On uniformly homeomorphic normed spaces II

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This paper continues the studies of the situation when two Banach spaces are uniformly homeomorphic (i.e., when there is a non-linear bijection f between them such that both f and f^{-1} are uniformly continuous). The question is how strongly the linear-topological structures of the two spaces must then be related to each other. Only very recently, Aharoni and Lindenstrauss [19] gave an example showing that the two spaces need not always be isomorphic. (This question of isomorphy, raised by Bessaga [3] and Lindenstrauss [7], [8], is still open in the general reflexive case and in the general separable case.)

The present author [18] has proved that for any two uniformly homeomorphic real normed spaces, the finite-dimensional subspaces are imbeddable into the other space by linear mappings T such that all the numbers $\|T\|\,\|T^{-1}\|$ have a common upper bound. This generalises some results of Enflo [5], [6] and Lindenstrauss [7]. Aharoni [1], [2] and Mankiewicz [13]—[16] have given nice results on some closely related problems. (For a recent survey, see Enflo [20].)

The purpose of this paper is to show how the mentioned result of [18] can be strengthened if one of the spaces is supposed to be uniformly rotund. As an application, it is proved that if $1<p<\infty$, then among all real Banach spaces only $\mathscr{L}_p$-spaces are uniformly homeomorphic to $\mathscr{L}_p$-spaces (Sect. 5).

Theorem 1. *Assume that E and F are normed spaces over the real field, that F is uniformly rotund, and that E and F are uniformly homeomorphic. Then there is a number $C>0$ such that for any integer $n\geqq 1$ and any finite-dimensional subspace K in E, there is a linear imbedding $T\colon K\to F$ with the following property:*

For every n-dimensional subspace L in F there is a linear mapping $S\colon (T(K)+L)\to E$ such that ST is the identity mapping on K and such that $\|S\|\,\|T\|\leqq C$.

The proof is given in Sect. 2—3.

Corollary 1. *Under the assumptions of Theorem 1, there is a number $C>0$ such that for any integer $n\geqq 1$ and any finite-dimensional subspace K in E, there is a linear* *imbedding* T: $K\to F$ *for which* $\|T\|\,\|T^{-1}\|\leqq C$ *and which has the following additional property:*

Consider any continuous linear projection P: $E\to K$; *then for every n-dimensional subspace* L *in* F *there is a linear projection* P_1: $(T(K)+L)\to T(K)$ *for which* $\|P_1\|\leqq C\|P\|$.

Proof. Take $P_1=TPS$.

The proof of Theorem 1 uses a variant of the technique for handling finite point-meshes introduced in the author's previous paper [18]. Like before, the "linearization" leading to S consists in forming averages of function values on a suitably selected point-mesh. The mapping T is obtained in a more direct manner, using "approximate affinity" of the uniform homeomorphism on a suitable point-mesh.

2. Finite point-meshes

For the proof of Theorem 1 we need some lemmas. This section contains those lemmas which do not depend on the uniform rotundity assumption.

Notation. Given some points $x_1, \ldots, x_d$ in a linear space and an integer $m\geqq 1$, we denote by $G(x_1, \ldots, x_d|m)$ the set of all linear combinations

$$\xi_1 x_1+\ldots+\xi_d x_d \quad \text{with } \xi_i \text{ integers}, \quad |\xi_i|\leqq m.$$

For a normed space E we let $\mathrm{S}(E)$ be the set of all d-tuples $(x_1, \ldots, x_d)\subset E$ such that $\|x_i\|=\|x_1\|\geqq 1$ and $\operatorname{dist}(x_i, \operatorname{lin}(x_1, \ldots, x_{i-1}))=\|x_1\|$ for all i. (This definition is somewhat wider than the one made in [18]; really, it ought to have been used there also.)

Assumptions. For this section, we assume that there are given two normed real linear spaces E and F, and a non-linear mapping f: $E\to F$ such that for some number $b>0$ we have

$$b^{-1}\|x-y\| \leqq \|f(x)-f(y)\| \leqq b\|x-y\|$$

$$\text{for} \quad x, y \quad \text{in} \quad E, \quad \|x-y\| \geqq 1.$$

Notation. With these assumptions, let x in E and u in F' be given points, and let $c>0$ be a given number. (F' is the conjugate space to F.) We denote by $\mathscr{A}(x, u|c)$ the class of all sets S in E such that whenever y is a point in S and k is any positive integer such that $y+kx$ is also in S, we have

$$u\big(f(y+kx)-f(y)\big) \geqq c\|u\|\,\|x\|\,k.$$

Further, we denote by $\mathscr{B}(x|c)$ the class of all sets S in E such that whenever y is a point in S and k is any positive integer such that $y+kx$ is also in S, we have

$$\|f(y+kx)-f(y)\| \leqq c\|x\|\,k.$$

The following lemma is a slight modification of Lemma 2 in [18]. The proof carries over with obvious changes and will not be repeated here.

Lemma 1. *With the Assumptions just made, let there be given an integer* $d\geqq 1$ *and a real number* $\theta>1$. *Then there is an integer* $m_0(d, b, \theta)=m_0\geqq 3$ *such that for* $m\geqq m_0$ *there is an integer* $j_0(d, m, b, \theta)=j_0\geqq 1$ *such that the following implication holds:*

Let there be given a d-tuple $(x_1, ..., x_d)$ *of* $\mathrm{S}(E)$, *a real number* c, $(2b)^{-1}\leqq c\leqq b$, *and integers* i, $1\leqq i\leqq d$, *and* $j\geqq j_0$. *Suppose that* y^0 *in* $G(x_1, ..., x_d|[m^{3j}/3])$ *and* u *in* F' *are points, and* n, $m^{3j_0}\leqq n\leqq m^{3j}$, *an integer for which*

$$u\big(f(y^0+nx_i)-f(y^0)\big) \geqq \theta cn\|u\|\,\|x_i\|.$$

Then the set $G(x_1, ..., x_d|m^{3j})$ *contains a subset which is of the form*

$$y^- + m^{j^- -1}G(x_1, \ldots, x_d|m)$$

(where $1\leqq j^-\leqq 3j-1$*), and which belongs to the class* $\mathscr{A}(m^{j^- -1}x_i, u|c)$.

Lemma 2. *With the Assumptions just made, let there be given an integer* $d\geqq 1$ *and a real number* $\theta>1$. *If* $(x_1, ..., x_d)$ *is a d-tuple of* $\mathrm{S}(E)$ *and* $m\geqq 1$ *an integer, there is a set which is of the form*

$$y+nG(x_1, \ldots, x_d|m)$$

(where $n\geqq 1$*), and which belongs to the class*

$$\bigcap_{1\leqq i\leqq d} \mathscr{A}(nx_i, u_i|c_i) \cap \bigcap_{1\leqq i\leqq d} \mathscr{B}(nx_i|\theta c_i)$$

for some elements $u_i\neq 0$ *in* F' *and some real numbers* $c_i>0$, $1\leqq i\leqq d$.

Proof. We shall prove that if $m\geqq 3$ is any given sufficiently large integer, then for all sufficiently large integers $j\geqq 1$ the set $G(x_1, ..., x_d|m^j)$ contains a set which is of the form

$$G^- = y^- + m^{j^- -1}G(x_1, \ldots, x_d|m)$$

and which is of class $\mathscr{A}(nx_1, u|c)\cap\mathscr{B}(nx_1|\theta c)$ for some $u\neq 0$ and $c>0$. The assertion of the lemma then follows from precisely the same iterative argument as was used to find G_n in the first half of the proof of Theorem 1A in [18].

For $k\geqq i\geqq 1$ let $r(k, i)$ be that integer r for which the set

$$m^i G(x_1, \ldots, x_d|m^{k-i})$$

belongs to the class $\mathscr{B}(m^i x_1|\theta^{(r+1)/2}) \setminus \mathscr{B}(m^i x_1|\theta^{r/2})$. Let $j_0(d, m, b, \theta)=j_0$ be the integer mentioned in Lemma 1. Now, the function $r(k, i)$ has finite range, and clearly it is increasing in k and decreasing in i. It follows that there is an integer $j_1(d, m, b, \theta)=j_1>3j_0$ such that one can always find some integers k', j' with $3j_0 \leqq 3j' < k' \leqq j_1$ and with

$$r(k'-1, 3j') = r(k', 3j'-3j_0) = r',$$

say.

Consider the set

$$G' = m^{3j'-3j_0} G(x_1, \dots, x_d|m^{k'-3j'+3j_0}).$$

This set, and thus every subset of it, belongs to the class $\mathscr{B}(m^{3j'-3j_0} x_1|\theta^{(r'+1)/2})$. But the relation enjoyed by r' also implies that in the set

$$m^{3j'} G(x_1, \dots, x_d|m^{k'-3j'-1})$$

there is a point y^0 such that

$$\left\|f\left(y^0+m^{3j_0}(m^{3j'-3j_0}x_1)\right)-f(y^0)\right\| \geqq \theta^{r'} m^{3j_0} m^{3j'-3j_0}\|x_1\|.$$

In view of this we can apply Lemma 1, which yields that in G' there is a subset G^- having the properties claimed above (with $c=\theta^{(r'-1)/2}$).

Notation. We denote by $\mathrm{W}(E)$ the set of all d-tuples $(x_1, \dots, x_d)$ of points in E such that

$$\|x_1\| \geqq 2b^2,$$

$$\|x_i\| \leqq 2b^2\|x_1\|,$$

and

$$\operatorname{dist}(x_i, \operatorname{lin}(x_1, \dots, x_{i-1})) \geqq (b^{-2}/2)\|x_1\|$$

for all i. (Remark: The choice of the constants here is motivated by our actual need in the next section.)

Lemma 3. *With the Assumptions made at the beginning of this section, for all integers $d, n \geqq 1$ there is an integer $M \geqq 1$ and a positive number δ such that the following implication holds:*

Let $(x_1, \dots, x_d)$ be a given d-tuple of $\mathrm{W}(E)$. Suppose that there is an affine mapping

$$a\colon G(x_1, \dots, x_d|M) \to F$$

such that

$$\|f(x)-a(x)\| \leqq \delta\|x_1\| \quad \text{for } x \text{ in } \quad G(x_1, \dots, x_d|M).$$

Then if K is any $(d+n)$-dimensional subspace in E which contains $G(x_1, \dots, x_d|M)$, there is a linear mapping $S\colon K \to F$ such that

1° $S(x) = a(x)-a(0)$ *for x in* $G(x_1, \dots, x_d|M)$.

2° $\|S\| \leqq 2b$.

Proof. The proof is analogous to the last half of the proof of Theorem 1A in [18]. Let M and δ be fixed, but suitably large resp. small to meet later requirements. Given K, we let $x_{d+1}, \ldots, x_{d+n}$ be such that $(x_1, \ldots, x_{d+n})$ becomes a $(d+n)$-tuple of $W(E)$ spanning K. Also, assume that $f(0)=a(0)=0$.

Let $m \leqq M$ be a fixed positive integer, to be specified later. Given an $\varepsilon > 0$ (specified later), and then supposing that M and δ were suitably chosen, we can construct a mapping $h\colon G(x_1, \ldots, x_{d+n}|m) \to F$ fulfilling the conditions

(i) $\|h(x)+h(y)-h(x+y)\| \leqq \varepsilon \|x_1\|$

(ii) $\|a(x)-h(x)\| \leqq \varepsilon \|x_1\|$ when x is in $G(x_1, \ldots, x_d|m)$

(iii) $\|h(x)\| \leqq b\|x\|$

for all x and y. Namely, let $N \leqq M$ be a fixed suitable positive integer and put

$$G' = G(x_1, \ldots, x_d|M)+G(x_{d+1}, \ldots, x_{d+n}|N).$$

Then we define the h-values as averages of differences of f-values in this way:

$$h(x) = (2M+1)^{-d}(2N+1)^{-n} \textstyle\sum_{x'} (f(x'+x)-f(x')),$$

where the summation index x' runs through G'.

Conditions (i)—(iii) are quickly verified. For (i), use the assumptions for f and $W(E)$, and note that if the defining sums for the h-values are written out, then a very large portion of all terms in the expression for $h(x)+h(y)-h(x+y)$ will cancel, if M/m and N/m were taken large enough. The assumptions for f and $W(E)$ clearly also imply (ii) if M/N is large and $\delta > 0$ is small enough. It is easily seen that these requirements on M, N, and δ do not depend on the choice of $(x_1, \ldots, x_{d+n})$, except on d and n.

We now define the linear mapping $S\colon K \to F$ by putting

$$S(\xi_1 x_1+\ldots+\xi_{d+n}x_{d+n}) = \xi_1 a(x_1)+\ldots+\xi_d a(x_d)+\xi_{d+1}h(x_{d+1})+\ldots+\xi_{d+n}h(x_{d+n})$$

for all reals ξ_i. Then S clearly coincides with a on the domain of a, as claimed. Further, it can be seen that conditions (i)—(iii) imply that $\|S\| \leqq 2b$, if m was chosen sufficiently large and ε sufficiently small, in view of the assumptions for $W(E)$; and these requirements on m and ε do not depend on $(x_1, \ldots, x_{d+n})$, except on d and n.

3. Uniform rotundity

First notice that the definition of uniform rotundity (cf. Day [4], Sect. VII. 2, Definition 2) can be rephrased thus: A space E is uniformly rotund if and only if for every $\varepsilon > 0$ there is a $\delta > 0$ such that if u is any element in E' with $\|u\|=1$, then the set of all points x in E with $\|x\| \leqq 1$ and $u(x) \geqq 1-\delta$ has diameter at most ε. In view of that, the following lemma is almost immediately obtained:

Lemma 4. *With the notations of the preceding section, suppose that F is uniformly rotund. For every $\varepsilon>0$ there is a $\theta>1$ such that the following implication holds:*

Let $(x_1, \ldots, x_d)\subset E$ be any linearly independent d-tuple and m any positive integer. Suppose that the set $G(x_1, \ldots, x_d|m)$ belongs to the class

$$\bigcap_{1\leqq i\leqq d} \mathscr{A}(x_i, u_i|c_i) \cap \bigcap_{1\leqq i\leqq d} \mathscr{B}(x_i, u_i|\theta c_i)$$

for some elements $u_i\neq 0$ in F' and some real numbers $c_i>0$, $1\leqq i\leqq d$. Then there is an affine mapping $a\colon G(x_1, \ldots, x_i|m)\to F$ such that

$$\|f(x)-a(x)\| \leqq \varepsilon dm \max \|x_i\|$$

for all x in $G(x_1, \ldots, x_d|m)$.

Proof. Define a putting $a(0)=f(0)$ and $a(x_i)=f(0)+y_i$, where y_i is the unique solution to the equation $u_i(y_i)=\|u_i\|\,\|y_i\|$ with $\|y_i\|=c_i\|x_i\|$.

Proof of Theorem 1. Clearly, the uniform homeomorphism f (say) fulfils the Assumptions at the beginning of Sect. 2 for some b. Let $(x_1, \ldots, x_d)$ be a d-tuple of $S(E)$ spanning K and with $\|x_1\|\geqq 3b^3$. Let m be a large positive integer, and let ε be a small positive number. Combining Lemmas 2 and 4, we find a set

$$G^- = y^- + rG(x_1, \ldots, x_d|m)$$

(where $r\geqq 1$) and an affine mapping $a\colon G^-\to F$ such that $\|f(x)-a(x)\|\leqq\varepsilon r\|x_1\|$ for x in G^-.

Let the integer $n\geqq 1$ be given; then if m and ε were chosen suitably, we can define the desired linear mapping $T\colon K\to F$ as the linear extension of the 0-preserving affine mapping $x\to a(x+y^-)-a(y^-)$. Namely, first note that if ε is small enough, then

$$\big(rT(x_1), \ldots, rT(x_d)\big)$$

is necessarily a d-tuple of $W(F)$. Now suppose that m was taken suitably large and ε suitably small; and let L be an arbitrary n-dimensional subspace in F. We can then apply Lemma 3, with f^{-1} in the place of f and a^{-1} in the place of a, to obtain a linear mapping $S\colon \big(T(K)+L\big)\to E$ with ST being the identity mapping on K and with $\|S\|\leqq 2b$. For m large and ε small we also have $\|T\|\leqq 2b$, which completes the proof.

4. Further observations

Corollary 1 can sometimes be given a stronger and more polished formulation with the aid of a recently studied notion, i.e., the uniform approximation property (u. a. p.). A space E has the u. a. p. if there is a number $C>0$ so that for every $d\geqq 1$ there is an $n\geqq 1$ such that for every d-dimensional subspace K in E, there is a linear mapping $T: E \to E$ with $T(x)=x$ for x in K, with $\dim T(E) \leqq n$, and with $\|T\| \leqq C$. The $L^p(\mu)$-spaces have the u. a. p., by Pelczynski and Rosenthal [17]; so do the reflexive Orlicz spaces, by Lindenstrauss and Tzafriri [12]. From Corollary 1, we immediately get:

Corollary 2. *With the assumptions of Theorem 1, also assume that F has the uniform approximation property. There is a number $C>0$ such that for every finite-dimensional subspace K in E, there is a linear imbedding $T: K \to F$ for which $\|T\| \|T^{-1}\| \leqq C$ and which has this property: If $P: E \to K$ is a linear projection, there is a linear projection $P_1: F \to T(K)$ with $\|P_1\| \leqq C\|P\|$.*

Let us notice that a sort of "approximate affinity" is generally possessed by uniformly continuous mappings into uniformly rotund spaces. For by Lemma 4 and an obvious modification of Lemma 2, we can obtain:

Corollary 3. *Let $f: E \to F$ be a uniformly continuous mapping from a real normed linear space into a uniformly rotund real normed linear space F. Let $(x_1, \dots, x_d)$ be a linearly independent d-tuple of elements in E and let m be a positive integer. For every number $\varepsilon > 0$ there is a set of the form*

$$G = y + nG(x_1, \dots, x_d|m)$$

(where $n \geqq 1$) and an affine mapping $a: G \to F$ such that $\|f(x) - a(x)\| < \varepsilon n$ for all x in G.

(Of course, this "approximate affinity" can be trivial, so that $a=0$ always suffices for n large.)

5. Application to $\mathscr{L}_p$

Concerning the $\mathscr{L}_p$-spaces, which were introduced by Lindenstrauss and Pelczynski [9], see Lindenstrauss and Rosenthal [10], or Lindenstrauss' and Tzafriri's book [11].

Theorem 2. *Let $1<p<\infty$. Then if a real Banach space is uniformly homeomorphic to an $\mathscr{L}_p$-space, it is an $\mathscr{L}_p$-space itself.*

Proof. Let E be a Banach space which is uniformly homeomorphic to an $\mathscr{L}_{p,\lambda}$-space F. It is known that every $\mathscr{L}_p$-space is isomorphic to a subspace of an L^p-space; so since $1<p<\infty$, F can be given an equivalent norm which is uniformly rotund.

According to a recent theorem of Pelczynski and Rosenthal [17], for $d \geqq 1$ there is an $n(d) \geqq 1$ such that every d-dimensional subspace in l^p is contained in an $n(d)$-dimensional subspace $N \subset l^p$ with $d(N, l^p_{n(d)}) \leqq 2$. Now apply Theorem 1 above, taking the finite-dimensional $K \subset E$ arbitrary and taking $n = n(d) - d$, where d is the dimension of K. It follows that there always are linear mappings $U: K \to l_n^p$ and $V: l_n^p \to E$ with $VU = \mathrm{id}_K$ and $\|V\| \|U\| \leqq 2C\lambda$. But the latter statement means precisely that E fulfils the hypothesis of Theorem 4.3 of Lindenstrauss and Rosenthal [10], whence E is an $\mathscr{L}_p$-space of an $\mathscr{L}_2$-space.

Now, the alternative of E being an $\mathscr{L}_2$-space can be ruled out when $p \neq 2$, by an application of the above argument with the roles of E resp. F interchanged (or by Theorem 6.3.1 of Enflo [6]).

Remark. In the separable case Theorem 2 can be restated thus (see [10] or [11]): *For* $1 < p < \infty$, *the class of all isomorphy types of complemented closed subspaces in* $L^p(0, 1)$ *is closed under uniform homeomorphy.* It might be pointed out that a corresponding statement holds for the class of isomorphy types of *all* closed subspaces in $L^p(0, 1)$. (Of course, the latter statement is of a "less precise" kind, since the latter class is so much wider than the former when $p \neq 2$.) Thus:

Let $1 < p < \infty$. *If a real Banach space is uniformly homeomorphic to a subspace in* $L^p(0, 1)$, *it is isomorphic to a subspace in* $L^p(0, 1)$.

Namely, this follows from the result of [18] cited at the beginning of this paper, combined with a known fact, which can be proved by a suitable diagonalisation: $L^p(0, 1)$ is a universal imbedding space for those separable Banach spaces all finite-dimensional subspaces of which can be imbedded into l^p by linear mappings T with $\|T\| \|T^{-1}\|$ having a common upper bound. (Alternative approach: It is possible to show that a uniform imbedding onto a subspace in $L^p(0, 1)$ can be "Lipschitzified", and hence a linear-topological imbedding then exists by a general theorem of Mankiewicz [13].)

Remark. As in [18] (cf. Sect. 5 there), it is a straightforward matter to get sharp quantitative forms of the results in this paper. E.g., to state such a form of Theorem 2, let E be a real Banach space which can be mapped onto an $\mathscr{L}_{p,\lambda}$-space by a bijection fulfilling the Assumptions stated at the beginning of Sect. 2 above; then E is an $\mathscr{L}_{p,b^2\lambda+\varepsilon}$-space for every $\varepsilon > 0$.

6. Locally bounded spaces

For E in Theorem 1, for the given space in Theorem 2, and for one of the two spaces in Theorem 1 of [18], it actually suffices to assume that it is a real locally bounded space. Namely, the proof of Theorem 1A in [18] clearly carries through if E is endowed with the Minkowski functional of a bounded 0-neighbourhood, and not necessarily with a norm. This generalization of Theorem 1A immediately implies the following (which is a generalization of Theorem 6.2.1 of Enflo [6]):

If a locally bounded space is uniformly homeomorphic to a normed space, it is normable.

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