0.2500	0.6198
	Strategic	0.8698*	0.2500	0.6198
	Dominated	0.1302**	0.7500	−0.6198
	Naively Sincere	0.5651*	0.1250	0.4401
AWP	Equilibrium Consistent	0.9297*	0.2500	0.6797
	Strategic	0.9297*	0.2500	0.6797
	Dominated	0.0703**	0.7500	−0.6797
	Naively Sincere	0.7188*	0.1429	0.5759
BWOP	Equilibrium Consistent	0.9115*	0.2857	0.6258
	Strategic	0.9766*	0.4286	0.5480
	Dominated	0.0234**	0.5714	−0.5480
	Naively Sincere	0.7578*	0.1429	0.6149
BWPP	Equilibrium Consistent	0.8932*	0.2857	0.6075
	Strategic	0.9714*	0.4286	0.5428
	Dominated	0.0286**	0.5714	−0.5428

* Behavior that occurred significantly more often than predicted by random behavior (95% level of confidence).

** Behavior that occurred significantly less often than predicted by random behavior (95% level of confidence).

rule elections. In order to most clearly illustrate the shifts in behavior, we have excluded "other" ballots from these tables.

Under plurality rule, the election results suggest that supporters of majority candidates are influenced by which of the majority candidates led in the preceding event. The data in Table XII show that, when one majority candidate led the other in the preceding event, Type O and G voters voted overwhelmingly for the leading candidate regardless of their preferences over these candidates. When the majority candidates tied on the preceding event, the majority voters tended to vote for their preferred candidate. However, they sometimes did vote for the other

Table X. Measure of predictive success for minority voters

Session	Vote Vector Classification***	Fraction of Votes Cast (r)	Area (a)	Measure of Predictive Success ($m = r - a$)
PWOP	Strategic	0.8750*	0.2500	0.6250
	Dominated	0.1250**	0.7500	−0.6250
PWP	Strategic	0.9479*	0.2500	0.6979
	Dominated	0.0521**	0.7500	−0.6979
AWOP	Strategic	0.7813*	0.1250	0.6563
	Dominated	0.2188**	0.8750	−0.6563
AWP	Strategic	0.8056*	0.1250	0.6806
	Dominated	0.1944**	0.8750	−0.6806
BWOP	Strategic	0.8125*	0.2857	0.5268
	Dominated	0.1875**	0.7143	−0.5268
BWP	Strategic	0.9514*	0.2857	0.6657
	Dominated	0.0486**	0.7143	−0.6657

* Behavior that occurred significantly more often than predicted by random behavior (95% level of confidence).
** Behavior that occurred significantly less often than predicted by random behavior (95% level of confidence).
*** Naively Sincere, Equilibrium Consistent and Strategic vote vectors are identical for minority voters under all voting rules.

Table XI. Tests for differences in explanatory power between naively sincere and equilibrium consistent behavior for majority voters

Session	Predictive Success of Naively Sincere Voting	Predictive Success of Equilibrium Consistent Voting
PWOP	0.3516	0.4691
PWP	0.3724	0.4896
AWOP	0.2813	0.6198
AWP	0.4401	0.6797
BWOP	0.5759	0.6258
BWP	0.6149	0.6075
Mann-Whitney Rank Sum (12 Obs):		26*
Wilcoxon Signed Rank Statistic (6 Prs.):		1.99*

* Significant at the 95% level of confidence in two sided tests.

Table XII. Effect of previous event on majority voters in plurality voting elections

Session	Voter Action if Either Orange or Green lead the Other in the Previous Event			Voter Action of Orange and Green Tied in the Previous Event	
	Voter Preferences	Voter Actions	Frequency	Voter Action	Frequency
PWOP (Effect of Previous Election)	Favor Leader	Vote for Leader Other Vote	158 (98.8%) 2 (1.3%)	Vote for Favorite	10 (62.5%)
	Favor Trailer	Vote for Leader Other Vote	119 (74.4%) 41 (25.6%)	Other Vote	6 (37.5%)
PWP (Effect of Previous Poll)	Favor Leader	Vote for Leader Other Vote	172 (97.7%) 4 (2.3%)	Vote for Favorite	22 (68.8%)
	Favor Trailer	Vote for Leader Other Vote	131 (74.4%) 45 (25.6%)	Other Vote	10 (31.3%)

candidate, possibly still attempting to avoid Borda's feared outcome even without the coordinating signal. A much more detailed investigation of poll and history effects under plurality voting, as well as the effect of ballot position, appears in Forsythe, Myerson, Rietz, and Weber [1993].

The data in Table XIII show that, under approval voting and Borda rule, the most influential aspect of the preceding event appears to be whether or not the minority candidate either leads or is tied for the lead. Approval double-voting was significantly more common when the minority candidate led and Borda vote-dumping (casting the single vote for one's least favored candidate) was significantly more common when that candidate trailed. Under approval voting, significantly more majority-candidate supporters double-voted when the minority candidate led, than they did when that candidate trailed the leader on the preceding event (although there was generally more double-voting in the absence of polls). Under the Borda system, significantly more vote-dumping occurred when the minority candidate trailed in the preceding event. Switching was relatively rare in all cases. Note that switching is reasonable only if $p_{OG} \geq 9/11$, i.e., a majority voter's perception is that the minority candidate has little chance of being in contention for victory.

V Conclusions and Discussion of Further Research

We reported the results of a series of three-candidate experimental elections. We argue the (single) parameter set we use is particularly interesting, having inspired scholarly debate for more than 220 years. Using these elections, we studied a variety of issues both at the aggregate outcome and individual behavior levels. Under three different voting rules, we asked how often Condorcet losers won elections, how often Duverger's law appeared to hold, whether particular game-theoretic equilibria were helpful when characterizing observed behavior, how polls and repeated elections affected outcomes, and whether individuals responded to polls and previous elections in predicted ways. Here, we summarize our results by voting rule starting with the most familiar, plurality rule.

Under plurality rule, the data from almost all groups was consistent with Duverger's law. Duverger effects arose significantly more often under plurality voting than under approval voting and Borda rule (which usually resulted in close three-way races). These effects came from a high degree of coordinated strategic voting. Such a level of coordination was not immediate. It often took several periods for a cohort to arrive at an equilibrium in which the Condorcet loser indeed lost. Thus, Condorcet losers won significantly more often under plurality voting than under the other two rules. However, support for the coordinated equilibria became stronger in later periods of each repeated election series with fixed electorates and candidates. (Of the 14 elections which do not support a coordinated equilibrium, only 3 were in the second half of a repeated election series.) Once established, the equilibrium selection process was simple: Most Type O and G voters used the previous poll or election ranking to decide

Table XIII. Effects of previous event on majority voters in approval voting and Borda Rule elections (Vote vectors for the voter's most favored candidated (x), second most favored candidate (y) and least favored candidate (z) in the order $[x, y, z]$.)

Session	Voting Rule	Minority Candidate Position	Majority Voter Actions: Vote Cast	Majority Voter Actions: Frequency	χ^2 Test for Independence of Vote Vector and Minority Candidate Position: χ^2 Stat.	dof	Pr[$\chi^2 > \bullet$]
AWOP (Effect of Previous Election)	Approval	Leading	[1,1,0]	50 (63.3%)	6.1612*	1	0.013
			[1,0,0]	29 (36.7%)			
		Trailing	[1,1,0]	100 (47.0%)			
			[1,0,0]	113 (53.0%)			
AWP (Effect of Previous Poll)	Approval	Leading	[1,1,0]	32 (49.2%)	4.2798*	1	0.039
			[1,0,0]	33 (50.8%)			
		Trailing	[1,1,0]	87 (35.2%)			
			[1,0,0]	160 (64.8%)			
BWOP (Effect of Previous Election)	Borda Rule	Leading	[2,1,0]	72 (83.7%)	9.5174*	2	0.009
			[2,0,1]	7 (8.1%)			
			[1,2,0]	7 (8.1%)			
		Trailing	[2,1,0]	204 (70.6%)			
			[2,0,1]	67 (23.2%)			
			[1,2,0]	18 (6.2%)			
BWP (Effect of Previous Poll)	Borda Rule	Leading	[2,1,0]	95 (87.2%)	7.8210*	2	0.020
			[2,0,1]	10 (9.2%)			
			[1,2,0]	4 (3.7%)			
		Trailing	[2,1,0]	196 (74.2%)			
			[2,0,1]	42 (15.9%)			
			[1,2,0]	26 (9.8%)			

* Significant at the 5% level.

who to vote for in the next election. Polls assisted with the coordination, seeming to provide Type O and G voters with clear signals about the candidates for whom they should vote. In contrast to poll accuracy under the other two rules, polls correctly forecast, not only the winner, but also, the exact ranking of the candidates the majority of the time under plurality rule. Also unlike the other two rules, plurality voting did not provide voters with apparent incentives to misrepresent their intentions in the polls. This resulted in a higher rate of truthful polling.

Under approval voting, outcomes usually were most consistent with close three-way races in which the Condorcet loser, Blue, won less often than the other two candidates. In the session without polls, voters appeared to act as if they generally expected this equilibrium to arise. Voters also seemed to respond to polls in a manner that supports this equilibrium. In particular, Type O and G voters tended to cast votes for their second favorite candidate if Blue was strictly ahead of their favorite candidate in the previous poll or election. Voters seemed aware of this response pattern. It implies that Type B voters would like Blue to finish last while Type O and G voters would like Blue to finish second or better. The first case leads Type O and G voters to cast votes only for their favorite candidates, allowing Blue to win. The latter case will result in more votes for Orange and Green, reducing Blue's chances of winning. Thus, voters of each type have incentives to misrepresent their preferences in the poll or abstain completely from it. Further, after being behind in the poll, Blue should rise in the standings and vice versa. The data support these results with voters abstaining or casting poll responses for their least favored candidate over 1/3 of the time. Further, out of 5 times that Blue won the poll, Blue won no elections outright and tied with Green once. Out of the 19 times that Blue lost the poll, Blue lost the election by more than two votes only twice, lost by one vote only 3 times, and won or tied for first 3 times.

Under Borda rule, outcomes were generally consistent with the equilibrium prediction of close three-way races in which the Condorcet loser, Blue, won less often than Orange or Green. We argued that simple best response behavior required that Type O and G voters would cast their "1" vote for Blue if Blue finished last in the previous balloting (either election or poll). As with approval voting, this behavior creates an incentive for Type B voters to vote so that their most preferred candidate finishes last in the previous poll. Type B voters tried to accomplish this by abstaining from polls an extraordinary amount (more than 2/3) of the time. This abstention strategy was apparently transparent to Type O and G voters who seemed to disregard poll results when voting. In the polls, Blue finished last in 26 of the 48 polls conducted, but was only able to win or tie for first in 4 of these instances.

We view this as an initial study of three-candidate elections rather than a in-depth study of particular issues that arise in these elections. Future research will examine the robustness of these results by examining different sets of preferences, different voting group sizes, and a wider range of voting rules. Nevertheless, the evidence already clearly shows a tendency for voters to cast strategic, equilibrium consistent vote vectors. This behavior gives rise to Duverger's law type effects under plurality voting and close, three-way races under approval voting and Borda rule in the elections we study.

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