Journal of Low Temperature Physics, Vol. 108, Nos. 3/4, 1997

Measurements in the Laminar and Turbulent Regime of Superfluid ^{4}He by Means of an Oscillating Sphere

Javier Luzuriaga

Comisión Nacional de Energia Atómica, Centro Atómico Bariloche and Instituto Balseiro, Universidad Nacional de Cuyo, 8400 Bariloche, Argentina

(Received January 27, 1997; revised March 17, 1997)

The translational oscillations of a sphere in liquid helium have been measured as a way of studying superfluid turbulence. Experiments were carried out in the laminar flow regime for reference purposes, and good agreement found between measured and calculated quantities. In the turbulent region, the dissipation is found to be proportional to the square of the velocity of the sphere, as found previously by other workers. For high vibration amplitudes there is an increase in the hydrodynamic mass. This seems to scale with the superfluid fraction in a way that strongly suggests that the superfluid component plays an important role in the turbulent regime.

I. INTRODUCTION

The present work reports experiments done with a sphere which oscillates in liquid helium, down to a temperature where a significant fraction of the liquid is in the superfluid state. It explores the onset of turbulence in the superfluid, a subfield of the long standing problem of turbulence, and a subject of continuing interest.[1]

The translational oscillations of a sphere have been studied recently by Jäger *et al.*,[2] as a means of exploring the onset of turbulence in superfluid helium. The spherical geometry has the advantage of simplicity. In this case all relevant parameters have been evaluated for laminar conditions, and good agreement is seen between theory and experiment. Therefore the starting point is well known, and departures from it can be well evaluated. Jäger *et al.* have used a magnetically levitated sphere of around 100 micron diameter which is part of a (non-linear) oscillator. In the present paper, results are shown which were obtained with a larger sphere, 7 mm in diameter, which is attached to a flexible stem, so that it is part of a linear mechanical oscillator. The linear oscillator is somewhat easier to analyze, while results seem to indicate that the perturbation due to the presence of the stem can be neglected. This experiment has evolved from a previous version with a vibrating reed, designed for advanced undergraduates.[3]

The results obtained confirm those of Jäger *et al.*,[2] over the temperature region where they overlap. As a new result, they show an increase in hydrodynamic mass in the turbulent regime. The fact that the oscillator is linear, and very stable in frequency ($\Delta f/f \sim$ a few parts per million) makes it possible to observe such effects.

II. EXPERIMENTAL DETAILS

The oscillator used in this study is a loaded cantilever, consisting of a flexible beam or stem of rectangular cross section, clamped at one end, and having a relatively heavy sphere attached at the free end as is shown in the inset of Fig. 1. The sphere is 7 mm in diameter. The whole oscillator, which is made of Be–Cu alloy, was turned on a lathe in one piece, and later, the stem which acts as the spring of the oscillator was filed to a rectangular cross section and to its final thickness of 0.3 mm. The stem is 2.5 mm long and 1 mm wide.

The oscillations are driven and detected capacitively, and the system is maintained at resonance by amplifying, phase shifting and feeding back the detected signal. In this way, the active element of the oscillator consists of the loaded cantilever beam plus sphere which oscillates at its natural resonance frequency. The system has been described by Kleinman *et al.*,[4] for high-Q mechanical oscillators. The quality factor Q of our oscillator is 3×10^3 when submerged in helium at 4 K and the resonance frequency is 319.5 Hz, with a reproducibility between runs of around 0.5 Hz. The driving and detection electrodes are disks 2 mm in diameter, located as shown in the inset of Fig. 1. The closest distance d between sphere and electrodes is around 0.1 mm. The voltage detected depends on this distance in a sensitive way, and is seen to change from run to run, presumably due to changes in d produced by thermal cycling.

The experiment was carried out in a pumped helium bath and no special precautions were taken as regards purity. Temperature was regulated by controlling the helium vapor pressure by means of a rubber diaphragm controller. On the first run, a carbon resistance thermometer (Speer) was calibrated against the LT 58 temperature scale by measuring the helium vapor pressure by means of a high precision manometer, and thereafter the temperature was found by reference to the carbon resistance.

Vibration isolation was achieved by hanging the relatively massive (13 gm) sample holder by cotton threads. This simple arrangement is effective

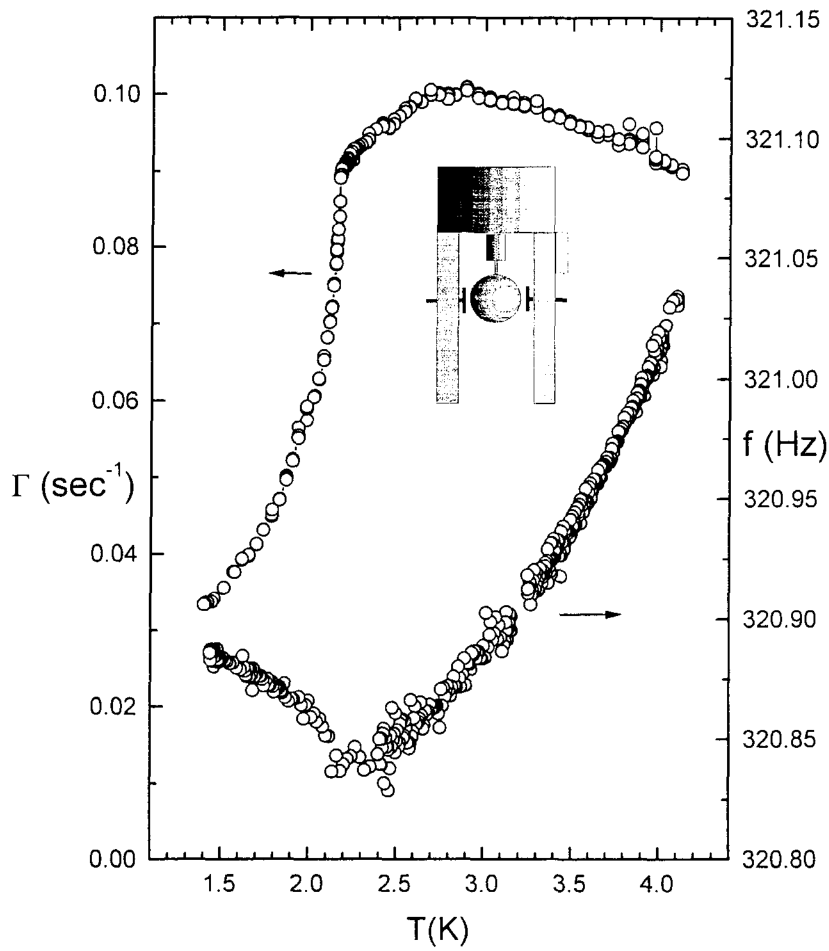

Fig. 1. Attenuation (upper curve) and resonance frequency (lower curve) of the oscillator as a function of temperature Inset: Experimental configuration.

enough to prevent the more troublesome vibrations transmitted through the cryostat. In an earlier arrangement, where the holder was rigidly attached to the cryostat, frequency stability was no better than one part in 10^4, while stability of a few parts in 10^6 was possible with the present setup. No attempts were made to reduce vibrations further since they did not appreciably alter the stability in frequency, however, there were small vibrations of the helium pressure ($\sim$0.2 torr at a few hertz) associated with the pressure regulator.

III. EXPERIMENTAL RESULTS

A. Laminar Regime

Figure 1 (lower curve) shows the resonance frequency as a function of temperature. The λ-point can be seen clearly as the minimum of the inverted cusp. As will be shown below, the shape of this curve is determined essentially by the changes in density of the liquid helium, since for a sphere of the present dimensions corrections due to changes in viscosity are negligible.

The effect of viscosity is seen, however, in the dissipation. This has been plotted in the upper curve of Fig. 1, which shows the dissipation term Γ as a function of temperature. It can be seen that there is a reduction in dissipation at temperatures slightly above the λ-transition (T_λ), and a large drop below it.

In the temperature range explored here, the mean free path of the quasiparticle excitations is always much smaller than the sphere dimensions, so that we are always in the hydrodynamic regime.

The response of an oscillating sphere immersed in a liquid of viscosity η and density ρ is well known.[5] For frequency of oscillation ω the force on a sphere of radius R is

$$F = 6\pi R\eta\left(1+\frac{R}{\delta}\right)v + 3\pi R^2\rho\delta\left(1+\frac{2R}{9\delta}\right)\dot{v} \tag{1}$$

with $\delta = \sqrt{2\eta/\rho\omega}$ the viscous penetration length. The first term ($\propto v$) in this force is dissipative while the second ($\propto \dot{v}$), is conservative and is usually referred to as the "hydrodynamic mass" of the sphere.

The equation for the damped oscillator has the usual form:

$$M\ddot{x} + \lambda\dot{x} + Kx = A\sin(\omega t) \tag{2}$$

Jäger *et al.*[2] have considered the changes in frequency and dissipation of such an oscillator with an additional force due to the presence of the liquid (given by Eq. 1), in the special case of the two fluid model of liquid helium. The change in frequency is given by:

$$\Delta f = \frac{f_0 - f}{f_0} = \frac{1}{4}\frac{\rho_{\mathrm{He}}}{\rho_{\mathrm{sphere}}} + \frac{9}{8}\frac{\delta}{R}\frac{\rho_N}{\rho_{\mathrm{sphere}}} \tag{3}$$

where f ($=\omega/2\pi$) is the measured frequency, $f_0 = (1/2\pi)\sqrt{K/M}$ is the corresponding frequency in vacuum, ρ_{He} is the total density of liquid helium, ρ_{sphere} the density of the material of the sphere and δ the penetration length. Because the waves described by δ are a viscous phenomenon, below T_λ, δ must be calculated by using ρ_N, the density of the normal fraction. Because we are dealing with a relatively large sphere, the value of δ/R in this experiment is always smaller than 10^{-3} and so can be neglected. It is then seen that the frequency change Δf is directly proportional to the density of the fluid. The density of Cu–Be is 8.23 gm/cm^3 and therefore ρ_{sphere} is available so that all the proportionality factors are known. A plot of ρ_{He} obtained from Eq. 3 and the data in Fig. 1 is shown in Fig. 2.

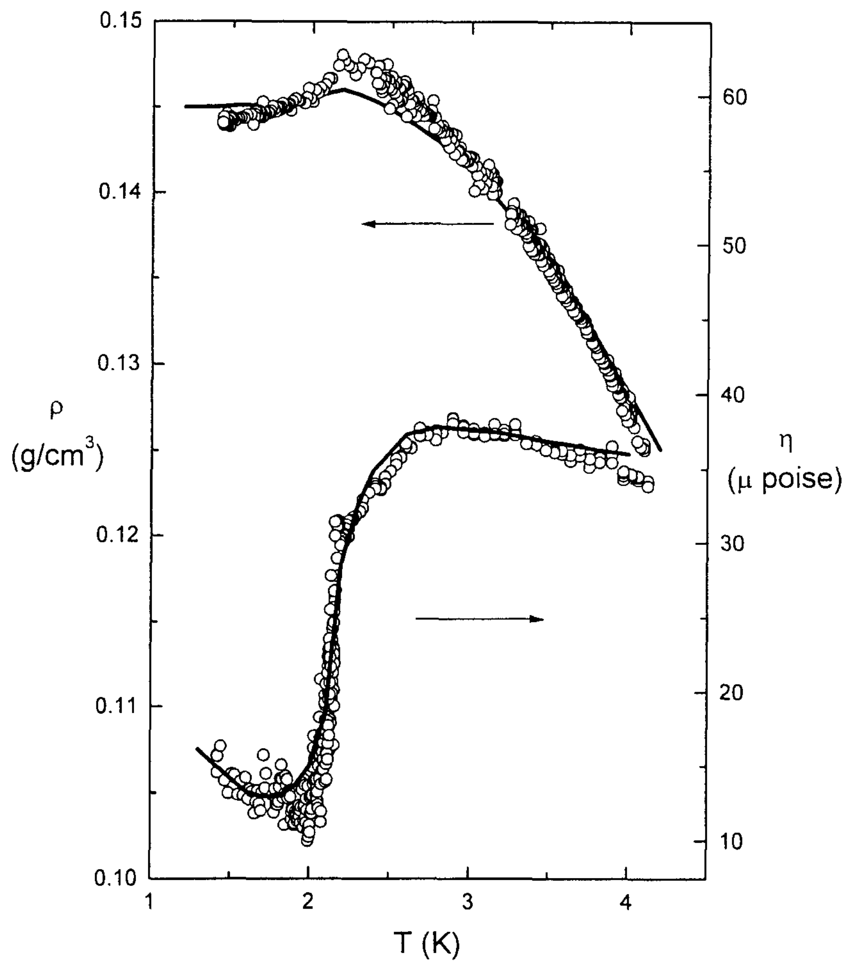

Fig. 2. Upper curve: Measured density (open circles) compared to literature values (full line). Lower Curve: Viscosity from the literature (full line) and the present measurements (circles).

The dissipation is given, in this linear approximation, by λ in Eq. 2. There are two terms in λ, one due to the liquid, λ_L and one being due to the residual damping of the material, λ_0. In general $\lambda_0 \ll \lambda_L$, but in the superfluid regime λ_0 cannot be neglected. The expression for λ_L has been calculated for the two fluid model[2]

$$\lambda_L = 6\pi\eta_n R\left(1+\frac{R}{\delta}\right) \tag{4}$$

where η_n is the viscosity of the normal component. In the experiment it is easier to obtain $\Gamma = (\lambda_L + \lambda_0)/M$. This, in practice is done by measuring a resonance curve at constant temperature T_0, and using the width at half maximum, which is a direct measure of $\Gamma(T_0)$.[6] In temperature sweeps, the amplitude of oscillation $A(T)$ is measured for a fixed driving force, and because this amplitude is proportional to the quality factor $Q = 2\pi f/\Gamma$ (in the high Q limit[6]) we can obtain $\Gamma(T) = \Gamma(T_0)\, A(T_0)/A(T)$.

Because of the numerical values of radius and frequency used in this experiment, in Eq. 4, one can use the approximation $(1 + R/\delta) \approx R/\delta$ to an accuracy close to 1%. Using this approximation:

$$\eta = \frac{2}{\omega\rho_N}\left(\frac{(\Gamma-\Gamma_0)\,M}{6\pi R^2}\right)^2 \tag{5}$$

The small changes in frequency due to the change in Young's modulus of the stem as a function of temperature, and the dissipation due to its internal friction (λ_0), were measured in a separate experiment, where the oscillator was kept in a vacuum, at a pressure smaller than 10^{-5} torr.

Using data for the density of the normal fraction ρ_N obtained from he literature,[10] the measured viscosity is plotted in Fig. 2, compared to published viscosity values.[10] As can be seen, the agreement is also good.

B. Non-Linear Regime

The non-linear regime was studied by increasing the driving force at constant temperature and recording the amplitude and frequency response of the oscillator. Temperature stability during these runs was between 0.5 and 0.2 mK.

The displacement of the sphere is detected capacitively, as a change in voltage read by a Lock In amplifier. The voltage reading can be converted into a distance measurement by the following argument:

The capacitance at the electrode C_e is proportional to $1/x$, where x is the distance between the electrode and the sphere. When the sphere moves a distance δx, the capacitance changes $\delta C_e = -C_e\,\delta x/x$. Because there is a charge Q_e on the capacitor, when the capacitance changes, a change in voltage $\delta V = Q_e\,\delta C_e$ appears on the electrode. On the other hand, $Q_e = V_{\rm BIAS}/C_{\rm TOT}$; where $V_{\rm BIAS} \cong 230$ V is the bias voltage applied to the electrode, and $C_{\rm TOT}$ is a capacitance which includes not only C_e, but other capacitances in parallel with the bias voltage, the main part of which corresponds to the cables which carry the signal to C_e. The ratio $C_e/C_{\rm TOT} \simeq 10^{-3}$, estimated from the geometry of the detection capacitor and the nominal capacitance of the coaxial cables. Combining the above expressions one finds:

$$\delta x = x\,\frac{C_{\rm TOT}}{C_e}\,\frac{\delta V}{V_{\rm BIAS}} \tag{6}$$

with $x \simeq 0.1$ mm, $\delta x \sim 0.36\,\mu$m per mV of detected voltage. The velocity of displacement, in the simple harmonic motion approximation, which is satisfied throughout the experiment is $v = v_0 \sin \omega t$, with $v_0 = 2\pi f\,\delta x$, therefore $v_0 \sim 0.072$ (cm/sec) per mV of detected voltage. The conversion factors have large uncertainties ($\sim 40\%$), and moreover are not constant from run to run because of the changes in x produced by thermal cycling. For this reason, in the graphs the values of displacement are expressed in mV, and the conversion factors can be used to get a rough idea of the values of displacement and velocity.

The amplitude sweeps at constant temperature are shown in Figs. 3 where we have plotted the amplitude of the response (A) against the excitation voltage (Exc). This, with the above mentioned conversion factors is equivalent to a velocity ($v \propto A$) *vs.* applied force ($F_0 \propto$ Exc), and shows the same behavior as found in the work of Jäger *et al.* A linear behavior is seen at low amplitudes, where $F_0 \propto v$, and at higher amplitudes the force $F_0 \propto v^2$. Because the damping is always light, one can use the principle of energy balance[2, 6] to infer that the drag force is proportional to v^2, as expected for turbulent drag from dimensional considerations.[5] It was further confirmed that the drag force can be fitted by an expression of the type

$$F_D = -\gamma(T)(v^2 - v_0^2) - \lambda(T)\,v \tag{7}$$

as was found in Ref. 2. The fits to a parabolic expression equivalent to Eq. 7 are plotted as full lines in Fig. 3. However, and in contrast to what is observed by Jaeger *et al.*,[2] no appreciable hysteresis could be observed when increasing or decreasing the amplitude. This is probably due to the

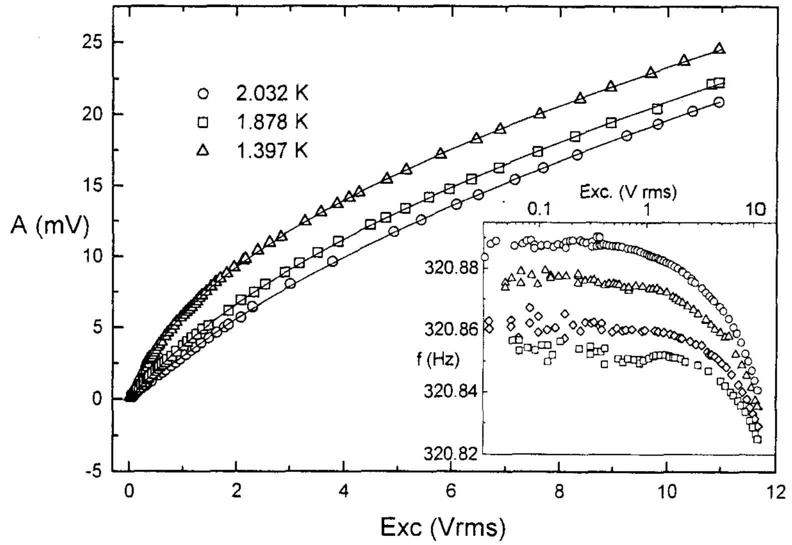

Fig. 3. Amplitude of vibration as a function of excitation voltage. The full lines are fits to Eq. 7 of the text. Inset: Frequency vs excitation voltage in semi-log plot. Squares correspond to $T = 2.032$ K, diamonds $T = 1.932$ K, triangles $T = 1.580$ K, circles $T = 1.397$ K.

higher level of the vibrations present in our cryostat due to the pressure regulator. For the same reason, the values of the velocity at which the turbulence sets in is probably an underestimate and lower than what could be measured in quiet conditions.

On the other hand, the fact that the oscillator is linear in vacuum, allows the study of the frequency of oscillation when the turbulent fluid interacts with the sphere.

The frequency remains constant at low amplitudes, but it starts to decrease at higher amplitude, with a maximum relative change of around 3×10^{-4}, as is shown in the inset of Fig. 3. A semilog plot is used so that the constant frequency region, corresponding to the linear region in amplitude can be better appreciated.

The Q value is rather high, being around 3×10^3 so that the correction in frequency due to damping $(f_{\text{damped}} = f_{\text{free}}(1 - 1/Q^2)^{1/2}$ is of order 10^{-7}. For similar amplitudes and with the sphere oscillating in a vacuum, no change in frequency is seen, so that effects due to imperfect elasticity of the spring can be ruled out. Thus, the frequency shift is due to an increase in the hydrodynamic mass of the sphere above the value found for laminar drag.

At low amplitudes the frequency f_0 is constant and $f_0 \propto \sqrt{K/M}$, but at higher amplitudes $f \propto \sqrt{K/M + \Delta M}$ where ΔM is the added effective mass. From this,

$$\frac{\Delta M}{M} = \left(\frac{f_0}{f}\right)^2 - 1 \tag{8}$$

The change in mass has been plotted against the excitation voltage and is shown in Fig. 4, for three selected runs taken at different temperatures. It can be seen that the effective mass increases linearly with excitation voltage (i.e., with the force applied to the oscillator) to a maximum of 300 parts per million (ppm) at the highest amplitudes measured. The slope of the curves is seen to increase when the amplitude sweeps are performed at lower temperatures.

A plot of these slopes as a function of temperature is shown in the inset of Fig. 4 and it can be seen that it roughly agrees with the temperature variation of the superfluid fraction, shown as a full line in the graph. This suggests that the superfluid has an important role in the turbulent regime, as was found to be the case in the analysis of the dissipation by Jaeger *et al.*[2]

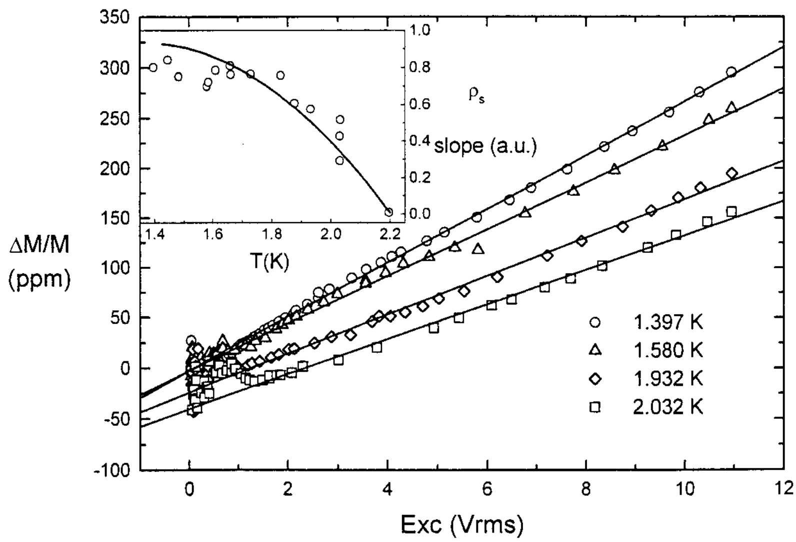

Fig. 4. Change in "hydrodynamic mass" as a function of excitation voltage. Full lines are least squares fits to a straight line. Inset: Superfluid density (full line) and slope of the straight lines (circles), as a function of temperature.

IV. DISCUSSION

Because of the link between superfluid fraction and slope of the $\Delta M/M$ curves it is tempting to attribute the apparent change in mass to vortices created in the superfluid wake that are accelerated together with the sphere because they are pinned in some way to its surface.

The change in effective mass is proportional to the excitation voltage, and in the turbulent regime the velocity at which the sphere moves in the liquid increases as the square root of the excitation voltage. Although no obvious physical reason for this comes to mind, empirically, this means that the change in mass is proportional to the square of the velocity at which the sphere moves in the liquid, i.e., to the kinetic energy of the sphere rather than to its momentum.

We can obtain a rough estimate of the critical velocity (v_c) from the amplitude *vs.* excitation graphs. This gives $v_c \sim 0.14$ cm/sec, and is of the right order of magnitude for a channel of width $W \sim 0.04$ cm[7,10] much larger than the viscous penetration depth $\delta \sim 0.001$ cm. This is in contrast to the behavior found in rotating spheres, as discussed in the book by Atkins[7] and it therefore seems that δ is not the length scale that is relevant for the onset of dissipation *i.e.* of the creation or growth of superfluid vortices. The estimated W is also much smaller than the radius R of the sphere, which is 0.35 cm. It therefore seems that the relevant length scale is set neither by R nor by W.

However, comparison with the data of Jäger *et al.* shows that the critical velocity times the radius have aproximately the same value in both experiments. In the present experiments, $v_c \sim 0.14$ cm/sec and $R = 0.35$ cm, therefore $v_c R \sim 0.03\ \mathrm{cm}^2/\mathrm{sec}$. Jäger *et al.*[2] the other hand obtain $v_c \sim 4$ cm/sec and $R = 0.01$ cm, giving $v_c R \sim 0.04\ \mathrm{cm}^2/\mathrm{sec}$. Considering the uncertainties in the values reported here, the agreement is relatively good. Both the quantum of circulation and the Reynolds number scale with the product vR, so from this alone, it is not possible to rule out a contribution from the normal component to the onset of turbulence.

There has been recent theoretical work which points out to the importance of cavitation in setting the maximum (critical) velocity for non-dissipative superfluid flow.[8] The importance of the interaction of vorticity and bubbles in the liquid[9] has also been pointed out. The present work is limited to the vapor coexistence curve because of the cooling method used, but if the critical velocity of the oscillator could be measured as a function of pressure, these theoretical proposals could be experimentally tested, because cavitation depends on the hydrostatic pressure of the liquid. A modified form of the present experiment where pressure can be varied would provide such a test.

ACKNOWLEDGMENTS

It is a pleasure to acknowledge the help given by J. C. Estrada Vigil, C. Grillo, and B. Maiorov for help in setting up this experiment and obtaining preliminary results as part of their undergraduate course work.

REFERENCES

1. R. J. Donnelly and C. E. Swanson, *J. Fluid Mech.* **173**, 387 (1986); J. T. Tough, in *Progress in Low Temperature Physics*, D. F. Brewer, ed. North-Holland, Amsterdam (1982), Vol. 8, pp. 133–216; K. W. Schwartz, *Phys. Rev. Lett.* **49**, 283 (1982).
2. J. Jäger, B. Shuderer, and W. Schoepe, *Phys. Rev. Lett.* **74**, 566 (1995); *Physica B* **210**, 201 (1995).
3. E. N. Martinez, P. Esquinazi, and J. Luzuriaga, *Am. J. of Physics* **58**, 1163 (1990).
4. R. N. Kleinman, G. K. Kaminsky, J. D. Reppy, R. Pindak, and D. J. Bishop, *Rev. Sci. Instr.* **56**, 2088 (1985).
5. L. D. Landau and E. M. Lifshitz, *Fluid Mechanics- Course of theoretical Physics Vol. 6*, Pergamon Press, Oxford (1959).
6. A. B. Pippard, *The Physics of Vibration, Vol. I*, Cambridge University Press, Cambridge (1978).
7. K. R. Atkins, *Liquid Helium*, Cambridge University Press, Cambridge (1959), p. 198.
8. C. Josserand, Y. Pomeau, and S. Rica, *Phys. Rev. Lett.* **75**, 3150 (1995).
9. H. J. Maris, *J. Low Temp. Phys.* **94**, 125 (1994); H. J. Maris, *Phys. Rev. Lett.* **66**, 45 (1991).
10. J. Wilks, *Liquid and Solid Helium*, Clarendon Press, Oxford (1967).