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Analysis of the Relationship between DBH and Crown Projection Area Using a New Model

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Relations of DBH-crown projection area (CPA) were studied for deciduous and coniferous trees with different models, one of which is newly derived this time. For DBH−CPA relations, a proposed power-sigmoid function was the most suitable one among four models because of its good fit and mechanistic meaning. This model contains the feature that CPA grows with the second power relation to DBH, and the increasing rate of CPA slows as DBH increases. With transformation, the power-sigmoid function for DBH−CPA relation can be applied for individual basal area (IBA)−CPA relations as single-saturate function, and these two functional models have high compatibility. Next, the differences of DBH−CPA between deciduous and coniferous tree groups were analyzed with power-sigmoid function. The initial increasing rates of CPA against DBH for each group were similar, though the CPA's increasing rate for the coniferous group tended to decrease earlier than for the deciduous group. Because the power sigmoid function has mechanistic meanings, one can separately analyze the attributes of the DBH-CPA relation: the initial increasing rate of CPA and final tree form.

Key words: crown projection area, DBH, mathematical model, power sigmoid function, tree form

In order to understand forest structure, competition and production, it is useful to know the crown-projection area (CPA) of trees. In a patch which is recovering from gap formation, for example, it is useful to know when individual trees begin to compete with neighboring trees, and the size of the tree at that time. In such patches, light competition usually will not occur until the total CPA of cohort trees in the patch exceed the patch area, because there is no overlap of CPA in these situations.

It is important to know when trees cause canopy closure in studies of patch dynamics of trees. It is also useful aid for forest-management practices, such as thinning, for estimating the critical size that will be affected by self-thinning. Furthermore, if we can easily estimate the CPA from the DBH of trees in a stand, appropriate tree density can be obtained by comparing the total of CPA in a stand and the stand area.

Thus, the estimation of CPA from their tree size is important on both forest ecology and silviculture. So, what kind of measurement should we do? If it is easy to estimate the individual basal area (IBA) of a tree, estimating the CPA using the IBA of a tree as a size index may be easy to do, because we can visualize the simple proportional relation between CPA and IBA. But actually, it is difficult to estimate IBA directly in fieldwork. Diameter at breast height (DBH) is the only index that we can directly measure with ease. Furthermore, we usually have many data sets which include the DBH-class distribution in forest management. With these reasons, using DBH as an index of tree size may be useful for describing the CPA and tree size relationship.

There have been many studies of the DBH-crown width (CW) relationship (Krajicek *et al.*, 1961; Curtin, 1964, 1970; Curtis, 1970; Curtis and Reukema, 1970; Arney, 1973; Strub, *et al.*, 1975; Zarnovican, 1982; Paine and Hann, 1982; Leech, 1984; Tabbush and White, 1988; Farr *et al.*, 1989; Smith *et al.*, 1992; Larocque and Marshall, 1994), but studies about DBH−CPA relations are scarce (Krajicek *et al.*, 1961). In this paper, I will present DBH−CPA relationships that can be easily measured in the field. Specifically this paper searches for a reasonable model to describe the DBH−CPA relation. The character and efficiency of various functional models will be presented and discussed. In addition, a new functional model will be presented specifically for DBH−CPA relations. To show the usefulness of the new model, another functional model for IBA−CPA relationships will be developed, derived from the new function for the DBH−CPA relation.

Another objective of this paper is to evaluate the difference of tree forms between deciduous broadleaved and coniferous tree groups, using the new model for DBH−CPA relation. These two groups were suitable for comparison, because these two groups have clearly different characteristics.

Materials and Methods

1 Materials

The data set was selected from natural deciduous broadleaved and coniferous forests from Tatewaki *et al.* (1966). These data were samples of natural deciduous broadleaved and coniferous forests in the cool-temperate zone in the Nikko area, central Japan. The data were collected by the patch-sampling method, which was suitable for my analysis. Because the method usually samples cohorts, which contain almost the same species and sizes, triggered by gap formation, the trees in a cohort are usually in light competition, and these data are desirable for tree-form analysis. Furthermore, because of the small plot size, such data will not be influenced by the topography of study site, which sometimes causes different tree forms.

The species were classified into two groups: deciduous broadleaved and coniferous trees. The deciduous group has six species and 247 individuals: *Fagus crenata*, *Quercus*

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crispula, *Ulmus propinqua*, *Prunus ssiori*, *Betula platyphylla* var. *japonica* and *Fraxinus mandshurica* var. *japonica*. The coniferous group has six species and 596 individuals: *Abies veitchii*, *Abies mariesii*, *Tsuga diversifolia*, *Picea jezoensis* var. *hondoensis*, *Pinus parviflora* and *Abies homolepos*.

2 Methods

The following models were used to estimate DBH-CPA relations: linear, second power, logistic and power-sigmoid models.

1) Linear functional model:

$$\mathrm{CPA} = a\,\mathrm{DBH} \qquad (1)$$

where a is a proportional constant. When a tree grows, CPA will also grow. In this linear functional model, DBH is used as a size index. As a result, it is assumed that CPA increases in simple proportion to the increase in DBH. The intercept of this function should be zero because CPA must be zero when DBH is zero.

2) Second power functional model:

$$\mathrm{CPA} = a\,\mathrm{DBH}^2 \qquad (2)$$

where a is a constant. IBA is an index of size, because it is proportional to DBH^2.

3) Logistic functional model:

$$\mathrm{CPA} = \mathrm{CPA}_{\max}/(1 + (\mathrm{CPA}_{\max}/n_0 - 1)/\exp\,(r\,\mathrm{DBH})) \qquad (3)$$

where $\mathrm{CPA}_{\max}$ is a constant which CPA can never exceed, r is the rate increase when CPA is the smallest, and n_0 is the smallest value of CPA. It is assumed that the rate of increase of CPA will slow when DBH is sufficiently large because of competition, though CPA will increase exponentially as DBH does. Here $n_0 = 1$, because the value n_0 should be the smallest natural number. r and $\mathrm{CPA}_{\max}$ are found from data fitting.

4) Power-sigmoid functional model:

$$\mathrm{CPA} = \mathrm{CPA}_{\max} - 1/\exp\,((a\,\mathrm{DBH}^2 - \mathrm{CPA}_{\max}\ln \mathrm{CPA}_{\max})/\mathrm{CPA}_{\max}) \qquad (4)$$

As in the logistic functional model, it was assumed that the increasing rate of CPA will slow down when DBH is sufficiently large. However, the power-sigmoid function has a power relation between DBH and CPA when the values are small enough, while the logistic function is exponential. As mentioned above, this power relation indicates that CPA is proportional to IBA, which is proportional to the square of DBH.

While other models are well known, this is the first time the power-sigmoid function has been used in this application. If a tree has no competition from neighboring trees, the crown diameter will grow with the linear relation to its stem diameter (Krajicek *et al.*, 1961; Curtin, 1964, 1970; Curtis and Reukema, 1970; Strub *et al.*, 1975; Zarnovican, 1981; Tabbush and White, 1988; Smith *et al.*, 1992). Therefore, when it is small enough, the crown area will increase in proportion to the square of DBH as follows:

$$y = ax^2 \qquad (5)$$

where y is the crown projection area, x is stem DBH, and a is a positive constant.

This function can be transformed as follows:

$$dy/dx = 2ax \qquad (6)$$

As the tree grows, the rate of increase of the crown area will slow because of competition with neighboring trees. However, the stem will continue growing until the tree dies (Shinozaki *et al.*, 1964a). Consequently, the crown area may converge to a constant value if stem diameter increases sufficiently. The convergence of crown area to larger stem sizes can be described as follows:

$$dy/dx = 2ax(1 - y/y_{\max}) \qquad (7)$$

where, $y_{\max}$ is a constant and maximum value of a crown projection area.

When y is small enough, it will increase in proportion to the square of x. When y is close to $y_{\max}$, it no longer increases, and becomes constant. This equation is solved as follows:

$$y = y_{\max} - 1/\exp(ax^2/y_{\max} + c) \qquad (8)$$

where, c is an integral constant. Supposing $y = 0$ when $x = 0$, the Eq. (8) is altered as follows:

$$y = y_{\max} - 1/\exp((ax^2 - y_{\max}\ln y_{\max})/y_{\max}) \qquad (9)$$

Figure 1 shows the loci of Eq. (9) with various parameters. Since the locus of this function forms a sigmoid curve, it is like the logistic function, although the function derived in this study has a power function in its early stage, and the logistic function is exponential in its early stage. From these characteristics, the derived function can be called the "power-sigmoid function." The power-sigmoid function is widely applicable to elements with an obvious power relationship between two terms such as stem DBH and the crown projection area.

The least squares method is used for model fitting with the Newton method.

Results

Figure 2 shows scattered diagrams that fit each DBH-CPA model. Residual variances are shown on Table 1.

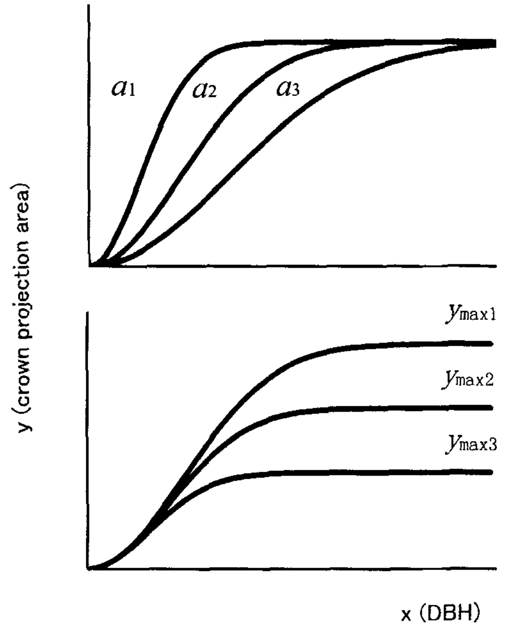

Fig. 1 Loci of the power-sigmoid function with various parameters ($a_1 > a_2 > a_3$, $y_{\max}1 > y_{\max}2 > y_{\max}3$).

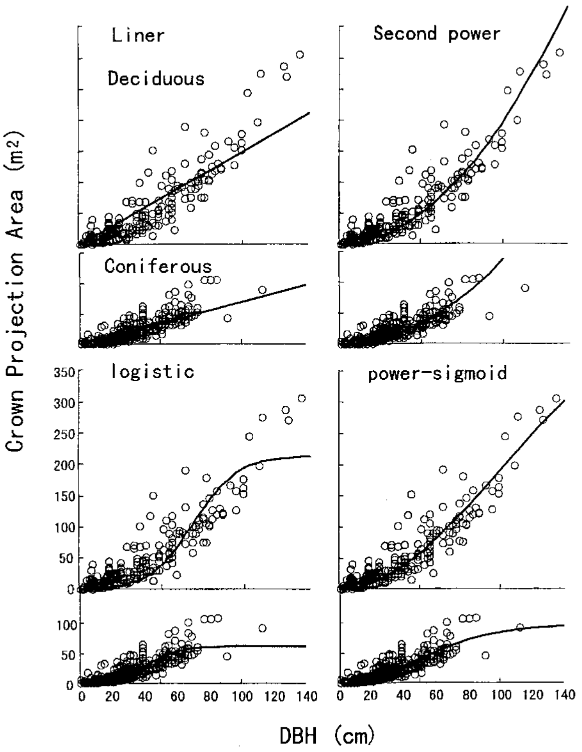

Fig. 2 DBH-CPA relations of deciduous broadleaved and coniferous trees with various models.

Table 1 Residual variances and multiple correlation coefficients (R) for each functional models.

Residual variance

	Linear	Second power	Logistic	Power sigmoid
Deciduous trees	639.4	482.9	723.9	428.2
Coniferous trees	92.3	96.6	79.4	70.0

R (multiple correlation coefficient)

	Linear	Second power	Logistic	Power sigmoid
Deciduous trees	0.909	0.935	0.901	0.934
Coniferous trees	0.860	0.855	0.861	0.876

Table 2 Parameters of each models.

	Linear	Second power	Logistic		Power sigmoid	
Equation	(1)	(2)	(3)		(4)	
Parameter	a	a	CPA_{max}	r	CPA_{max}	a
Deciduous trees	1.511	0.019	214.0	0.077	482.3	0.024
Coniferous trees	0.699	0.014	62.0	0.099	95.6	0.021

The residual variance of power-sigmoid-function model of the DBH−CPA relation is less than all other models for both deciduous broadleaved and coniferous trees. The second model is another power function. On the other hand, variances of linear and logistic models are large, suggesting that they do not fit well. The parameters found by the least squares method are shown in Table 2.

Figure 3 shows the relations between actual (observed) CPA and an estimated one. On this figure, when the estimated value of CPA is similar to the observed one, the dots lie on a straight line with an inclination of 45 degrees. Table 1 shows the correlation coefficient "R" that represent the fit between actual and estimated CPA. The fits of the second power function and the power-sigmoid function on the DBH−CPA relation are higher than those of the two other functions. For coniferous trees, the power-sigmoid functional model has the best fit.

In the coniferous data (Fig. 2), the largest two points of DBH did not correspond with other points, though these two points were from natural forests. Therefore, the fit of each

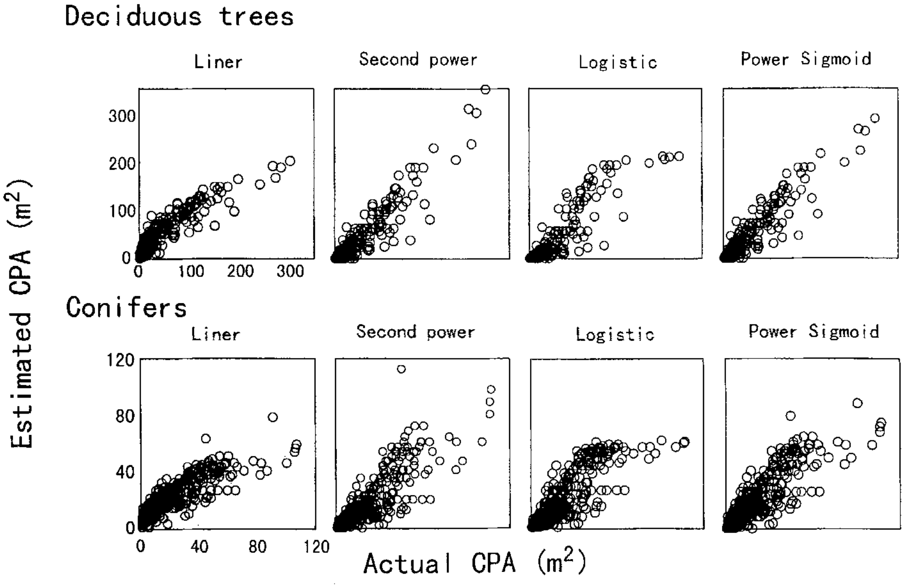

Fig. 3 Relations between CPA and the estimated on for deciduous broadleaved and coniferous trees from DBH-CPA analysis.

Table 3 Residual variances and multiple correlation coefficients (R) for each functional models in coniferous date without larger two points in DBH.

	Linear	Second power	Logistic	Power sigmoid
Residual variance	91.5	74.2	77.5	67.7
R	0.857	0.879	0.859	0.876

function was examined without these two points. In this case, the power-sigmoid function had the best fit (Table 3).

Discussion

Shinozaki *et al.* (1964a, b) and Kira (1965) hypothesized that there was a proportional relation between leaf amount and non-photosynthetic tissue, called the pipe-model theory. This theory holds that in order to maintain and sustain a quantity of leaves, pipes with proportional cross-section areas are needed, and the pipes should lead to the ground (Kira, 1965). Shinozaki *et al.* (1964a) called this "the unit-pipe system," and showed that an individual plant is a unit-pipe system. According to this concept, an increase of leaf quantity should be proportional to the IBA of the tree trunk. This relation is the second power relation of DBH−CPA.

In nature, however, there is the effect of competition with neighboring trees. The rate of increase of CPA is restricted, though CPA will expand as DBH grows. Shinozaki *et al.* (1964a) indicated that this phenomenon also could be explained by the pipe-model theory. When a tree is small enough that light competition is not an important factor, the tree can be identified as a unit-pipe system. After that, lower branches in the canopy will begin to die because of shade from neighboring trees. While branches and leaves fall off, the pipes which run from the branch to the ground will remain in the trunk (Yoda, 1971). In other words, the discarded pipes remain. As a result, the actual IBA will have a much larger cross-section area than necessary to produce leaves (Shinozaki *et al.*, 1964a). Such a relation has been widely confirmed in many tree species (Yoda, 1971).

The increasing rate of CPA will slow when tree size is affected by competition, while IBA will grow with a similar rate of increase as time goes by. This is why it is assumed that there is a reduction in the rate of CPA in power-sigmoid and logistic models in DBH−CPA relations. This decrease of the rate of crown growth has been shown in many studies (Curtin, 1970; Maeda and Miyakawa, 1971; Arney, 1973; Paine and Hann, 1982; Leech, 1984; Maeda *et al.*, 1989; Farr *et al.*, 1989; Smith *et al.*, 1992; Larocque and Marshall, 1994).

DBH−CPA Relations

It is clear from the graphs that the CPA value rises with DBH. Can the linear relation be found? Even if it fits well, the linear relation should not be acceptable because CPA should be in proportion to the square of DBH, as emphasized above; the linear relation has no mechanistic meaning. In fact, it does not fit well.

The second power model is not only a good fit but also has mechanistic meaning because it assumes the unit-pipe system. Further, this model agrees with many past models showing the linear relation between DBH and CW (Krajicek *et al.*, 1961; Curtin, 1964; Curtis and Reukema, 1970; Strub *et al.*, 1975; Zarnovican, 1982; Tabbush and White, 1988; Smith *et al.*, 1991). The fits are somewhat high with low variances and high multiple correlation coefficients. In this model, however, it is not assumed that the rate of increase will slow due to competition when the stand matures. As mentioned above, many studies treating the DBH−CW relation indicated a decrease in the rate of increase of CW as DBH increased after canopy closure.

The power-sigmoid model based on the second power function, moreover, shows that increases in CPA will slow as DBH increases. Therefore, the power-sigmoid model should fit much better than the second power model, if a decrease in the rate of CPA occurs. The value "R" of power-sigmoid-function model was the highest in the DBH−CPA relation in conifer trees because of the decrease of the rate of CPA. Further, the variance of the power-sigmoid model was also the lowest in coniferous and deciduous forests data sets.

The decrease of the increasing rate of CPA is assumed in the logistic model, as in the power-sigmoid model. While the power-sigmoid function has a power relation between DBH and CPA when those values are small, the logistic function has an exponential relation between DBH and CPA. Therefore, the logistic model is suitable for relations which have a clear exponential relation between two terms. In this analysis, however, the logistic model is not suitable because there was a clear power relation between the two terms. Actually, the logistic model fit worse than the power-sigmoid model in this case.

Because the power-sigmoid function is mechanistic, we can examine each parameter. Parameter "a" in Eqs. (4) and (9) is the proportional constant of squared DBH against CPA when there is no competition from neighboring trees. The larger "a" is, the greater the rate of increase of CPA against DBH. The parameter "$y_{\max}$" in formula (9) or "$\mathrm{CPA}_{\max}$" in formula (4) is the maximum value possible for CPA; because of competition from neighboring trees, CPA cannot increase infinitely. In the case of deciduous trees, CPA will not always reach its maximum value. Hence, parameter "$\mathrm{CPA}_{\max}$" indicates the degree of decrease of the rate of CPA, rather than the maximum CPA. Consequently, small "$\mathrm{CPA}_{\max}$" indicates a large decrease in the CPA rate.

IBA−CPA Relations

We can specifically transform the power-sigmoid function for the IBA−CPA relation. It is assumed that CPA is in proportion to the square of DBH until competition with neighboring trees begins. On the other hand, CPA is assumed to be in proportion to the IBA till competition begins, as follows:

$$\mathrm{CPA} = \mathrm{CPA}_{\max} - 1/\exp \left((a\,\mathrm{IBA} - \mathrm{CPA}_{\max} \ln \mathrm{CPA}_{\max}) / \mathrm{CPA}_{\max} \right) \quad (10)$$

where a is a proportional constant and $\mathrm{CPA}_{\max}$ is also a constant which the CPA value cannot exceed. This functional model can be called the "single-saturate function" because: 1) the basic relation between IBA and CPA is linear (single powered) and, 2) the value of CPA will saturate in the end (Fig. 4).

These two models, the power-sigmoid and single-saturate models, have compatible characteristics. Consequently, it is expected that the value of $\mathrm{CPA_{max}}$ from the power-sigmoid model is equal or very close to that of the single-saturate model. Furthermore, since

$\mathrm{CPA} = a_1\,\mathrm{DBH}^2$ in power-sigmoid (a_1 is a proportional constant) and $\mathrm{CPA} = a_2\,\mathrm{IBA}$ in single-saturate model (a_2 is a proportional constant) in the original assumption, the relation is expected to be:

$$\mathrm{CPA} = a_1\,\mathrm{DBH}^2 = a_2\,\mathrm{IBA} \tag{11}$$

Because $\mathrm{IBA} = \pi\,(\mathrm{DBH}/2)^2$, the relation is expected to be:

$$a_1\,\mathrm{DBH}^2 = a_2\,\pi\,(\mathrm{DBH}/2)^2 \tag{12}$$

therefore,

$$a_1 = (\pi/4)\,a_2 \tag{13}$$

The parameter values of $\mathrm{CPA_{max}}$ in the power-sigmoid model are very close to those of the single-saturate model: 482.31 and 482.30 for deciduous trees, 95.64 and 95.65 for conifers, respectively. For deciduous trees, the value $(\pi/4)a_2$ in Eq. (13) is 0.02422, is close to a_1 (0.02418) of the power-sigmoid function. For coniferous trees, the value $(\pi/4)a_2$ is 0.0205818, similar to the a_1 (0.0205819) of the power-sigmoid function.

This compatibility shows that the power-sigmoid function can be used for the DBH−CPA relation and the single-saturate function for the IBA−CPA relation. Because the power-sigmoid function for the DBH−CPA relation is easily applied to the single-saturate function of the IBA−CPA relation, the compatibility of values is also high. Furthermore, the single-saturate function should be suitable for the DBH−CW relation as well.

Deciduous *versus* Coniferous Trees

Next, the DBH−CPA relation was analyzed with special attention to the differences and similarities between deciduous broadleaved and coniferous trees. The power-sigmoid model was used for analysis because of its efficiency.

Curves regressed by the power-sigmoid function are shown in Fig. 5. In the early developmental stage, until about 10 or 20 cm in diameter, the loci of the two regressions are apparently the same. This is similar to parameter "a" in the power-sigmoid function. The values of parameter "a" are 0.024 and 0.021 in coniferous and deciduous groups, respectively. Because the difference of these two values is fairly small, there will be few differences in the DBH−CPA relation when there is no competition due to their small size, though their final tree forms are very different. Their final tree forms can be found by using $\mathrm{CPA_{max}}$. The conifers with low value of $\mathrm{CPA_{max}}$ have smaller CPA than the deciduous trees. Hence it follows that the decrease of CPA's increasing rate occurs in much smaller DBH than that of deciduous trees.

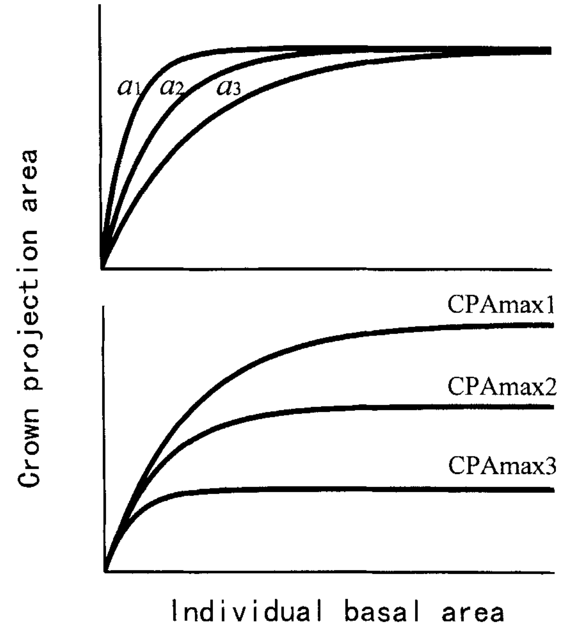

Fig. 4 Loci of the single-saturate function with various parameters ($a_1 > a_2 > a_3$, $\mathrm{CPA_{max}1} > \mathrm{CPA_{max}2} > \mathrm{CPA_{max}3}$).

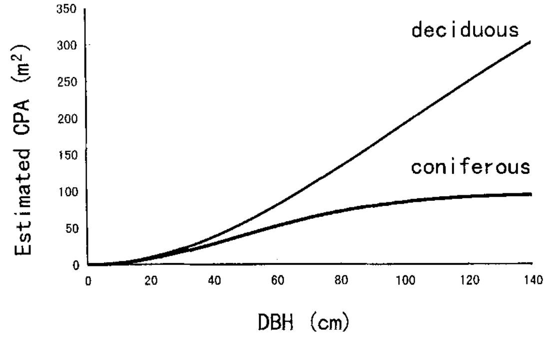

Fig. 5 Loci regressed by power-sigmoid function for deciduous broadleaved and coniferous trees.

Conclusion

It was confirmed that the tree form could be effectively analyzed using the power-sigmoid model. In the case of the coniferous and deciduous trees, the model indicated that the rates of increase of CPA against DBH were not so different though their final tree forms were different. Such characteristics were understood because the model is not statistical but mathematical.

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