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EVALUATION OF THE SEISMIC EFFECT OF AN UNDERGROUND EXPLOSION

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1. Introduction. In order to evaluate the seismic effect created by underground explosion on the surrounding objects apart from the normal results of experimental studies it is possible to use mathematical modelling for studying these processes. In modelling the effect of an explosion in solid material around a cavity with explosive it is normal to separate zones of displacement (zone 1), crushing (zone 2), crack formation (zone 3), and elastic deformation (zone 4) [1-3]. From the analysis of actual data [4] it is known that the radius of zone 1 does not exceed two and a half explosive charge radii, the radius of zone 2 reaches ten explosive charge radii, and the radius of zone 3 is 3-100 explosive charge radii (Fig. 1, r_0 is radius of a cavity with explosive, r_1 is the radius of zone 1, r_2 is that for zone 2, r_3 is that for zone 3). Use of the results of theoretical studies makes it possible to determine the size of these zones more accurately for a specific material or rock. A quite complete account of the main characteristics of the effect of an explosion and calculation schemes which make it possible to predict the size of zones around a cavity with explosive are provided in [2, 3, 5-9].

In [10-13] the dimensions of zone 3, the sizes of lumps into which the material is broken and their distribution with respect to the axis of the explosion are determined. Work in [11, 14] is devoted to studying the development of star-shaped cracks under conditions of antiplane deformation.

In the majority of these works either the features of stresses close to the edge of a propagating crack or the size of the different zones of explosive effect are determined. In the main the explosive source studied was either a spherical charge or a balsthole charge of infinite length (plane charge). A study of the seismic effect of an explosion rested mainly on analyzing the experimental data. In order to construct a more physically real model of an underground explosion it is necessary to consider the finite length of a blasthole charge and different orientation of it with respect to surrounding objects.

2. Model Description. In order to construct a three-dimensional model of an underground explosion we make the following assumptions.

1. The process occurs in an unbounded isotropic elastic material.

2. In this material as a result of detonation of a blasthole charge of finite length a star-shaped system of separation cracks starts to propagate (Fig. 2, W is blasthole length, AB is the blasthole itself, the width of the star-shaped crack is assumed to equal the blasthole length, and the length of the crack is assumed to be unchanged in any section of the blasthole charge).

3. Crack parameters such as length and opening are connected with explosive parameters and characteristics of the surrounding material, and they are determined on the basis of supplementary calculations.

4. In performing supplementary calculations for the overall scheme of an explosion (Fig. 1, here in contrast to Fig. 2 for simplicity only a section of the blasthole charge is shown) the radius of crack formation zone r_3 is determined which is assumed to equal the length of the model crack (Fig. 2). Also for the general scheme (Fig. 1) an estimate is made of the volume of cavities which form as a result of expansion of the cavity with explosive and the increase in the randomness of oriented cracks in zone 2 and radical cracks in zone 3. Opening of model cracks (Fig. 2) is selected so that the overall volume of the cavity obtained as a result of developing these cracks equal the volume of cavities obtained by the general scheme (Fig. 1).

5. Zones 1 and 2 do not markedly affect formation of the wave field in zone $r > r_3$ because as follows from the work given above the radius of zone 3 is much greater than that of zones 1 and 2 and larger radial cracks which develop carry more information about the explosive source than a crack developing randomly in zone 2 (considerably shorter length).

6. In order to describe the crack system we use a dislocation approach developed in [14-17, 18] with which in the whole area of failure as a boundary condition we prescribe the value and direction of the movement vector [**B** (B_x, B_y, B_z) on

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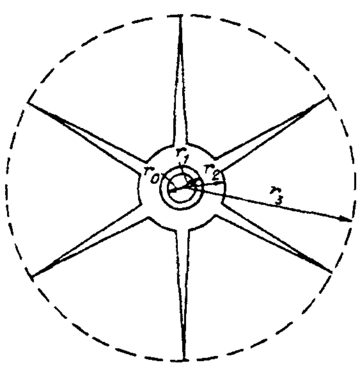

Fig. 1

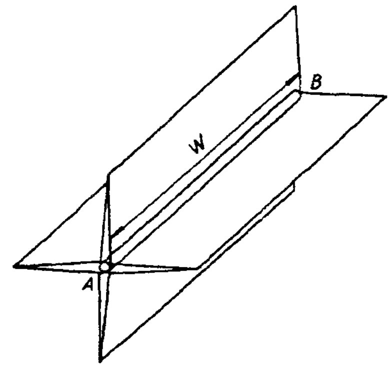

Fig. 2

a Cartesian coordinate system or **B** (B_r, B_α, B_z) on a cylindrical coordinate system, B_x, B_z (correspondingly B_r, B_z) are shear components of the movement vector and B_y and B_α are separation components]. Since cracks comprising a star-shaped system are assumed to be separation cracks, then shear components of the movements vector will equal zero.

As far as the number of large radial cracks and their propagation rate are concerned, then answers to these questions may be obtained from analyzing experimental data. It follows from [10] that the 'process ceases when 4-6 cracks remain.' Estimates of crack propagation rates with an explosion for different materials are provided in [19].

In contrast to many existing models of an explosive source, in the model suggested, first, we consider a system of separation radial cracks, and second, for them we determine quite accurately such parameters as length and opening.

These assumptions make it possible to formulate a mathematical statement of the problem for studying features of seismic emission from an explosive source with a radial crack system.

3. Statement of the Problem. Let in an elastic unbounded isotropic material at the initial instant of time start to propagate at a constant rate v a system of n (n > 1) discontinuities with width W distant from each other at the same angle α_0 (Fig. 3).

In this case the equations for material movement will be wave equations

$$\Delta\Phi = \frac{1}{c_p^2}\frac{\partial^2\Phi}{\partial t^2},\ \Delta\Psi_i = \frac{1}{c_s^2}\frac{\partial^2\Psi_i}{\partial t^2},$$

where

$$\Delta = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2};\quad i = x, y, z;$$

c_p and c_s are longitudinal and transverse wave velocities; potentials Φ and $\Psi(\Psi_x, \Psi_y, \Psi_z)$ are connected with the displacement vector by the relationship

$$\mathbf{U} = \text{grad}\ \Phi + \text{rot}\Psi,\ \text{div}\Psi = 0.$$

Boundary conditions at the edges of discontinuities are prescribed in the form

$$U_\alpha = B_\alpha H(t - r/v),\ \sigma_{r\alpha} = \sigma_{z\alpha} = 0,$$

when 0 < z < W (Fig. 3), and

$$U_\alpha = 0,\ \sigma_{r\alpha} = \sigma_{z\alpha} = 0,$$

when $z < 0$ and $z > W$ with $\alpha = 2\pi k/n$, $k = 0, \ldots, n - 1$, $r^2 = x^2 + y^2$, $\alpha = \operatorname{arctg}(y/x)$. Here H is the Heaviside function; B_α is the component of the movement vector at the discontinuity ($B_\alpha = \text{const}$); σ_{ij} are stress tensor components; r, α, z are cylindrical coordinates. The initial conditions are zero, and at infinity the material is at rest.

4. Construction of the Solution. In order to obtain a stringent mathematical solution of the problem it is sufficient to consider the sector $0 \leq \alpha \leq 2\pi/n$ (in view of symmetry). However, this approach is often connected with serious mathematical problems. Therefore, following the principles developed in [15, 16, 18] based on using the superposition principle we present a general wave field caused by propagation of a set of n discontinuities in the form of the sum of wave fields created by each of these discontinuities individually. In view of this in this case it will be sufficient to consider the problem when $n = 1$ and the solution for each subsequent discontinuity with $n > 1$ will have the same form taking account of changes in variables connected with rotation by angle α_0. This simplified approach leads to the situation that there is no consideration of surface and reflected waves which arise around discontinuities comprising the star-shaped system. These waves may only have a marked effect in the immediate vicinity of discontinuities and it is possible to assume that the overall picture of the wave field does not change at a distance from them. A solution for the problem of a discontinuity starting with velocity v and which ends up complex (shear with separation) was obtained by means of the Kanyar method in [17] and in the notation of this work considering that shear components of the movement vector for equal zero have the form

$$
\begin{gathered}
U_i = U_i^p + U_i^s,\ i = x, y, z,\\
U^{p,s} = U^{p,s}(x, y, z, t) - U^{p,s}(x, y, z - W, t)\\
- U^{p,s}(x - \mathscr{L}, y, z, t - \mathscr{L}/v) + U^{p,s}(x - \mathscr{L}, y, z - W, t - \mathscr{L}/v),\\
U_x^p(x, y, z, t) = A\left\{\left[(1 + \beta_1)\ln_1^p + \frac{\beta_1\gamma}{2\gamma_p}\mathrm{Ln}_1^p + f_p\right](H_1^p - H_2^p) + \left[\Gamma_2^p + \frac{\beta_1\gamma}{2\gamma_p}\mathrm{Ln}_2^p\right]H_2^p\right\},\\
U_x^s(x, y, z, t) = -A\{[\beta_1\ln_1^s + \gamma\gamma_s\mathrm{Ln}_1^s + f_s](H_1^s - H_2^s) + [\Gamma_2^s + \gamma\gamma_s\mathrm{Ln}_2^s]H_2^s\},\\
U_y^p(x, y, z, t) = A\{[F_p - \beta_1\mathrm{arc}_1^p](H_1^p - H_2^p) + [\Gamma_3^p - \beta_1\mathrm{arc}_2^p]H_2^p\},\\
U_y^s(x, y, z, t) = A\{[2\gamma^2\mathrm{arc}_1^s - F_s](H_1^s - H_2^s) + [2\gamma^2\mathrm{arc}_2^s - \Gamma_3^s]H_2^s\},\\
U_z^p(x, y, z, t) = A[(\gamma_s^2 + 1)\ln_2^p + \Gamma_4^p]H_2^p,\\
U_z^s(x, y, z, t) = -A[\gamma_s^2\ln_2^s + \Gamma_4^s]H_2^s.
\end{gathered}
\tag{4.1}
$$

Here

$$
\begin{gathered}
\sin\psi = y/\rho_1,\ \cos\psi = x/\rho_1,\ \sin\eta = y/\rho_2,\ \cos\eta = x/\rho_2,\\
\rho_1 = \alpha_z R,\ \rho_2 = \alpha_x R,\ R^2 = x^2 + y^2 + z^2,\ \gamma = c_p/v,\\
\alpha_x = \sqrt{1 - \nu_x^2},\ \alpha_z = \sqrt{1 - \nu_z^2},\ \nu_x = \frac{x}{R},\ \nu_y = \frac{y}{R},\ \nu_z = \frac{z}{R},\\
\Gamma_1 = -\nu_y\nu_z[\tau\nu_x(3 - \nu_z^2) + 2\gamma\alpha_z^2]\tau\alpha_z^{-4},\\
\Gamma_2^{p,s} = -\tau\nu_z[\tau\alpha_x^2(2\nu_y^2 - \nu_y^2\nu_z^2 - \nu_x^2) - 2\gamma\nu_x(1 + \nu_y^2)\alpha_z^2]\alpha_x^{-2}\alpha_z^{-4} + 2\gamma\alpha_x^{-1}\nu_z T_2^{p,s},\\
\Gamma_3^{p,s} = -\Gamma_1 - \tau\nu_y\nu_z\alpha_x^{-4}[2\gamma(1 + \nu_x^2) - \tau\nu_x(3 - \nu_x^2)] + 2\nu_y\nu_z\alpha_x^2\tau_2 T_2^{p,s},\\
\Gamma_4^{p,s} = -\tau[\tau\nu_x(2\nu_y^2 - \nu_x^2\nu_y^2 - \nu_z^2) + 2\gamma(\nu_z^2 - \nu_x^2\nu_y^2)]\alpha_x^{-4} + (\nu_z^2 - \nu_y^2)\frac{\tau_2}{\alpha_x^2}T_2^{p,s},\\
F_{p,s} = (\tau_1\sin2\psi + 2\gamma\sin\psi)T_1^{p,s},\ f_{p,s} = (\tau_1\cos2\psi + 2\gamma\cos\psi)T_1^{p,s},\\
\mathrm{arc}_1^{p,s} = \operatorname{arctg}\frac{T_1^{p,s}\sin\psi}{\gamma - \tau_1\cos\psi},\ \mathrm{arc}_2^{p,s} = \operatorname{arctg}\left(\frac{\tau_2}{T_2^{p,s}}\operatorname{ctg}\eta\right),\\
\mathrm{Ln}_1^{p,s} = \ln\frac{(\tau_1\gamma_{p,s} - \gamma T_1^{p,s})^2\sin^2\psi + [\beta_{p,s}^2 - (\tau_1\gamma - \gamma_{p,s}T_1^{p,s})\cos\psi]^2}{\beta_{p,s}^2[(\gamma - \tau_1\cos\psi)^2 + (T_1^{p,s}\sin\psi)^2]},\\
\mathrm{Ln}_2^{p,s} = \ln\frac{(\gamma_{p,s} - T_2^{p,s}\cos\eta)^2 + \tau_2^2\sin^2\eta}{\tau_2^2 + \gamma_{p,s}^2\sin^2\eta},\ \ln_1^{p,s} = \ln\frac{\tau_1 + T_1^{p,s}}{\beta_{p,s}},\\
\ln_2^{p,s} = \ln[(\tau_2 + T_2^{p,s})\alpha_x],\ T_1^{p,s} = \sqrt{\tau_1^2 - \beta_{p,s}^2},
\end{gathered}
$$

$$T_2^{p,s} = \sqrt{\tau_2^2 + \gamma^2 - \beta_{p,s}^2},\ \tau = tc_p/R,\ \tau_1 = tc_p/\rho_1,$$
$$\tau_2 = (tv - x)\gamma/\rho_2,\ \beta_p = 1,\ \beta_s = \beta = c_p/c_s,$$
$$\gamma_{p,s} = \sqrt{\gamma^2 - \beta_{p,s}^2},\ \beta_1 = 2\gamma^2 - \beta^2,\ A = B_\alpha/(4\pi\beta^2),$$
$$H_1^{p,s} = 2H(z)H(\tau_1 - \beta_{p,s}),\ H_2^{p,s} = H(\tau - \beta_{p,s})$$

($\mathcal{L}$ is discontinuity length, W is width).

In order to use solution (4.1) it is necessary to determine the length of cracks and their intrinsic opening. For this it is necessary to find the radius of zone 3 (Fig. 1) and the volume of the cavity which forms as a result of expansion of the cavity with explosive and crack growth around it.

5. Determination of Crack Length and Average Opening. We assume that external mine pressure P_0 operates at a distance from the explosion cavity and in the future we shall assume that as a result of the final stage of development of a camouflet explosion from a blasthole charge in a brittle material in the explosion cavity r_1 and within the system emerging from it consisting of n radial cracks of length r_3 there is some detonation product pressure P_1. We also assume that the temperature of the detonation products is close to that of the material. Then gas pressure may be determined from the volume of the cavity and cracks by means of the gas law on condition that the volume of gases which forms as a result of detonation under normal conditions is known. Assuming that crack length r_3 is much greater than cavity radius r_1 (at least $r_3 \geq 3r_1$), in future we shall assume that the whole volume of gases which form with detonation of an explosive charge equals the volume of the cavities formed by a system of n radial cracks.

In order to determine crack length it is also necessary to introduce an additional condition which would describe the final failure process, i.e. crack stopping. We assume that with completion of the development of a system of n radial cracks each of them will be in limiting equilibrium and the stress intensity factor K_I at their tips will equal the critical factor for crack stopping K_{Ic}. We also assume that in each section perpendicular to the blasthole charge (Fig. 1) the material is in a condition indistinguishable from plane strain.

We give briefly an approximate solution of this problem following from the main suggestions of an approach given in [13].

In order to determine gas pressure P_1 we estimate the volume of the cavity and cracks. We assume that on a circle of radius r_3 passing through the tips of cracks radial stress with a value P_1 operates. The material within the circle ($r < r_3$) will be compressed by all-round pressure P_1. Then the change in rock volume within the circle ($r < r_3$) along the blasthole length has the form

$$\Delta V_1 = 2\pi P_1 W(1 - 2\nu)(1 + \nu)(r_3^2 - r_1^2)/E,$$

where W is blasthole length (width of discontinuities); ν is Poisson's ratio; E is Young's modulus.

Movement of a circle of radius r_3 under the action of internal pressure P_1 and external mine pressure P_0 leads to an additional change in volume

$$\Delta V_2 = 2\pi W r_3^2(1 + \nu)\{P_1 - 2P_0(1 - \nu)\}/E.$$

The total increase in gas volume due to expansion of the cavity and crack opening along the length of the blasthole with $r \geq 3r_1$ is estimated by the expression

$$\Delta V = \Delta V_1 + \Delta V_2 = 4\pi W r_3^2(1 - \nu^2)(P_1 - P_0)/E. \tag{5.1}$$

Unknown pressure P_1 in (5.1) is found from the gas law

$$P_1(\pi W r_1^2 + \Delta V) = P_n V_n. \tag{5.2}$$

Here V_n is the volume of explosion products under normal conditions (pressure $P = P_n$ and temperature $T = T_n$).

By substituting (5.1) in (5.2) we have

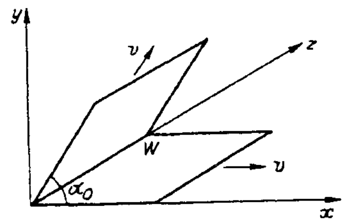

Fig. 3

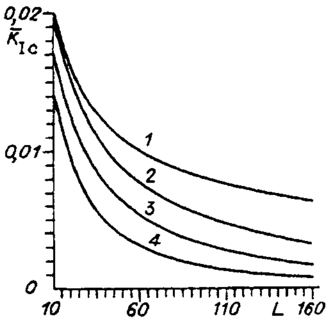

Fig. 4

$$P_1 = E[A_1 - 1 + (1 + 2A_2(2F_1 - \bar{P}_0)L^2 + A_1^2)^{1/2}]/(2A_2L^2), \tag{5.3}$$

where

$$F_1 = P_n V_n/(\pi E W r_1^2);\ \bar{P}_0 = P_0/E;\ L = r_3/r_1;$$
$$A_1 = A_2\bar{P}_0L^2,\ A_2 = 4(1 - \nu^2).$$

In the case when $r_3 \geq 3r_1$ from (5.3) we obtain

$$P_1 = \frac{P_0}{2} + \frac{1}{2}\sqrt{\frac{P_n V_n E}{\pi W(1 - \nu^2)r_3^2} + P_0^2}.$$

Then in order to determine the crack length we use a solution [13] for the stressed state of a star-shaped system consisting of n discontinuities equal in length distributed through the same angles and under the effect of internal pressure equal to the difference in gas pressure P_1 and external mine pressure P_0. In the case of limiting equilibrium we have the condition [13]

$$K_1 \approx 2\sqrt{\pi r_3/n}\,(P_1 - P_0) = K_{Ic}. \tag{5.4}$$

In dimensionless form relationship (5.4) may be presented as:

$$\bar{K}_{Ic} = 2\sqrt{\pi L/n}\,(\bar{P}_1 - \bar{P}_0). \tag{5.5}$$

Here

$$\bar{K}_{Ic} = K_{Ic}/(E\sqrt{r_1});\ \bar{P}_1 = P_1/E.$$

Relationship (5.5) connects the charge parameters (V_n, r_1) and rock parameters (P_0, K_{1c}, E, ν) with crack length r_3 and the number of cracks n.

For $P_0 = 0$ and $r_3 \geq 3r_1$ relationship (5.4) is simplified.

$$K_{Ic} = \sqrt{\frac{P_n V_n E}{W n r_3(1 - \nu^2)}},\ r_3 = \frac{P_n V_n E}{W n K_{Ic}^2(1 - \nu^2)}.$$

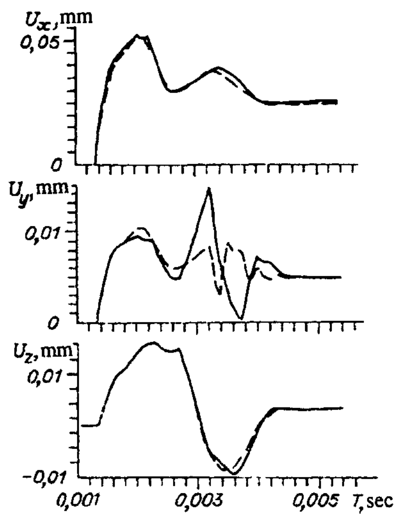

Fig. 5

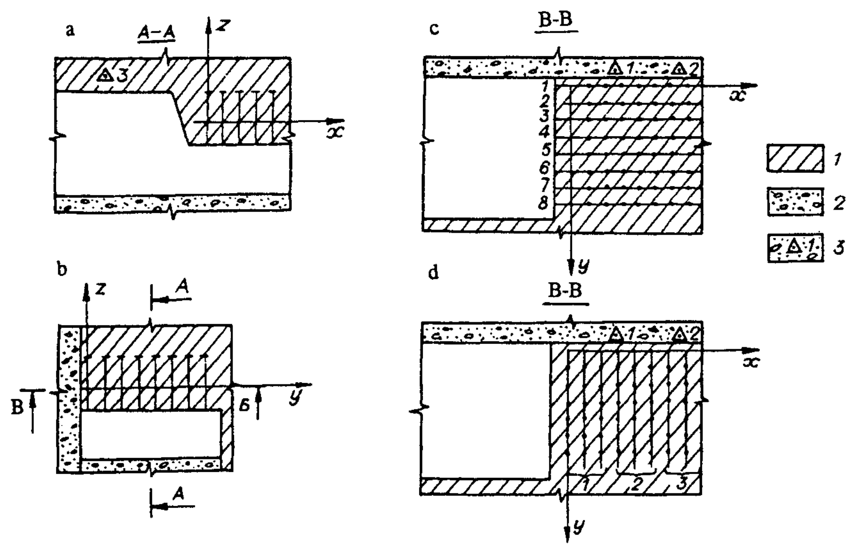

Fig. 6

In determining average crack opening it should be noted that in [18] it was shown that the form of seismic emission does not depend markedly on a function describing the jump in displacement in a rectilinear section of the failure surface, and with an equal volume of cracks and dislocation discontinuities the form of the seismic signal is almost the same. Therefore in future we shall assume that the average vector for the jump in displacements in each of n discontinuities is the same and it may be determined from the relationship

$$B_\alpha^{av} = \frac{\Delta V}{nr_3 W} = \frac{4\pi r_3}{En}(1 - \nu^2)(P_1 - P_0).$$

6. Analysis of the Results. The expressions obtained entirely characterize the test model and make it possible to determine the length and opening of radial cracks as a function of material, explosive charge, and external mine pressure parameters. If as an explosive we take granulite AS-8 with V_n = 850 liter/kg, r_0 = 0.024 m, W = 2.7 m, and explosive weight in the blasthole of 5 kg, then for marble with E = $3 \cdot 10^{10}$ Pa and K_{Ic} = 3 MPa·m$^{1/2}$ [13] we obtain with external mine pressure $P_0 = 1.5 \cdot 10^7$ Pa r_3 = 4.2 m, and opening B_α^{av} = 0.08 m when the number of radial cracks n = 4 and B_α^{av} = 0.053 m when n = 6.

Given in Fig. 4 are curves for the dependence of the value $L = r_3/r_0$ on material parameters $\bar{K}_{Ic}$ obtained from relationships (5.3) and (5.5). Curves 1-4 correspond to $\bar{P}_0 = P_0/E$ = 0; 0.0005; 0.001; 0.002. In order to plot the curves in

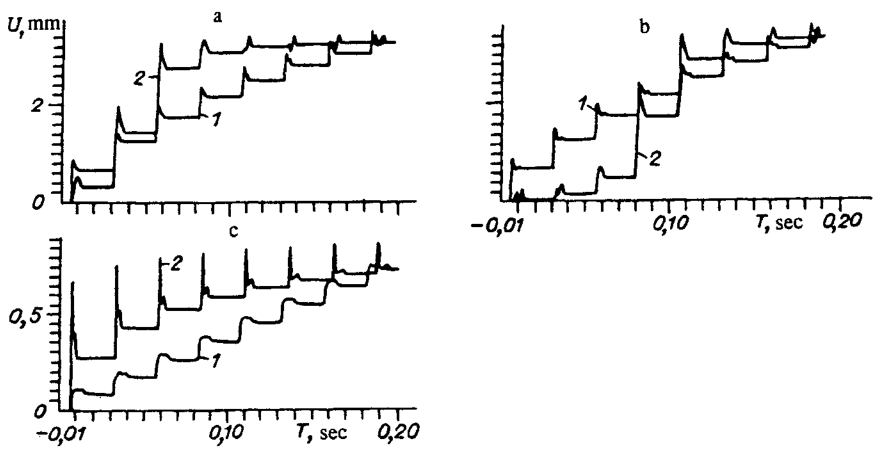

Fig. 7

Fig. 4 the explosive was granulite AS-8, the blasthole was 2.7 m long, the blasthole radius was 0.024 m, and the explosive weight in the blasthole was 5 kg. Thus, from Fig. 4 knowing previously or determining by experiment the material parameters, external mine pressure, and initial blasthole radius, it is possible to find the length of the radial cracks which form. The reverse is also true: by measuring the length of radial cracks formed as a result of an explosion it is possible from Fig. 4 to determine such an important material parameter as K_{Ic}.

Given in Fig. 5 are theoretical seismograms plotted from solution (4.1) for the number of radial cracks n = 4 (solid lines) and n = 6 (broken lines). Crack length was assumed to equal 0.64 m, the width was 2.7 m, opening was 0.0015 m when n = 4, and 0.001 m when n = 6, and their propagation rate was 700 m/sec. Velocities were c_p = 2940 m/sec, and c_s = 1650 m/sec. The coordinates of the point of observation are x = 5 m, y = 1 m, z = 3 m (the coordinate system shown in Fig. 3).

From Fig. 5 it is possible to conclude that the form of the seismograms obtained is governed to a considerable extent by the explosive energy and the parameters of the surrounding material.

This algorithm for calculating seismic emission from an explosive source may be used effectively for estimating the seismic effects in pits during extraction of economic minerals where simultaneously several rows of blastholes are exploded by some scheme. We dwell on analyzing the results of blasting with simultaneous initiation of blasthole charges.

Explosive breaking of ore with a layered system of working is carried out mainly using successive-transverse and successive-longitudinal explosive initiation schemes. Presented in Fig. 6 is a general scheme for explosive breaking of ore with a continuous layered system of working and stowing of the worked-out space (*a* is view from the side, b is view from in front, c and d are views from below for successive-longitudinal and successive-transverse schemes, A–A, B–B are vertical and horizontal sections of the working respectively, 1 is ore mass, 2 is stowed mass, 3 are observation points for which seismic effects will be calculated).

For a successive-longitudinal scheme (Fig. 6c) numbers 1-8 denote the sequence of initiation of rows of blasthole charges: at the start the first row of charges is exploded, then the second, etc. With a successive-transverse scheme charges are exploded in groups consisting of several rows of blasthole charges in the order shown by numbers in Fig. 6d. The delay time between the explosion of rows and groups of charges is 0.025 sec, and the distance between blastholes is 0.9 m.

The equations provided above entirely characterize a single explosive source. In carrying out drilling and blasting work it is normal to explode at one stroke a large number of blasthole charges (in Fig. 6c, d the overall breaking scheme is presented; normally the number of blastholes along axis x is 24 and the total number of blastholes in a layer is 24 × 8 which are exploded in groups of 24 charges simultaneously with a delay of 0.25 sec). Construction of the explosion models for successive longitudinal and successive-transverse schemes also rests on the superposition principle: the overall wave picture created by explosion of blasthole charges is represented by the sum of wave fields created by each blasthole individually. It is assumed that explosion of a single charge does not affect the nature of the occurrence of the explosive process of another charge. The parameters of radial cracks which arise near a blasthole are determined by the relationships provided above as if it were a single charge.

Presented in Fig. 7 are theoretical seismograms plotted by solution (4.1) for successive-longitudinal and successive-transverse initiation schemes for explosive charges (curves 1 and 2 respectively). The coordinate system is shown in Fig. 6. The number of blastholes is 24 × 8. The coordinates of the points of observation were as follows (in meters): x = 5.175, y

$= -1.5$, $z = 2.65$ (Fig. 7*a*); $x = 10.35$, $y = -1.5$, $z = 2.65$ (b); $x = -5$, $y = 4$, $z = 3.7$ (c). The first two observation points are at the boundaries of the broken layer in the stowing mass, and the third is in the roof. The points of observation are shown in Fig. 6 by triangles and numbers 1-3.

From Fig. 7 it is possible to draw the following conclusions:

– in theoretical seismograms it is easy to reveal areas corresponding to the arrival of a seismic wave from each delayed group (in this case eight of them) which makes it possible to analyze the wave field as a whole and individually for each delayed group;

– the amount of displacement in the rock mass (seismic effect) with initiation of explosive charges by a successive-transverse scheme appears to be greater than by a successive-longitudinal scheme. This may be observed for the overall displacement vector $U = (U_x^2 + U_y^2 + U_z^2)^{1/2}$.

The second conclusion may be important for practice with the aim of introducing into production a successive-longitudinal scheme since according to production instructions it is normal to use a successive-transverse breaking scheme.

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