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VALIDITY AND RELIABILITY OF BIOSTEREOMETRIC MEASUREMENT OF THE HUMAN FEMALE BREAST

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A measurement technique has been developed for application in the area of noninvasive breast cancer detection. The measurement process involves the use of close-range stereophotogrammetry as a data acquisition device necessary for determination of breast volume and volume distribution. This report details the methodology used to acquire and analyze stereopair photographs necessary to document the validity and reliability of this application. The volume of a test object was determined by both water displacement and stereophotogrammetric analysis to estimate the precision of the proposed methodology. Additionally, the reliability component of the study was documented by analyzing variability of coordinates representing a series of locations marked on the surface of an irregularly shaped object. Both tests confirm that this stereometric analysis is a reliable and valid method of measurement and may be well suited for further development in the field of breast cancer detection.

Keywords—Stereophotogrammetry, Validity, Reliability, Breast volume.

INTRODUCTION

It has long been recognized by both women and physicians that the human female breast undergoes volumetric changes during many physiologic and pathologic situations. Pathologists, for example, have demonstrated that changes in cancerous breasts, in addition to the tumor itself, such as increased fibrous tissue, collagen deposits, aci-

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nar and ductal dilatation with epithelial proliferation and skin thickening, exist that should be reflected by a change in the volume (3,4,12,15,19). Historically, in spite of the general recognition and acceptance of these claims, factual documentation of volumetric changes in the breast has been difficult to attain (due to the lack of a simple, accurate and reproducible method of breast volume determination). Ingleby (11) in 1949 reported in a study designed to determine whether measurable changes in breast volume took place during the menstrual cycle and whether such changes conformed to any patterns, that the actual measurements of the breast were far more difficult and time consuming than anticipated. The method used in her study included use of paraffin models made from plaster molds of the breast. These models were weighed and the volume calculated. This work is a classic study of breast volume in spite of the fact it was noted to be cumbersome, time consuming, and therefore, not applicable to the study of the large numbers of women necessary to determine scientifically whether or not the breast volume changes did occur and could be correlated to various pathologic conditions. Other methods have been developed to determine breast volumes and include direct measurement and computations made from mammograms in an effort to determine the incidence of cancer as related to the volume of the right and left breasts (13). Milligan, *et al.* (14) as well as Hamilton and Rankin (6) measured breast volumes by use of water displacement for the purpose of documenting volumetric changes during the menstrual cycle. Hamilton and Rankin noted, however, in their study that repeated breast volume measurements were within ±6%. Therefore, based on the apparent paucity of information regarding accurate breast volume measurements in the literature, a study was developed for the purpose of determining if a biostereometric measurement procedure would be a sufficiently sensitive and reliable measurement of breast shape as represented by volume and volume distribution measurements.

Biostereometrics is defined as the spatial and spatio-temporal analysis of biological form and function based on the principles of analytic geometry. Herron (7) based the need for human biostereometrics on the fact that humans, like all organisms, are comprised of irregular shapes and configurations which can often be better quantified by three-dimensional measurement rather than the two-dimensional parameters used in traditional physical anthropometry.

Many three-dimensional form sensors exist such as Moire interferometry, holography, light split techniques and mechanical or electromechanical devices. However, the most widely used and versatile stereometric sensor is stereophotogrammetry.

In describing the stereophotogrammetric technique and its many biomedical applications, Herron (8) discussed advantages with relation to in vivo studies of the human body. First, the technique is a non-contact procedure that will not deform sensitive or soft tissue during measurement. This advantage has also been cited by Pierson (16). Burke (2) and Atkinson and associates (1), who noted the importance of the non-contact advantage, stressed the ability to examine sensitive tissues such as the eyes and open wounds using the stereophotogrammetric technique. Second, measurement can be made to a set level of accuracy using stereophotogrammetry. Terada (18), Burke (2), and Atkinson and associates (1) have also noted the accuracy in a variety of measurement tasks. Hertzberg and others (10) stated after comparing anthropometric and photographic measurements, "In our opinion, stereoanthropometry is much more precise than hand anthropometry, the question is no longer how accurate is the system, but how accurate is the man." Third, the "contact time" with the subject is greatly reduced over that of traditional anthropometric measures (8). The fourth advantage noted by Herron (7) is the ease with which data can be mathematically manipulated to generate desired numerical parameters. Coupled with this advantage is the fact that the exposed stereopair contains all of the data necessary for reconstructing a complete spatial model of the body for further retrospective examination.

The role of stereophotogrammetry is to produce and quantify a spatial model of a three-dimensional object. This methodology can be compared to the principle of human depth perception, where an object is viewed simultaneously from two points by cameras rather than eyes. When the photographs are properly oriented in a stereoplotter, a precise model of the object is projected and can be measured accurately.

Hallert (5) described the geometric principles responsible for accurate determination of the third-dimension (depth) as a relationship between image parallax and the height of an object (Fig. 1). Mathematically the relationship is expressed as:

$$\Delta P = \Delta p \frac{(H - \Delta H)}{c}$$

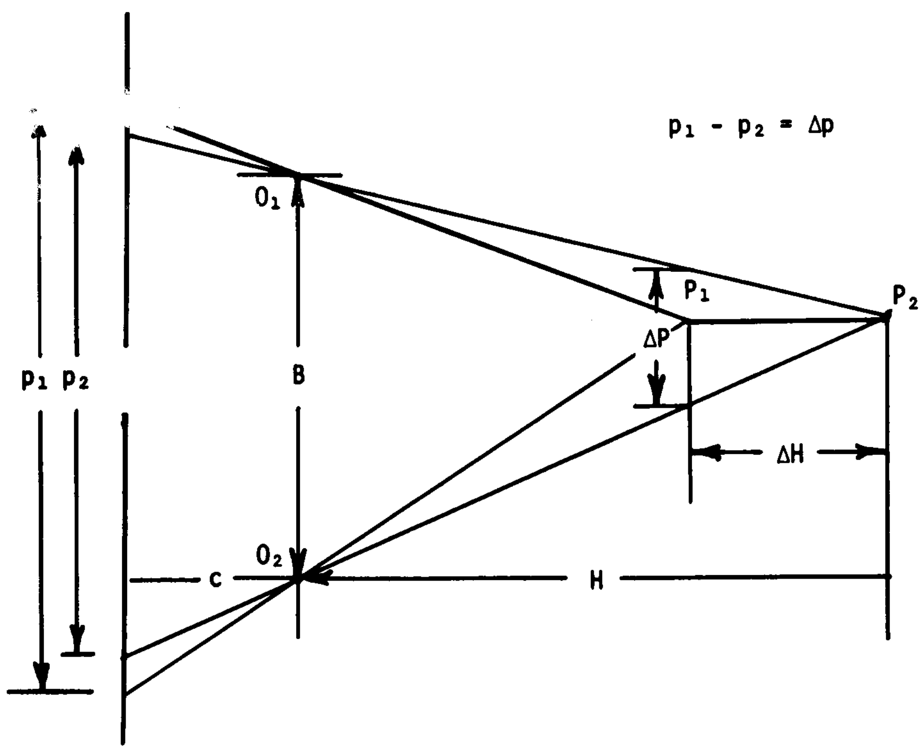

FIGURE 1. Image parallax difference (ΔP) is the basis for measuring the third dimension (ΔH). Horizontal parallax (Δp) is measured in the data reduction process. O_1 and O_2 are the camera lenses. Camera to reference plane (H), principal distance (c) and camera base (B) are known values.

where ΔP is the image parallax, Δp is the horizontal parallax, and ΔH is the object height, H is the camera to object distance and c is the principal distance (lens to image plane).

Also from similar triangles:

$$\frac{\Delta H}{\Delta P} = \frac{H}{B}$$

where B is the camera base.

Substituting for ΔP,

$$\Delta H = \frac{\Delta p(H - \Delta H)H}{Bc}$$

Once the horizontal parallax is measured (Δp), ΔH can be calculated since H, B, and c are known values.

Specifically this paper deals with determination of measurement error variations expressed as validity and reliability of the biostereometric measurement technique and protocol for breast volume documentation.

METHODS

Data for the determination of both the validity and reliability of close-range stereophotogrammetry required a series of photographic recordings of test objects using stereometric cameras specially designed for close-range photogrammetry. Wide angle stereometric cameras[1] were used to satisfy the need for metric (precision) quality imagery (Fig. 2). The cameras were equipped with biogon lenses having an effective focal length of 90 mm and a maximum aperture of f/5.6. Special features of the Kelsh K-460 cameras included a vacuum film plane that insured a perpendicular relationship between the film emulsion and the principal axis of the lens as well as specially designed fiber optics which produced fiducial marks at the moment of film exposure (Fig. 3). The fiducials provided information regarding internal geometry of the camera needed for proper orientation of the films for stereo-plotting data processing. The individual cameras were mounted on a double rail support to insure stability and to provide suitable external orientation of the stereometric camera to the test object.

The primary source of illumination of the test objects, the Surface Contrast Optical Projector (SCOP) was mounted on the double rail support between the two cameras. This projector served not only to illuminate the object being photographed but also projected a random contrast pattern on the surface of the test object, thereby increasing the ability of stereopsis of the monochromatic surface and therefore increasing the accuracy of measurement. The photographic imagery was recorded on Kodak Linograph Shellburst film using a format of 105 × 178 mm. This film was selected because of its fine grain and high resolving power, both features necessary for accurate recording of small details.

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[1]Kelsh K-460 stereometric cameras, Danko-Arlington Kelsh, Inc., Baltimore, MD.

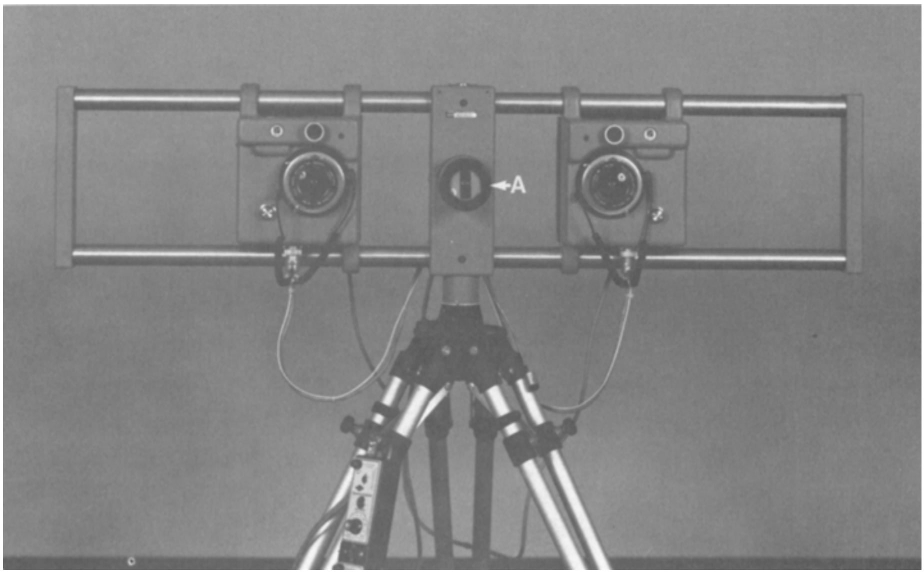

FIGURE 2. Kelsh K-460 stereometric camera with surface contrast optical projector (B).

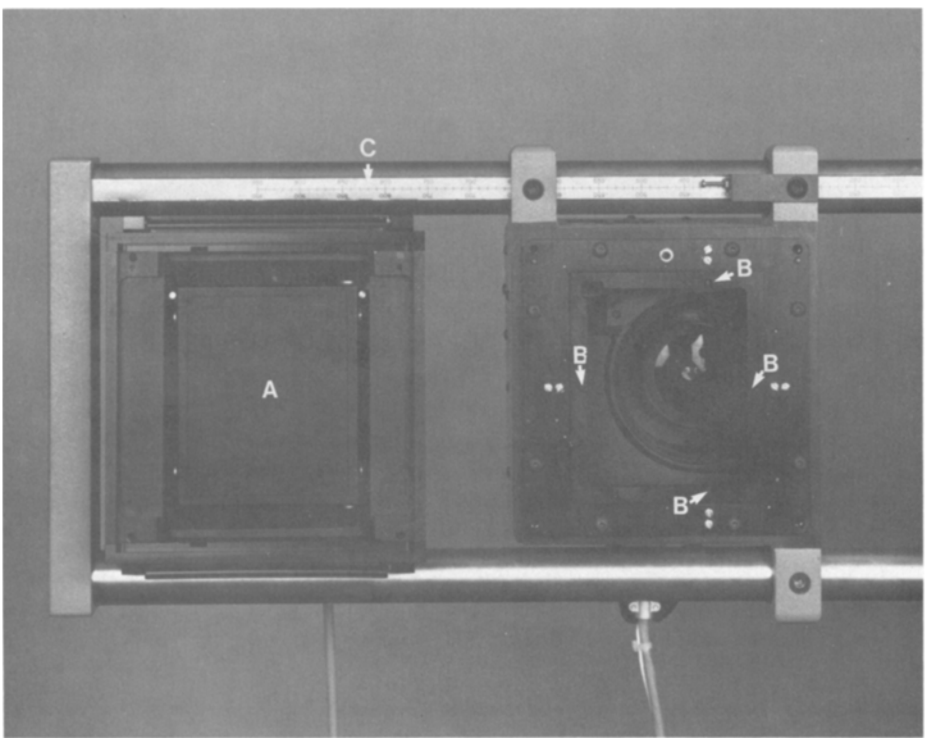

FIGURE 3. Individual camera opened to show (A) vacuum film plane, (B) fiber optic terminations for exposing the fiducial marks on the film, and (C) the scale for presetting the cancer base distance.

CONTROL REFERENCE

The test objects were photographed between two control stands (Fig. 4) which provided knowledge of the object with reference to the stereometric camera. This positioning allowed the transformation of the surface coordinates to a single external system for analysis. The control stands also provided information that ultimately allowed for the completion of leveling, scaling and parallax removal of the stereomodels. Of particular importance were the features that provided scaling information. Each reference stand supported graduated steel tapes. The determination of the length of the known distance on the tape provided the necessary scale factor for the measurement of height or the y axis of the stereomodel. Rods of precisely known lengths were also attached to the control stand and enabled the operator to determine the z or depth scale.

Data Acquisition

For the purpose of this study, two solid objects were selected as the test subjects. A cone was used for the validation of the volumetric computations. Second, the torso of a mannequin with a series of 32 points located by pressure sensitive markers on the surface was provided for the examination of point location repeatability. Each

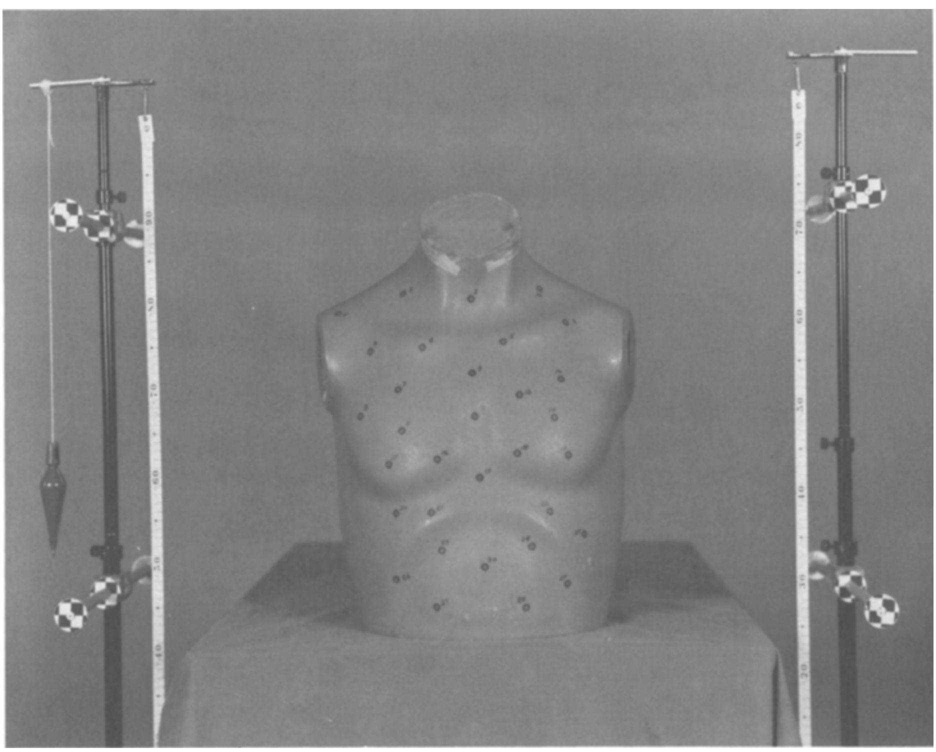

FIGURE 4. Control reference stands with steel tapes for scaling in the x and y directions and the four rods used in determining the depth or z axis scaling factor as well as mannequin torso used in the reliability study.

of these test objects was placed between the control stands and was photographed using the stereometric cameras. These photographs served as input for the data reduction phase of the biostereometric process. Additionally, the conical test object was subjected to repeated water displacement procedures for determination of a criterion volume measurement.

Data Reduction

Following film development, the data were reduced on a modified stereo-plotter.[2] The plotter allowed the operator to obtain coordinates for a series of points lying on the surface of the test objects. In this study the Cartesian coordinate system was used, i.e. x, y, and z being orthogonally oriented at 90 degrees to one another. A continuous digital readout of all three coordinates appeared on an H. Dell Foster RSS-4-MGT Metric Graphic Terminal. The operator then transferred the representative coordinates for each desired surface point to a magnetic tape storage device which served as input to the IBM 3033 computer at the University of Akron used for production of the desired volume and point reliability measurement output.

Proper orientation of the enlarged stereopairs required the replacement of the camera by stereoplotter optics. Because of the camera design and the information yielded by the reference or control stands, restitution of the spatial relationship between images, cameras and the coordinate system enabled production of an accurately scaled model for measurement. This restitution required three basic steps. First using the fiducial marks made on the corners of the images, the individual enlargements were centered properly on the stereoplotter. The principal distance of the stereoplotter was set at a value of 97.58 mm which equalled the lens to film distance of the camera. This step, known as internal orientation, reproduced the relationship between the fiducials and the lens during the initial exposure of the film.

Second, the relative orientation step of the plotting process consisted of restoring the exact position of the two photographs that comprise the stereoscopic image to the geometric relationship of the taking cameras at the moment of film exposure. Basically, relative orientation is achieved by orienting the two photographs so that corresponding pairs of rays from each image will intersect in space. For our purpose, five rays representing points on the control stand tapes are made to intersect by rotating the individual images about their axes in κ, ϕ, and ω (Fig. 5) as well as translating the two image coordinate systems in the y and z directions until the origins coincide. The intersection of a sixth pair of rays is used to check the results of this orientation step.

Again, using the steel tapes for the final step (absolute orientation) the three-dimensional image was rotated in omega (Ω) about the x axis (Fig. 6), about the y axis in phi (Φ) and about the z axis in kappa (K); scaled and translated in x, y and z. This step made the chosen image plane parallel with that of the x, y plane of the stereoplotter (20).

A systematic approach for acquiring coordinates representing the surface of the cone was used. The points were read in parallel contours separated by a .25 cm interval. These contours lay parallel to the x,y or frontal plane of the control stand. The

[2]Kern PG-2 Stereoplotter, Kern Instrument Co., Aarau, Switzerland.

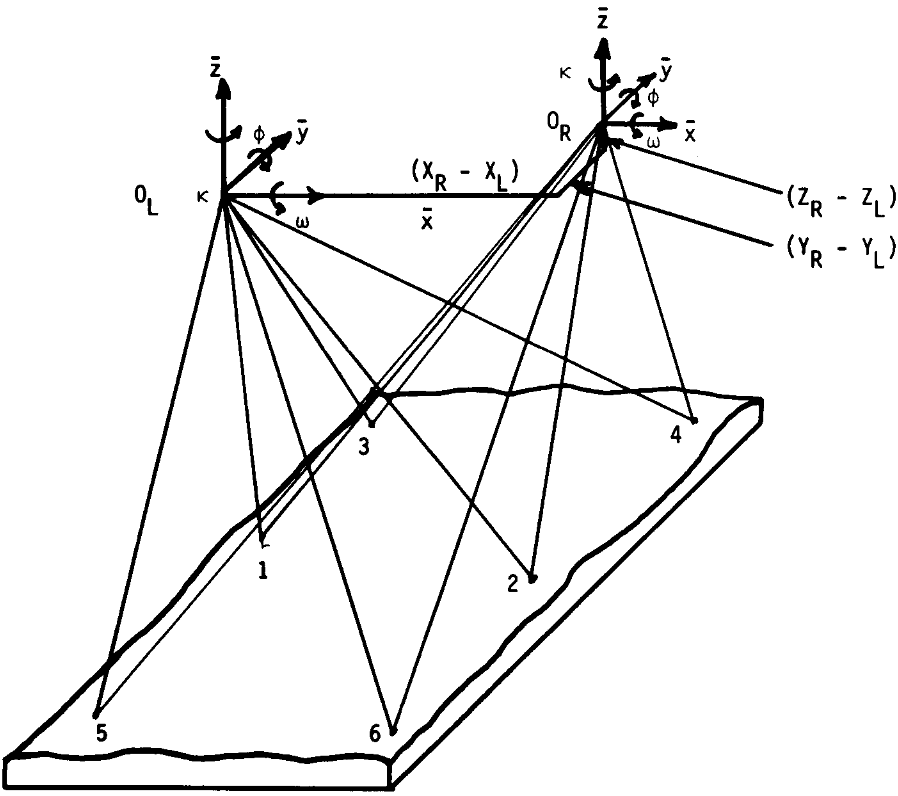

FIGURE 5. Geometric conditions showing the possible rotations and translations necessary for relative orientation. (Adapted with permission from *Manual of Photogrammetry*, Fourth Edition.)

z or depth coordinate was fixed at the predetermined level and then the image was scanned in closed contours and the x, y coordinates for each contour were recorded. The z coordinate was then repositioned and the image again scanned. This procedure was repeated until all predetermined levels of z from the apex to the base of the cone were examined.

The approach used for acquiring coordinates of the reliability portion of the study involved the digitization of the 32 markers located on the surface of the mannequin. These targets were digitized a total of 32 times each. The sequencing for recording these points was randomized.

Data Analysis

Validity Study. The data obtained from the cone were analyzed using an algorithm to determine the volume of the cone from the apex to the base. The algorithm makes use of Green's Theorem for reducing a double integral to a line integral in order to compute the area which is contained within each contour. The resulting areas are then subjected to the trapezoidal rule approximation for determination of area.

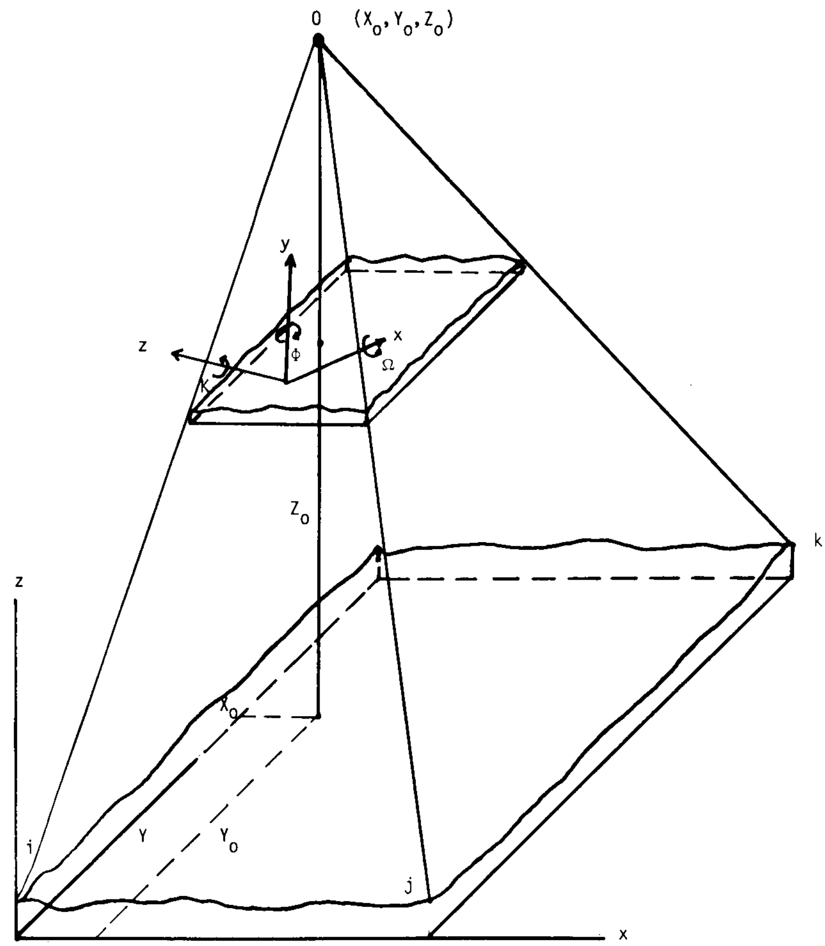

FIGURE 6. Absolute orientation, final rotation and translations to bring the image plane parallel to that of the stereoplotter. (Adapted with permission from *Manual of Photogrammetry*, Fourth Edition.)

Computing the Area of a Region R

Let C be a smooth simple closed curve and let R be the region consisting of the interior of C. The purpose of this section is to describe an algorithm which can be used to approximate the area A of R. This technique is based on Green's Theorem.

Green's Theorem. Let C and R be as described above. If $M(x,y)$ and $N(x,y)$ are functions that are continuous and have continuous first partial derivatives throughout an open region D containing R, then

$$\oint_C M\mathrm{d}x + N\mathrm{d}y = \int\int_R \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right)\mathrm{d}A \qquad (1)$$

where dA is the element of area.

Now, let $M=0$ and $N=x$ in Eq. 1 to obtain

$$A=\int\int_R \mathrm{d}A=\oint_C x\mathrm{d}y \ . \tag{2}$$

Likewise, letting $M=-y$ and $N=0$ yields

$$A=\int\int_R \mathrm{d}A=\oint_C y\mathrm{d}x \ . \tag{3}$$

Upon adding Eqs. 2 and 3 one finds that

$$2A=\oint_C x\mathrm{d}y-y\mathrm{d}x \ ,$$

or, more conveniently,

$$A=\frac{1}{2}\oint x\mathrm{d}y-y\mathrm{d}x \ . \tag{4}$$

Equation 4 indicates that the problem of approximating the area of the region R is reduced to approximating the integral

$$\frac{1}{2}\oint x\mathrm{d}y-y\mathrm{d}x \ .$$

To this end let $\alpha_1(t)$ and $\alpha_2(t)$, $t\epsilon[0,1]$, be parametric equations for C, i.e., C is continuously described as t ranges over the values in the interval $[0,1]$. Using the changes of variables $x=\alpha_1(t)$ and $y=\alpha_2(t)$, and the fact that $0\le t\le 1$, Eq. 4 becomes

$$A=\frac{1}{2}\int_0^1(\alpha_1(t)\alpha_2'(t)-\alpha_1'(t)\alpha_2(t))\mathrm{d}t \ . \tag{5}$$

This last integral can be approximated by the n point repeated Trapezoidal Rule to obtain

$$A=\frac{h}{2}\sum_{i=1}^{n}(\alpha_1(t_i)\alpha_2'(t_i)-\alpha_1'(t_i)\alpha_2'(t_i))+\mathcal{O}(h^2) \ ,^3 \tag{6}$$

where $t_i=ih$, $i=1,2,\ldots,n$ with $h=1/n$.

Next, approximations for α_1' and α_2' are needed.

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[3]This means that there is a number $K>0$ such that

$$\left|A-\sum_{i=1}^{n}\alpha_1(t_i)\alpha_2'(t_i)-\alpha_1'(t_i)\alpha_2(t_i)\right|\le Kh^2 \ .$$

To be consistent, the following formulas are used:

$$\alpha_1(t_i) = \frac{\alpha_1(t_{i+1}) - \alpha_1(t_{i-1})}{2h} + \mathcal{O}(h^2) \qquad (7a)$$

and $\quad i = 1,2,\ldots,n$

$$\alpha_2(t_i) = \frac{\alpha_2(t_{i+1}) - \alpha_2(t_{i-1})}{2h} + \mathcal{O}(h^2) \qquad (7b)$$

where $t_0 := 0$ and $t_{n+1} := t_1$. Let $(x_i, y_i) := (\alpha_1(t_i), \alpha_2(t_i))$ for $i = 0,1,\ldots,n+1$. Using Eqs. 7a and 7b in Eq. 6 then yields

$$A = \frac{h}{2}\sum_{i=1}^{n}\left[x_i\frac{y_{i+1}-y_{i-1}}{2h} - \frac{x_{i+1}-x_{i-1}}{2h}y_i\right] + \mathcal{O}(h^2)$$

$$= \frac{1}{2}\sum_{i=1}^{n}\left[x_i(y_{i+1}-y_{i-1}) - (x_{i+1}-x_{i-1})y_i\right] + \mathcal{O}(h^2) \qquad (8)$$

$$= \frac{1}{2}\sum_{i=1}^{n}(x_{i-1}y_i - x_iy_{i-1}) + \frac{1}{4}\left[(x_ny_{n+1} - x_{n+1}y_n) - (x_0y_1 - x_1y_0)\right] + \mathcal{O}(h^2)$$

Since $x_n = x_0$, $x_{n+1} = x_1$, $y_n = y_0$ and $y_{n+1} = y_1$ (so that $(x_ny_{n+1} - x_{n+1}y_n) - (x_0y_1 - x_1y_0) = 0$), Eq. 8 becomes

$$A = \frac{1}{2}\sum_{i=1}^{n}(x_{i-1}y_i - x_iy_{i-1}) + \mathcal{O}(h^2)\ . \qquad (9)$$

The sum given in Eq. 9 is used as an approximation to A.
Note: The points (x_i, y_i), $i = 1,2,\ldots,n$, *must* be the image of a set of parametric equations for C at *equally spaced* points in their domain. This requires that the original data points be approximated by a curve from which the ordered pairs (x_i, y_i) can be obtained.

Reliability Study. The reliability component of the stereometric process was determined by examining the variability of the digitized coordinates representing the series of points located on the surface of the mannequin torso. Each surface point was digitized a total of 32 times. Each triplet (x, y, and z coordinates) for each point measured was converted to a single value d where

$$d = \sqrt{(x_n - \bar{x})^2 + (y_n - \bar{y})^2 + (z_n - \bar{z})^2}$$

An analysis of variance was performed to determine if the individual values of d varied significantly for each point read over the total series of 32 trials.

Results

Validity. The results of the volumetric determinations of the cone as measured by water displacement yielded a mean volume of 646.0 ml. The volume computed using the stereophotogrammetric technique was found to have a value of 641.3 ml. This accounted for variation of 0.73% between the two methods of measurement. This result correlated very positively with an earlier study in which the volume of a whole body mannequin was measured using stereophotogrammetry and water displacement. The volume difference between the two methods was noted to be 0.74% (7).

Reliability. Following the collection of the 32 series of values representing the locations of the 32 different surface points on the mannequin, the mean value of d for each trial was computed. An analysis of variance was performed to determine if these mean values ($\bar{d}$) differed significantly from trial to trial. The analysis of variance yielded an F ratio of 1.05 which indicated a probability of 0.38 that these differences could have occurred by chance alone. Therefore it was concluded that the intrapoint variabilities from one trial to the next were not significantly different and the measurement process could be considered reliable. Of additional interest it was to be noted that the standard deviation for each point which was read 32 times ranged from ±.008 mm to ±.027 mm. This slight amount of variability documented the resolution of the stereometric measurement technique.

DISCUSSION AND CONCLUSIONS

One goal in the development of methods to measure volume and volume distribution of the human female breast has been to provide a process capable of being rapid and non-contact and comfortable for the subject. It is reasonable to assume that casting methods are conducive to deformation of the breasts and therefore may alter not only volume distribution (shape) but the total volume as well. Water displacement has been recognized as being both cumbersome and time consuming as well as questionable in its total accuracy. However, the biostereometric method described in this paper has the distinct advantage of offering a procedure that is noninvasive, non-contact and rapid with respect to the patient/subject time involvement. These advantages have been viewed as conditions necessary for development of methodologies for the rapid examination or screening of large numbers of women for breast disease, namely, cancer.

The purpose of this paper was to further document the variability of stereophotogrammetry as a means of specifically measuring both shape and volume of the female breast. The results appear to satisfy the accuracy and precision requirements for continued development of breast disease diagnosis using biostereometrics.

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NOMENCLATURE

ΔP = image parallax

Δp = horizontal parallax

ΔH = object height

H = camera to object distance

B = base, distance between camera lenses

C = simple closed curve

R = region consisting of the interior of C

D = region containing R and C

κ = rotation about z-axis for individual image

ϕ = rotation about y-axis for individual image

ω = rotation about x-axis for individual image

K = rotation about z-axis for absolute orientation

Φ = rotation about y-axis for absolute orientation

Ω = rotation about x-axis for absolute orientation

$\alpha_1(t), \alpha_2(t)$ = parametric equations describing the curve C

$\oint$ = contour integration

$\int$, $\int\int$	= standard integration
∂	= symbols used to denote partial differentiation
$\sum$	= summation symbol
$\mathcal{O}$	= denotes order of convergence, Eq. 6