Financial Engineering and the Japanese Markets **1**: 55–66, 1994.

Estimating Unknown Join Points: Determination of the Yen-Dollar Exchange Rate

HIROKI TSURUMI
Department of Economics, Rutgers University, New Brunswick, N.J. 08903 U.S.A.

and

CHYONG LIN CHEN
Department of Economics, Feng Chia University, Taiwan

Abstract. We propose a Bayesian procedure to estimate a switching regression in which the number of switching points (i.e. join points) is not known. We apply the Bayesian procedure to a regression model for the yen-dollar exchange rate using monthly data from January 1973 to June 1992. We identify three join points in January 1978, September 1988, and March 1990. We compare the post-sample forecast performances of our switching regression model to those of other regression models. The post-sample forecasts show that the Bayesian switching model performs better than the other models.

Key words. Bayesian procedure, join point, switching regression, ARCH, GARCH, cointegration, error correction.

1. Introduction

In this paper we first introduce a Bayesian procedure to estimate unknown join points of the switching regression model in which the heteroscedastic error term follows a first order autoregressive process. We apply the procedure to the monthly data on the yen-dollar exchange rate from January 1973 to June 1992. We compare the Bayesian model to other regression models that are often used in financial studies: ARCH-M(1), GARCH-M(2, 1), and error correction model (ECM). A post sample forecast exercise shows that the Bayesian switching regression performs better than the other models.

The organization of the paper is as follows. In section 2 we derive a Bayesian procedure to estimate unknown join points. In section 3 after a brief discussion on the existing empirical studies of exchange rate determination, we apply the Bayesian switching regression model to the monthly data on the yen-dollar exchange rate. ARCH-M(1), GARCH-M(2, 1) and ECM are given in section 4, and in section 5 we present a post sample forecast exercise.

2. Bayesian Estimation of Unknown Join Points

Suppose that there is one join point, t^*, in the sample. Then the switching regression model in which the heteroscedastic error term follows the first order autoregressive process may be specified by

$$y_t = x_t\beta_1 + u_t, \tag{1}$$

$$u_t = \rho_1 u_{t-1} + \varepsilon_t \quad t = 1,\dots,t^* \tag{2}$$

$$y_t = x_t\beta_2 + u_t \tag{3}$$

$$u_t = \rho_2 u_{t-1} + \varepsilon_t \quad t = t^*+1,\dots,n \tag{4}$$

$$\varepsilon_t \sim \mathrm{N}(0,\sigma_1^2), \quad \text{for} \quad t = 1,\dots,t^* \tag{5}$$

$$\varepsilon_t \sim \mathrm{N}(0,\sigma_2^2), \quad \text{for} \quad t = t^*+1,\dots,n \tag{6}$$

where $x_t = (x_{t1},\dots,x_{tk})$ and β_i is a $k \times 1$ vector of regression coefficients ($i = 1, 2$). Equations (1)–(4) may be written as

$$\begin{aligned} y_t - \rho_1 y_{t-1} &= (x_t - \rho_1 x_{t-1})\beta_1 + (u_t - \rho_1 u_{t-1}), \quad t = 1,\dots,t^* \\ y_{t^*+1} - \rho_2 y_{t^*} &= (x_{t^*+1} - \rho_2)\beta_1 + x_{t^*+1}\delta + (u_{t^*+1} - \rho_2 u_{t^*}), \quad t = t^*+1 \\ y_t - \rho_2 y_{t-1} &= (x_t - \rho_2 x_{t-1})\beta_1 + (x_t - \rho_2 x_{t-1})\delta + (u_t - \rho_2 u_{t-1}), \\ & \qquad t = t^*+1,\dots,n \end{aligned}$$

or in matrix form

$$\begin{aligned} Z &= X\beta_1 + V\delta + \varepsilon \\ &= W\zeta + \varepsilon \end{aligned} \tag{7}$$

where

$$Z = (Y_1', \omega Y_2')', \quad Z = (X_1', \omega X_2')', \quad V = (0', \omega \bar{X}_2')', \quad W = (X, V),$$
$$\zeta = (\beta_1', \delta')', \quad \delta = (\beta_1' - \beta_2')', \quad \omega = \sigma_1/\sigma_2,$$

$$Y_1 = \begin{bmatrix} y_1 - \rho_1 y_0 \\ \vdots \\ y_{t^*} - \rho_1 y_{t^*-1} \end{bmatrix}, \quad Y_2 = \begin{bmatrix} y_{t^*+1} - \rho_2 y_{t^*} \\ \vdots \\ y_n - \rho_2 y_{n-1} \end{bmatrix}, \quad X_1 = \begin{bmatrix} x_1 - \rho_1 x_0 \\ \vdots \\ x_{t^*} - \rho_1 x_{t^*-1} \end{bmatrix}$$

$$X_2 = \begin{bmatrix} x_{t^*+1} - \rho_2 x_{t^*} \\ x_{t^*+2} - \rho_2 x_{t^*+1} \\ \vdots \\ x_n - \rho_2 x_{n-1} \end{bmatrix}, \quad X_2 = \begin{bmatrix} x_{t^*+1} \\ x_{t^*+2} - \rho_2 x_{t^*+1} \\ \vdots \\ x_n - \rho_2 x_{n-1} \end{bmatrix}, \quad \varepsilon = \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \vdots \\ \varepsilon_n \end{bmatrix}$$

and $\varepsilon \sim \mathrm{N}(0, \sigma_1^2 I_n)$.

Let the prior pdf be given by

$$\mathrm{p}(t^*, \beta_1, \delta_\omega, \sigma_1, \rho_1, \rho_2) \propto \sigma_1^{-1}\omega \tag{8}$$

After integrating out β, δ, and σ_1, we have the joint posterior pdf for t^*, ρ_1, ρ_2, and ω:

$$\mathrm{p}(t^*, \rho_1, \rho_2, \omega \mid \text{data}) \propto |W'W|^{-1/2} \omega^{n_2-1} (vs^2)^{-v/2} \tag{9}$$

where $n_2 = n - t^*, v = n - 2k$, and $vs^2 = Z'[I - W(W'W)^{-1}W']Z$. The marginal posterior pdf of t^*, ρ_1, ρ_2, and ω may be obtained by numerical integration. As the point estimate of the join point t^*, let us use the posterior mode, $\hat{t}^*$.

There is no guarantee that $\hat{t}^*$ lead to the rejection of the null hypothesis H:

$$\mathrm{H}: \delta = 0 \text{ versus } \mathrm{K}: \delta \neq 0$$

and thus we may test the hypotheses by the highest posterior density region (HPDR) for δ conditioned on $\hat{t}^*, \hat{\rho}_1, \hat{\rho}_2$ and $\hat{\omega}$:

$$\mathrm{p}(\delta \mid \hat{t}^*, \hat{\rho}_1, \hat{\rho}_2, \hat{\omega}) \propto \left[1 + \frac{(\delta - \hat{\delta})'V'M_x V(\delta - \hat{\delta})}{vs^2}\right]^{-(n-k)/2} \tag{10}$$

where $\hat{\delta} = (V'M_x V)^{-1} V'M_x Z$, and $M_x = I - X(X'X)^{-1}X'$. If we set $\delta = 0$, then $\hat{\delta}'V'M_x V\hat{\delta}/vs^2$ is distributed conditionally on $\hat{t}^*, \hat{\rho}_1, \hat{\rho}_2$, and $\hat{\omega}$ as $\mathrm{F}(k, n - 2k)$ *a posteriori*. We take $\hat{a}$ to be the posterior mode of a.

We proceed as follows:

(1) For the entire sample period, find $\hat{t}^*$, and test the null hypothesis H by the F test.
(2) If we reject the null hypothesis H, then we declare $\hat{t}^*$ as a join point, $\hat{t}_1^*$.
(3) For the sample period from 1 to $t^* - 1$, we repeat Steps (1) and (2). Also for the sample period from $t^* + 1$ to n we repeat Steps (1) and (2), and find join points, $\hat{t}_2^*, \ldots, \hat{t}_r^*$
(4) Using any *a priori* qualitative information that may explain why a join point occurred, we evaluate each of the join points, $\hat{t}_j^*$, $j = 1, \ldots, r$. If we cannot explain a join point, then we may discard it as a statistically significant but economically meaningless join point.

Item (4) above indicates that a search of an unknown number of join points should not be left as a mechanical exercise. It should be a combination of a statistical procedure and qualitative economic judgment.

If the number of join points is known, the estimation of the join points is easier than the case when it is not known. The Bayesian procedures for estimating one join point are available in the literature. See Tsurumi (1988), for example. The procedure for estimating the unknown number of join points, $t_1^*, t_2^*, \ldots, t_r^*$, that is proposed in this paper is new. Our procedure is a conditional one in the sense that given $\hat{t}_1^*$, we estimate $\hat{t}_2^*$, and $\hat{t}_3^*$ is estimated given either $\hat{t}_1^*$, or $\hat{t}_2^*$. Kashiwagi (1991) proposes an unconditional posterior probabilities of join points. Since the number of combinations of possible join points increases in factorial fashion as the sample size increases, he proposes an approximation procedure, but its computational burden increases as the number of structural changes and sample size increase.

3. Application of the Bayesian Estimation Procedure to the Foreign Exchange Rate of Yen

Let us apply the Bayesian procedure explained in the previous section to the monthly data on the foreign exchange rate of yen from January 1973 (1973.01) to June 1992 (1992.06). Before we do so, we present a brief discussion on the empirical studies of foreign exchange rates.

Empirical studies of foreign exchange rates may be classified into three groups. The first group consists of the studies that analyze time series data on foreign exchange rates either to fit distribution functions or to test random walk hypotheses. The main point of interest is to analyze time series *per se* without examining the relationship between a set of variables and a foreign exchange rate. Boothe and Glassman (1987), for example, fit three non-normal probability density functions (pdf's) to daily data on the changes in logarithms of exchange rates and conclude that there is evidence that distribution parameters vary over time. Westerfield (1977) and So (1987) also fit time series data on foreign exchange rates to some pdf's. Hakko (1986) argues that one of the reasons why the tests of random walk of foreign exchange rates have yielded mixed results is that the tests on a random walk have low power. Kariya and Matsue (1989) test to see whether daily and weekly foreign exchange rates follow a random walk. They use nonlinear conditional heteroscedastic variance models of Taylor (1986) and find that daily exchange rates follow a stochastic trend, and that Monday rates follow a random walk while other weekly rates follow autoregressive processes.

The second group of studies consists of regression models of foreign exchange rates. Based on economic theory a relationship between a set of regressors and a foreign exchange rate is specified and estimated. Comprehensive surveys are given in Levich (1985) and Isard (1988). Many regression models of foreign exchange rates use some form of monetary theory, and depending on how it is used they may be labeled as the *flexible-price monetary models* (Frenkel, 1976; Bilson, 1978; 1979) *sticky-price monetary models* (Dornbusch, 1983), and *sticky-price asset models* (Frenkel, 1983; Hooper and Morton, 1982). Kariya and Fukao (1988) develop a *rational expectation model* based on a simultaneous equations system.

The third group of studies is the combination of the first two groups: within regression framework random walk or cointegration tests are carried out or regression models are specified in state space form in which regression coefficients follow a random walk. Enders (1988), Bleaney (1992), and Pippenger (1993), among others, test the purchasing power parity under fixed and flexible exchange rate regimes and show that the regression error processes have unit roots. Baille and Selover (1987) estimate the flexible monetary model of Frenkel (1976) and sticky price monetary model of Dornbusch (1983) in addition to other monetary models and conclude that many of the regression coefficients are significant but the sign and magnitude are different from what one expects from theory and that the error terms tend to have unit roots. They state that the regression models do not exist as a stable long-run relationship. State space models of exchange rates have been made by Wolff (1987) and Fukao (1988), among others. Wolff used the Kalman filter to improve the predictive performance of monetary exchange rate models. Fukao, on the other hand, used the Kalman filter to trace the changes in the regression parameters associated with interest rate differentials and trade balances. He concludes that the importance of the interest rate differential has increased while the influence of the trade balances on the foreign exchange rate has been reduced in recent years relecting the internationalization of the major financial markets of the world.

More recently, the autoregressive conditional heteroscedastic (known as ARCH and GARCH) models in which the variance of the error term is specified by an autoregressive or autoregressive and moving average process have been applied to analyze primarily the volatility or risk premium of the foreign exchange rates. The papers by Lastrapes (1989), Deibold and Nerlov (1989), Ballie and Bollerslev (1990), and Kroner and Lastrapes (1993) are notable examples. Cointegration and error correction procedures have recently become popular among econometric time series analysts, and they have been applied to foreign exchange rate models. Some representative papers are Copeland (1991), Bleaney (1992), Clifarelli (1992), Booth and Chowdhury (1992), and McNown and Wallace (1992).

Common findings among the three groups of studies appear to be that the parameters of the distributions of the foreign exchange rates are not stable and they appear to change over time. In short, the foreign exchange rates exhibit characteristics of nonstationarity. One way of modeling a non-stationary process is to use, after confirming unit roots, cointegrated error correction models, or to specify a regression model with changes in regression coefficients over time. We introduce a switching regression model with a heteroscedastic and autocorrelated error term. The switching regression experiences regime changes at unknown join points.

The regression model we consider is

$$\begin{aligned} s_t = \beta_1 + \beta_2(m_t - m_t^*) + \beta_3(y_t - y_t^*) \\ + \beta_4(r_t - r_t^*) + \beta_5(p_t - p_t^*) + \beta_6 f_t + u_t \end{aligned} \tag{11}$$

where

s_t = natural logarithm of the nominal exchange rate at time t

m_t = natural logarithm of money supply at time t

y_t = natural logarithm of output at time t

r_t = natural logarithm of the short-term interest rate at time t

p_t = natural logarithm of the wholesale price index at time t

f_t = natural logarithm of the foreign currency reserves at time t

and asterisks denote foreign quantities. The data sources are explained in the appendix. Equation (11) is assumed to change regimes at unknown join points.

The model specification in (11) is similar to the one used by Ballie and Selover (1987). Their model does not include f_t, and uses the long-term interest rate as a proxy for the expected rate of inflation. In general we would expect that $\beta_2 > 0, \beta_3 < 0, \beta_5 > 0$, and $\beta_6 < 0$. The sign of β_4 is unclear since if one follows the Keynesian persuasion a rise in the domestic interest rate leads to a currency appreciation, implying $\beta_4 < 0$. However, if a rise in the domestic interest rate is due to inflationary expectation, then $\beta_4 > 0$. If one follows the sticky price model, then $\beta_4 = 0$ and $\beta_5 > 0$. (See Dornbusch, 1976) Mishkin (1992) and Frenkel (1993) give comprehensive discussions on the exchange rate determination. The logarithm of the foreign currency reserves, f_t, is included in equation (11) as a proxy for the accumulation of trade balances which will exert a pressure on the appreciation of yen. One could compute the accumulation of trade balances given an initial value, but since the data on the foreign currency reserves are available, we use the available data rather than generating the data on the cumulative trade balances.

The Bayesian procedure yields the estimates of three join points: $\hat{t}_1^* = 78.01$ ($F = 8.39$), $\hat{t}_2^* = 88.09$ ($F = 6.16$), and $\hat{t}_3^* = 90.03$ ($F = 8.39$), where ($F = \cdot$) denotes the F statistic. Table I presents the regression coefficients and their standard errors obtained conditionally on the posterior modes of $\hat{\rho}_j$, and $\hat{\omega}_j$.

The results in Table I show that the regression coefficients, the coefficient of the AR(1) process, and the ratio of standard deviations tend to change from one regime to another, and among the regression coefficients, β_6 is consistently negative and highly significant. The plots of the logarithm of the spot exchange rate, LEXCH, and the logarithm of the foreign exchange reserves, LFRSV, against time are given in

Table I. Regression results for four regimes

Coefficient	73.01–78.01	78.02–88.09	88.10–90.03	90.04–92.06
β_1	9.473	8.073	7.215	9.961
	(0.535)	(0.699)	(4.746)	(1.841)
β_2	0.496	0.208	0.600	0.090
	(0.115)	(0.188)	(0.743)	(0.111)
β_3	0.019	0.142	1.611	−0.266
	(0.158)	(0.271)	(0.852)	(0.277)
β_4	0.045	−0.068	0.126	−0.129
	(0.025)	(0.036)	(0.175)	(0.091)
β_5	−0.243	0.759	3.22	2.154
	(0.181)	(0.468)	(1.900)	(0.543)
β_6	−0.422	−0.259	−0.216	−0.360
	(0.058)	(0.070)	(0.422)	(0.170)
ρ	0.620	0.946	0.306	0.887
σ_1, ω	$\sigma_1 = 0.018$	$\omega = 0.528$	$\omega = 1.214$	$\omega = 1.505$
R^2	0.925	0.961	0.852	0.931

The figures in parentheses are standard errors.
The regression is estimated given $\hat{\rho}_j$, and $\hat{\omega}_j$.

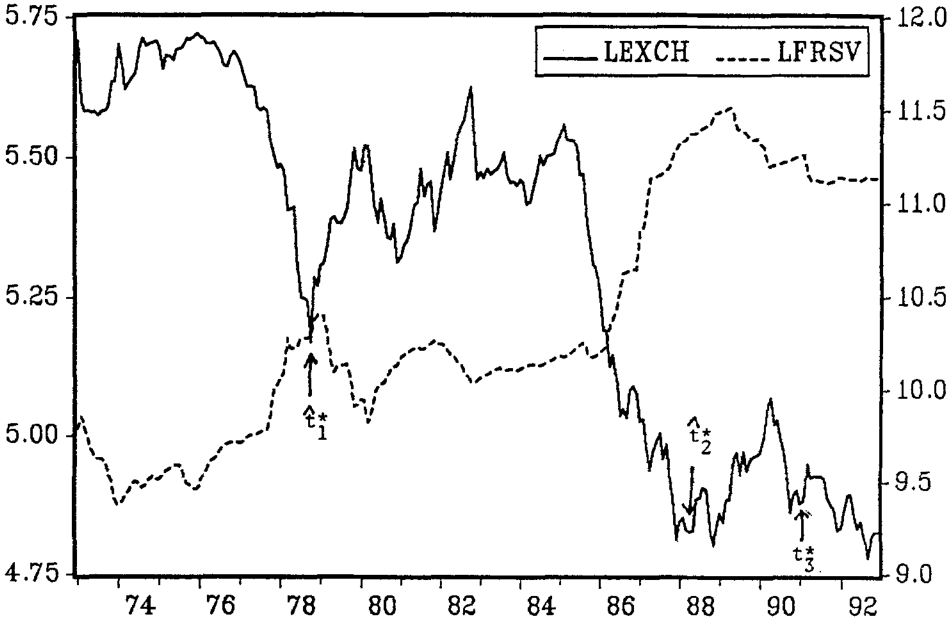

Fig. 1. Log of exchange rate (LEXCH) and log of foreign currency reserves (LFRSV) 1973.01–1992.12.

Fig. 1. We observe that the first two join points occurred after sharp increases in the foreign exchange reserves.

4. ARCH, GARCH, and Error Correction Models of the Foreign Exchange Rate of Yen

The switching regression model in the preceding section is one way of taking care of nonstationary time series data, but there are other models to handle nonstationary processes. Among the most notable of these models are the ARCH, GARCH and error correction (EC) models. Let us estimate these models using the specification of equation (11) and compare the post sample prediction performances of the different models.

Instead of the straight ARCH and GARCH models, let us estimate ARCH-M (ARCH-in-Mean) and GARCH-M (GARCH-in-Mean) models in which a function of the conditional variance is included as an additional regressor to represent a time varying risk premium.

An ARCH-M(p) model is given by

$$s_t = \beta_1 + \beta_2(m_t - m_t^*) + \beta_3(y_t - y_t^*) + \beta_4(r_t - r_t^*) + \beta_5(p_t - p_t^*) + \beta_6 f_t + \gamma g(h_t) + u_t \tag{12}$$

$$u_t \sim \mathrm{N}(0, h_t) \tag{13}$$

$$h_t = \alpha_0 + \sum_{i=1}^{p} \alpha_i u_{t-i}^2 \tag{14}$$

while a GARCH-M(p, q) model is given by (12), (13), and the h_t specified by

$$h_t = \alpha_0 + \sum_{i=1}^{p} \alpha_i u_{t-i}^2 + \sum_{j=1}^{q} \phi_j h_{t-j} \tag{15}$$

As possible candidates for $g(h_t)$, often $g(h_t) = \log h_t$ and $g(h_t) = \sqrt{h_t}$ are chosen. Let us use the AIC criterion to choose one of these two specifications of $h(h_t)$ as well as the orders of the lag lengths p and q in (14) and (15). The AIC indicates that the ARCH-M(1) with $g(h_t) = \log h_t$ is preferred. As for the GARCH-M(p, q) models, the AIC indicates that the GARCH(2, 1) with $g(h_t) = \log h_t$ is preferred.

Table II reports the estimated results for the ARCH(1) and GARCH(2, 1) models. We also include the results for the AR(1) model, i.e. the regression model with an AR(1) error process. All these models are estimated by the maximum likelihood procedures.

Table II. AR(1), ARCH-M(1), and GARCH-M(2, 1) Models, 1973.01–1992.06

Coefficient	AR(1)	ARCH-M(1)	GARCH-M(2, 1)
β_1	8.789	9.249	9.256
	(0.455)	(0.109)	(0.123)
β_2	0.097	0.023	0.024
	(0.096)	(0.034)	(0.072)
β_3	0.156	0.397	0.461
	(0.147)	(0.100)	(0.149)
β_4	−0.068	−0.084	−0.091
	(0.024)	(0.010)	(0.014)
β_5	0.448	0.564	0.629
	(0.172)	(0.073)	(0.135)
β_6	−0.332	−0.373	−0.371
	(0.046)	(0.011)	(0.014)
γ		−0.006	−0.006
		(0.002)	(0.002)
AR(1)	0.917		
	(0.023)		
α_0		0.00031	0.00052
		(0.00008)	(0.00016)
α_1		1.1016	1.0762
		(0.1470)	(0.1496)
α_2			−0.6855
			(0.3264)
ϕ_1			−0.6136
			(0.2840)

The figures in parentheses are standard errors.
γ is the coefficient of $g(h_t) = \log h_t$ in (12).

The results in Table II show that all three models yield more or less similar regression coefficient estimates; in particular the estimated coefficients of the ARCH-M(1) and GARCH-M(2, 1) models are very close to each other, except the estimates of β_2.

Let us turn to an error correction model (ECM). An error correction model arises when a combination or combinations of difference stationary (often denoted as $I(1)$) variables yields a stationary process. The six variables in equation (11) are first tested by the augmented Dickey-Fuller (ADF) test as well as by the 95% highest posterior density interval (HPDI) test for a unit root to see whether they are $I(1)$ variables or not. The HPDI test for a unit root is discussed in Tsurumi and Wago (1993). Table III presents the results. Judged both by the ADF and HPDI tests, we concude that all of the six variables are $I(1)$ processes.

Given that all the six variables are $I(1)$, we proceed to carry out cointegration tests by Johansen's canonical correlation tests (Johansen 1988; 1991) and by Bayesian HPDI tests for singular values (Tsurumi and Wago, 1993). In the literature the ADF test is often used as a test of cointegration. However, since the test of cointegration is a simultaneous test, it is preferable to use the canonical correlation or the HPDI tests. Table IV presents the results of the cointegration tests.

The results in Table IV show that according to Johansen's test, there are two cointegrating relationships. According to the 95% HPDI for a singular value, there are two nonzero singular values, indicating that there are two cointegrating relationships. Hence, we conclude that a cointegrating relationship exists. We shall set up a vector autoregressive model as the error correction model:

$$\Delta y_t = \Gamma + \Pi y_{t-1} + B_1 \Delta y_{t-1} + \cdots + B_{p-1} \Delta y_{t-p+1} + v_t \tag{16}$$

where $y_t = (s_t, m_t - m_t^*, y_t - y_t^*, r_t - r_t^*, p_t - p_t^*, f_t)'$, and $\Delta y_t = y_t - y_{t-1}$. Γ is a 6×1 vector of constant terms.

Table III. Augmented Dickey-Fuller (ADF) and Highest Posterior Density Interval (HPDI) tests of a unit root

Variables	ADF t-statistic	95% HPDI
s_t	−0.703	(0.94, 1.04)
$m_t - m_t^*$	−0.720	(0.93, 1.05)
$y_t - y_t^*$	−0.424	(0.95, 1.03)
$r_t - r_t^*$	−2.218	(0.93, 1.03)
$p_t - p_t^*$	−0.359	(0.98, 1.01)
f_t	−1.416	(0.97, 1.02)

The lag length is set at 10.
The critical values are

−3.459 for 1%; −2.874 for 5%, and −2.573 for 10%.

95%HPDI = 95% highest posterior density interval (see Tsurumi and Wago (1993)).

Table IV. Cointegration tests

Johansen's Test		95% HPDI for singular values	
# of coint. vec	Test statistic	Singular value	95% HPDI
0	117.00**	λ_1	(0.22, 0.47)
1	69.95*	λ_2	(0.10, 0.33)
2	39.25	λ_3	(0.0, 0.17)
3	19.99	λ_4	(0.0, 0.10)
4	6.78	λ_5	(0.0, 0.03)
5	0.25	λ_6	(0.0, 0.01)

** = significant at 1% level
* = significant at 5% level
95% HPDI = 95% highest posterior density interval for a singular value (see Tsurumi and Wago (1993)).

5. Post Sample Forecast

Since one of the primary reasons for estimating time series models of the yen-dollar exchange rate is to predict future values of the exchange rate, let us conduct a post sample prediction exercise for six months beyond the end of the sample period: 1992.07–1993.12. Table V presents the forecast errors for each of the switching regression, AR(1), ARCH-M(1), GARCH-M(2, 1), and error correction models. We observe from the table that the forecast performance ranking is the switching regression, ECM, AR(1), ARCH-M(1), and GARCH-M(2, 1) models.

Appendix: Data Sources

s = logarithm of the yen price of the U.S. dollar (spot rate), taken from the *Economic Statistics Monthly*, Bank of Japan.

Table V. Comparison of the post sample forecast errors

Month	Actual s_t	Forecast errors				
		AR(1)	SRM	ARCH-M(1)	GARCH-M(2, 1)	ECM
1992.07	4.846	0.013	0.018	−0.040	−0.070	0.020
0.08	4.811	−0.016	−0.008	−0.060	−0.086	0.005
0.09	4.781	−0.047	−0.033	−0.096	−0.129	−0.026
0.10	4.813	−0.023	0.007	−0.047	−0.076	0.011
0.11	4.826	−0.015	0.004	−0.040	−0.065	0.029
0.12	4.826	−0.020	0.001	−0.039	−0.063	0.034
RMSEF		0.025	0.016	0.057	0.084	0.023

s_t = logarithm of the yen-dollar exchange rate
AR(1) = AR(1) model
SRM = switching regression model
ECM = error correction model
RMSEF = root mean squared errors of forecast.

$m - m^*$ = logarithm of the ratio of the Japanese money supply to the U.S. money supply. Money supplies are seasonally adjusted M1 figures. The U.S. money supply data are taken from the *Citi Base data* and the Japanese money supply data are from the *Economic Statistics Monthly*, Bank of Japan.

$y - y^*$ = logarithm of the ratio of Japanese to U.S. real income. Seasonally adjusted industrial production indexes are used for real income, taken from the *Citi Base data*, and *Economic Statistics Monthly*, Bank of Japan.

$r - r^*$ = logarithm of short-term interest rate differential. r is the call rate and r^* is the U.S. treasury bill rate, taken from *Citi Base data* and *Economic Statistics Monthly*, Bank of Japan.

$p - p^*$ = the logarithm of the ratio of the Japanese wholesale price index to the U.S. wholesale price index, seasonally adjusted, taken from the *Citi Base data*, and *Economic Statistics Monthly*, Bank of Japan.

f = the logarithm of the foreign currency reserves, seasonally adjusted, taken from the *Economic Statistics Monthly*.

References

Ballie, R.T. and D.D. Selover, (1987), 'Cointegration and model of exchange rate determination', *International Journal of Forecasting*, **3**, 43–51.

Ballie, R.T. and Bollerslev, T. (1990), 'A multivariate generalized ARCH approach to modeling risk premia in forward foreign exchange rate markets', *Journal of International Money and Finance*, **9**, 309–324.

Bleaney, M. (1992), 'A test of long-run purchasing power parity, using annual data for seven countries, 1900–88', *Economic Internazionale*, **45**, 180–196.

Bilson, J.F.O. (1978), 'Rational expectations and the exchange rates', J. Frenkel and H. Johnson (eds.) *The Economics of Exchange Rates*, Addison-Wesley, Reading, Mass.

Bilson, J.F.O. (1979), 'The Deutsch mark/dollar rate – a monetary analysis', Brunner, K. and Meltzer A.H. (eds.) *Policies for Employment, Prices and Exchange Rates*, (North-Holland, Amsterdam).

Booth, G.G. and Chowdhury, M. (1992), 'Canadian foreign exchange policies: intervention, control, cointegration', *Weltwirtschaftliches Archiv*, b128, h2, 21–33.

Boothe, P. and Glassman, D. (1987), 'The statistical distribution of exchange rates', *Journal of International Economics*, **22**, 297–319.

Cifarelli, G. (1992), 'Exchange rate market efficiency tests and cointegration analysis', *Economia Internazionale*, 45, 197–208.

Copeland, L.S. (1991), 'Cointegration tests with daily exchange data', *Oxford Bulletin of Economics and Statistics*, **53**, 2, 185–198.

Diebold, F.X. and Nerlove, M. (1989), 'The dynamics of exchange rate volatility: a multivariate latent factor ARCH model', *Journal of Applied Econometrics*, **4**, 1–21.

Dickey, D.A. and Fuller, W.A. (1981), 'Distribution of the estimators for autoregressive time series with a unit root', *Journal of the American Statistical Association*, **74**, 427–431.

Dornbusch, R. (1976), 'Expectations and exchange rate dynamics', *Journal of Political Economy*, **84**, 1161–1176.

Dornbusch, R. (1983), 'Exchange rate economics: Where do we stand?', Bhandari, J.S. and Putnam B.H. (eds.) *Economic Interdependence and Flexible Exchange Rates*, (the MIT Press, Cambridge Mass.).

Enders, W. (1988), 'ARIMA and cointegration tests of PPP under fixed and flexible exchange rate regimes', *Review of Economics and Statistics*, **55**, 251–276.

Frenkel, J.A. (1979), 'On the mark: a theory of floating exchange rates based on real interest differentials', *American Economic Review*, **69**, 610–622.

Frenkel, J.A. (1983), 'Monetary and portfolio-balance models of exchange rate determination', *Economic Interdependence and Flexible Exchange Rates*, Bhandari, J.S. and B.H. Putnam (eds.) the MIT Press, Cambridge Mass.

Frenkel, J.A. (1993), *On Exchange Rates*, MIT Press, Cambridge, Mass.

Fukao, M. (1988), 'Kinyu no kokusaika ga kawase rate no hendo oyobi kokusai shushi ni ataeru eikyo ni tsuite' (Influences of liberalization of financial markets on the foreign exchange rates and on balances of payments) *Kinyu Kenkyu*, **7**, 1–42.

Hakkio, C.S. (1986), 'Does the exchange rate follow a random walk? A Monte Carlo study of four tests for a random walk,' *Journal of International Money and Finance* **5**, 221–229.

Hooper, P. and J. Morton (1982), 'Fluctuations in the dollar: a model of nominal and real exchange rate determination', *Journal of International Money and Finance*, 1, 39–56.

Isard, P. (1988), 'Exchange rate modeling: an assessment of alternative approaches', Bryant, R.C., D.W. Henderson, G. Holthar, P. Hooper, and S.A. Symansky (eds.) *Empirical Macroeconomics for Interdependence Economies*, (Brookings Institution, Washington D.C.) **1**, 183–201.

Johansen, J. (1988), 'Statistical analysis of cointegration vectors'. *Journal of Economic Dynamics and Control*, **12**, 231–254.

Johansen, J. (1991), 'Estimation and hypothesis testing of cointegration vectors in Gaussian vector autoregressive models'. *Econometrica*, **59**, 1551–1580.

Kariya, T. and Fukao, K. (1988), *Goriteki yosokeisei ni yoru inflationkawase bunseki* (Rational Expectation Analysis of foreign exchange rates), Yuhikaku, Tokyo.

Kariya, T. and Matsue, Y. (1988), 'Hisenkei bunsan hendo model ni yoru nichiji shuji kawase rate hendo bunseki', (Analysis of daily and weekly foreign exchange rates within the framework of nonlinear heteroscedastic variance models), *mimeograph.*

Kashiwagi, N. (1991), 'Bayesian detection of structural changes', *Annals of the Institute of Statistical Mathematics*, **43**, 77–93.

Kroner, K.F. and Lastrapes, W.D. (1993), 'The impact of exchange rate volatility on international trade: reduced form estimates using the GARCH-in-mean model', *Journal of International Money and Finance*, **12**, 298–318.

Lastrapes, W.D. (1989), 'Exchange rate volatility and U.S. monetary policy: An ARCH application', *Journal of Money, Credit, and Banking*, **31**, 66–77.

Levich, R.M. (1985), 'Empirical studies of foreign exchange rates: price behavior, rate determination and market efficiency', Jones, R.W. and P.B. Kenen (eds.) *Handbook of International Economics II* North-Holland, Amsterdam.

McNown, R. and Wallace, M.S. (1992), 'Cointegration tests of a long-run relation between money demand and the effective exchange rate', *Journal of International Money and Finance*, **11**, 107–114.

Mishkin, F. (1992), *The Economics of Money, Banking and Financial Markets*, 3rd edition, Harper Collins Publication.

Mussa, A. (1979), 'Empirical regularities in the behavior of exchange rates and theories of the foreign exchange rates', *Carnegie-Rochester Conference Series on Public Policy*, Autumn, **11**, 9a–57.

Pippenger, M.K. (1993), 'Cointegration tests of purchasing power parity: the case of Swiss exchange rates', *Journal of International Money and Finance*, **12**, 46–61.

So, J.C. (1987), 'The sub-Gaussian distribution of currency futures: stable Paretian or nonstationary?' *Review of Economics and Statistics*, **69**, 100–107.

Taylor, S.J. (1986), *Modeling financial time series*, John Wiley and Sons, New York.

Tsurumi, H. (1988), 'Survey of Bayesian and non-Bayesian tests of model stability in econometrics', J. Spall ed., *Bayesian Analysis of Time Series and Dynamics*, Marcel Dekker, New York, 75–99.

Tsurumi, H. and Wago, H. (1994), 'Bayesian tests of a unit root and cointegration', mimeograph.

Wolff, C.C.P. (1987), 'Time-varying parameters and the out-of-sample forecasting performance of structural exchange models,' *Journal of Business and Economic Statistics*, **5**, 87–97.