Journal of Radioanalytical and Nuclear Chemistry, Vol. 242, No. 3 (1999) 617–622

An empirical expression for the full energy peak efficiency of an N-type high purity germanium detector

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(Received May 17, 1999)

An empirical expression for the full energy peak efficiency (ε) in terms of the gamma-ray energy (E) and the vertical distance from the detector surface (z) (i.e.,$\varepsilon=\varepsilon(z,E)$) has been obtained for an N–type high purity germanium (HPGE) detector using an extended mixed standard radionuclide solution. Comparison of the calculated efficiencies and the experimentally measured values for the energy range of 59.5–1332.5 keV and a source-to-detector distance of 1.2–7.2 cm showed that the theoretical values agree with that of the experiment within 1.5–3.7% standard deviations. This shows a good agreement between the theory and the experiment.

Introduction

The absolute method of quantitation employed in neutron activation analysis and gamma-ray spectrometry requires the accurate knowledge of the efficiency of the detector system for the particular sample-detector geometry. There are generally two main types of sample-detector geometry; (1) The fixed sample-detector geometry and (2) the variable sample-detector geometry.

The fixed sample-detector geometry

For the fixed sample-detector geometry, the full energy peak efficiency (ε) depends only on the gamma-ray energy (E) i.e., $\varepsilon=\varepsilon(E)$. However, the energies of the gamma active radionuclides produced by thermal neutron activation are in the range of 20–4000 keV. Since there are not sufficient gamma-ray standard sources to cover this wide range, the most convenient method for the determination of the efficiency within this energy range is by interpolation.[1–3] This method involves choosing as primary standards, some radionuclides of relatively long half-lives with known emission probabilities and well resolved gamma energy peaks within the energy range. Using these standard sources the full energy peak efficiency of the detection system was experimentally measured. A theoretical curve was then fitted to the experimental data from which the efficiency at any energy within the range can be obtained by calculations.

The variable sample-detector geometry

For the variable sample-detector geometry, the full energy peak efficiency of the detector system does not depend only on the energy of the gamma-ray (E) but also on the sample-to-detector distance (z) i.e., $\varepsilon=\varepsilon(z,E)$. Thus by adopting the fixed sample-detector geometry approach, experimental measurements were made at different source-detector distances. The measured efficiencies, as a function of E, are plotted on the same graph and a family of curves is obtained, each curve representing a different z distance from the detector surface. An analytical function for the efficiency, in terms of E, was then obtained for each z distance. Using these analytical functions the efficiency values may be obtained for any of the z positions and at any energy E within the energy range. Some of the neutron activation analysis and the gamma-ray spectroscopy software packages adopt this approach.[4,5] Others adopt a purely theoretical approach for obtaining the full energy peak efficiency by making use of the mechanisms involved in the interaction of electromagnetic radiation with matter together with the detector characteristics and specifications.[3] Others, however, adopt the Monte Carlo Code approach to calcudate the efficiencies.[2,3,6]

The present work, however, which is based on the variable sample-source geometry aims a single analytical function for the full energy peak efficiency in terms of both z and E (i.e., $\varepsilon=\varepsilon(z,E)$) instead of series of analytical functions for the different z positions. Measurements were made using an N–type high purity germanium (HPGE) detector system at the Neutron Activation Analysis Laboratory of Ghana Atomic Energy Commission at GHARR-1 Centre.

Theory

The full energy peak efficiency (ε) of a high purity germanium (HPGE) detector may be expressed in the form of a polynomial with respect to the gamma-ray energy (E) as:[7,8]

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0236–5731/99/USD 17.00

Elsevier Science B. V., Amsterdam
Akadémiai Kiadó, Budapest

$$\ln(\varepsilon) = \sum_{i=0}^{n} a_i (\ln(E))^i \quad (1)$$

where a_i are the coefficients of the polynomial and are different for different source-to-detector distance, z. These coefficients, a_i, may be obtained for each z distance by fitting Eq. (1) to the experimentally measured efficiency values for that particular z distance. Assuming that the coefficients, a_i, can also be expressed in a polynomial form involving, z, then we may write

$$a_i \equiv a_i(z) = \sum_{j=0}^{m} a_{ij} z^j \quad (2)$$

where a_{ij} are the coefficients of the polynomial. These coefficients, a_{ij} may also be obtained by fitting the graphs of $a_i(z)$ versus z with Eq. (2). Thus by combining Eqs (1) and (2) a general equation for the efficiency, ε, may be expressed as;

$$\ln(\varepsilon) = \sum_{i=0}^{n} \sum_{j=0}^{m} a_{ij} z^j (\ln(E))^i \quad (3)$$

Hence knowing the constants, a_{ij}, the full energy peak efficiency ε, may be obtained for a wide range of gamma-ray energies, E, and for various z distances from the detector surface.

Experimentally, the full energy peak efficiency for a particular sample-to-detector geometry is obtained by measuring the net counts under the photopeak energy of interest and using the formula

$$\varepsilon = \frac{A_{FEP}}{A_{STD} f C_{ABS} C_{SEA}} \quad (4)$$

where A_{FEP} is the net activity (couns/second) under the photopeak, A_{STD} is the activity of the standard source, f is the emission probability per decay for the particular gamma-transition (gamma-yield), C_{ABS} and C_{SEA} are the respective correction factors for self-absorption and summing effect.

Experimental

A Canberra N-Type detector Model GR2518 of relative efficiency of 25% and an energy resolution of 1.8 keV at 1332.5 keV gamma-ray of ^{60}Co was used for the efficiency measurements. The other associated electronics consisted of an H.V power supply model 3105, a spectroscopy amplifier model 2020, all manufactured by Canberra Industries Inc. and a NIM power supply unit model PS01-B manufactured by SILENA. A mixed standard radionuclide solution containing 10 different fadioactive nuclides of known activities, supplied by the International Atomic Energy Agency (IAEA), provided the necessary gamma-ray energies for the measurements. The specifications of the characteristics of the mixed standard source are given in Table 1.

An amount of 0.5 ml of the solution was carefully pipetted into a polyethylene irradiation capsule of 1.3 cm internal diameter to form a thin layer disk source. The capsule was then placed at a distance, $z = 1.2$ cm in front of the detector such that the vertical axis through the centre of the source and normal to the plane of the source coincided with that of the detector.The mixed solution was counted for 2000 seconds by means of an 8K EMCA PLUS multichannel analyzer (MCA) emulation software. By means of Eq. (4) the full energy peak efficiency values were calculated for the various radionuclides in the mixed solution using SPAN 5.0 gamma-analysis software[5] on a 486 micro-computer. Measurements were taken for distances of 2.2, 3.5, 4.7, 6.0 and 7.2 cm and the full energy peak efficiency are displayed as dots in Fig. 1.

Table 1. Specifications of the IAEA mixed radioactive solution used for the efficiency measurements

Radionuclide	Gamma-ray energy, (E), keV	Gamma yield (f), %	Half–life	Activity (A) at 1–03–1995, kBq/ml
^{241}Am	59.54	35.70	432.21 y	4.56±1.0%
^{109}Cd	88.03	3.61	1.27 y	21.2±4.5%
^{57}Co	122.06	85.20	270.0 d	0.82±1.7%
^{203}Hg	279.21	81.46	46.61 d	4.25±1.2%
^{85}Sr	514.00	99.27	64.84 d	3.18±1.5%
^{137}Cs	661.66	85.21	30.17 y	4.23±2.4%
^{54}Mn	834.84	99.98	312.12 d	5.41±1.8%
^{60}Co	1173.20	99.90	5.27 y	6.29±1.0%
	1332.50	99.98	5.27 y	6.29±1.0%

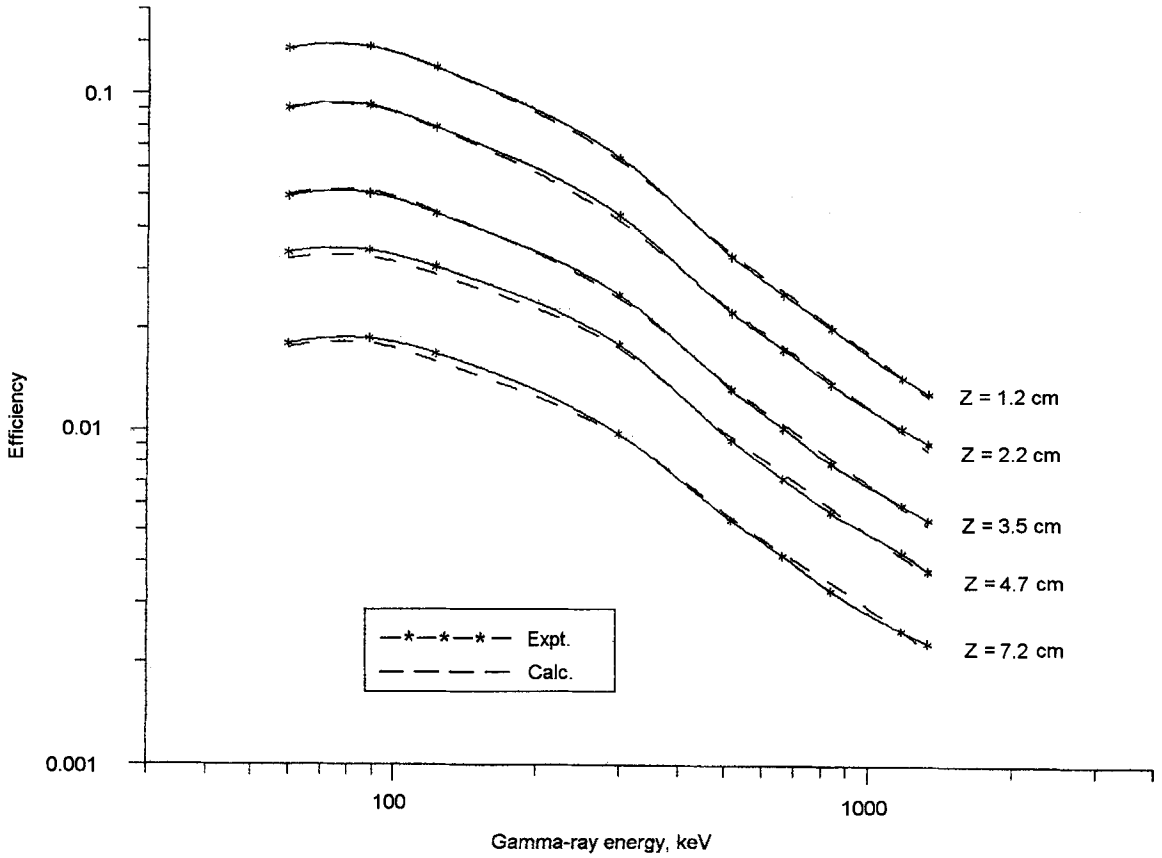

Fig. 1. Comparison of the experimental and calculated efficiencies of an N-type HPGE detector

Results

Determination of the coefficients, $a_i(z)$

To obtain the coefficients $a_i(z)$, the experimentally measured efficiencies for the different z-position were fitted with a theoretical function. Since it was not possible to cover the full energy range of 59.5–1332.5 keV with a single polynomial, the range was divided into two portions, a lower energy portion with $E \leq$ 130 keV and a higher energy portion, with $E<130$ keV. A theoretical fit was then applied to each portion separately. For the lower energy portion a second order polynomial in E could fit the experimental data;

$$\varepsilon = \sum_{i=0}^{2} a_i(z) E^i \quad E \leq 130 \text{ keV} \qquad (5)$$

where $a_i(z)$, are the coefficients of the polynomial. For the higher enertgy portion a power function (first order polynomial in ln(E)) was adequate to fit the experimental data, i.e.:

$$\ln(\varepsilon) = \ln(a_0(z)) + a_1(z)\ln(E) \quad E>1130 \text{ keV} \qquad (6)$$

The $a_i(z)$ values obtained from the fit for the different z positions are shown in Table 2. The solid lines in Fig. 1 are the calculated curves to the experimental data for different z values.

Determination of the coefficients, a_{ij}

To obtain the ciefficients, a_{ij}, graphs of the energy coefficients, $a_i(z)$, versus z, were plotted for the two separate portions. The results of the fit showed that for the two portions a fourth order polynomial function in terms of z could fit the data; i.e.:

$$a_i(z) = \sum_{j=0}^{4} a_{ij} z^j \qquad (7)$$

The values of the coefficients, a_{ij} are shown in Table 3.

Table 2. Coefficients for the energy polynomial, $a_i(z)$ for the different z positions

Sample to detector distance (z), cm	Coefficients of energy polynomial $a_i(z)$ Gamma-ray energy, $E \leq 130$ keV $a_0(z)$ $\times 10^{-2}$	$a_1(z)$ $\times 10^{-3}$	$a_2(z)$ $\times 10^{-6}$	Gamma-ray energy, $E > 130$ keV $a_0(z)$ –	$a_1(z)$ –
1.2	7.968	1.500	–9.931	3.032	–1.029
2.2	5.188	1.060	–6.821	2.466	–0.999
3.5	2.863	0.566	–3.593	1.924	–0.998
4.7	2.109	0.342	–2.163	1.537	–0.991
6.0	1.398	0.265	–1.639	1.163	–0.979
7.2	0.998	0.210	–1.295	0.658	–0.941

Table 3. Coefficients of the z polynomial, a_{ij}, corresponding to each of the energy coefficients $a_i(z)$

Coefficients of energy polynomial $a_i(z)$	Coefficients of the z polynomial a_{ij} a_{i0}	a_{i1}	a_{i2}	a_{i3}	a_{i4}
Gamma-ray energy, $E \leq 130$ keV					
$a_0(z)$	0.1336	$-5.536 \cdot 10^{-2}$	$9.653 \cdot 10^{-3}$	$-7.123 \cdot 10^{-4}$	$1.501 \cdot 10^{-5}$
$a_1(z)$	$2.066 \cdot 10^{-3}$	$-3.535 \cdot 10^{-4}$	$-1.105 \cdot 10^{-4}$	$3.352 \cdot 10^{-5}$	$-2.267 \cdot 10^{-6}$
$a_2(z)$	$1.352 \cdot 10^{-5}$	$-2.480 \cdot 10^{-6}$	$-6.484 \cdot 10^{-7}$	$2.067 \cdot 10^{-7}$	$1.411 \cdot 10^{-8}$
Gamma-ray energy, $E > 130$ keV					
$a_0(z)$	41.93	–24.16	6.27	–0.7614	$3.462 \cdot 10^{-2}$
$a_1(z)$	1.120	–0.1145	$3.705 \cdot 10^{-2}$	$-4.544 \cdot 10^{-3}$	$1.984 \cdot 10^{-4}$

Table 4. Comparison of the experimental and the calculated efficiencies

Gamma-ray energy, (E), keV	Detector efficiency (ε), $\times 10^{-2}$ Experimental	Calculated $z = 1.2$ cm	Deviation, %	Experimental	Calculated $z = 2.2$ cm	Deviation, %
59.54	13.59	13.57	–0.1	9.06	8.98	–0.9
88.0	13.79	13.76	–0.2	9.20	9.12	–0.9
122.0	11.92	11.86	–0.5	7.92	7.84	–1.0
297.2	6.47	6.33	–2.2	4.38	4.22	–3.7
514.0	3.32	3.38	1.8	2.25	2.29	1.8
661.66	2.55	2.61	2.4	1.75	1.78	1.7
834.85	2.02	2.05	1.5	1.38	1.41	2.2
1173.2	1.44	1.45	0.7	1.02	1.00	–2.0
1332.5	1.30	1.27	–2.3	0.92	0.88	–4.3
Standard deviation (σ):			1.57	–	–	–2.35

A generalized expression for the efficiency as a function of both z and E

From the theoretical fit to the experimental data, the following expressions were obtained for the efficiency of the detector for the energy range of $59.5 \leq E \leq 1332.5$ keV and for a source-to-detector distance of the range of $1.2 \leq z \leq 7.2$ cm.

$$\varepsilon = \sum_{i=0}^{1} \sum_{j=0}^{4} a_{ij} z^j E^i \quad \text{for } E \leq 130 \text{ keV} \qquad (8)$$

$$\varepsilon = \exp\left[\ln\left(\sum_{j=0}^{4} a_{0j} z^j \right) + \left(\sum_{j=0}^{4} a_{1j} z^j \right) \ln(E) \right] \qquad (9)$$

for $E > 130$ keV

Thus by combining Eqs (8) and (9), a general analytical function was obtained for calculating the full energy peak efficiency value of the detector as a function of both E and z. Tables 4–6 show the experimental efficiencies and the corresponding values obtained using the analytical functions represented by Eqs (8) and (9). The calculated efficiencies are shown as dotted lines in Fig. 1.

Table 5. Comparison of the experimental and the calculated efficiencies

Gamma-ray energy, (E), keV	Detector efficiency (ε),$\times 10^{-2}$					
	Experimental	Calculated $z = 3.5$ cm	Deviation, %	Experimental	Calculated $z = 4.7$ cm	Deviation, %
59.54	4.96	5.02	–2.0	3.38	3.23	–4.4
88.0	5.06	5.14	–1.6	3.44	3.28	–4.7
122.0	4.42	4.47	–1.1	3.06	2.90	–5.2
297.2	2.54	2.48	–2.4	1.80	1.76	–2.2
514.0	1.33	1.35	1.5	0.94	0.96	2.1
661.66	1.02	1.05	2.9	0.72	0.75	4.2
834.85	0.80	0.83	3.7	0.57	0.59	3.5
1173.2	0.60	0.59	–1.7	0.43	0.42	–2.3
1332.5	0.54	0.52	–3.7	0.38	0.37	–2.6
Standard deviation (σ):			2.46	–	–	3.65

Table 6. Comparison of the experimental and the calculated efficiencies

Gamma-ray energy, (E), keV	Detector efficiency (ε),$\times 10^{-2}$					
	Experimental	Calculated $z = 6.0$ cm	Deviation, %	Experimental	Calculated $z = 7.2$ cm	Deviation, %
59.54	2.40	2.44	1.7	1.80	1.76	–2.2
88.0	2.48	2.49	0.4	1.87	1.81	–3.2
122.0	2.20	2.21	0.5	1.69	1.60	–5.3
297.2	1.33	1.28	–3.8	0.98	0.97	–1.0
514.0	0.70	0.71	1.4	0.54	0.55	1.9
661.66	0.54	0.55	1.9	0.42	0.43	2.4
834.85	0.43	0.44	2.3	0.33	0.35	6.1
1173.2	0.32	0.32	0.0	0.25	0.25	0.0
1332.5	0.29	0.28	–3.4	0.23	0.22	–4.3
Standard deviation (σ):			2.11	–	–	3.48

Analysis

From Tables 4–6, it can be seen that the calculated efficiencies agree very well with the experimental values for the range of E and the z covered. For $z = 1.2$ cm the largest percentage deviation was only 2.4%, whilst those for $z = 2.2$, 3.5, 4.7, 6.0 and 7.2 cm were 4.3%, 3.7%, 5.2%, 3.8% and 6.1%, respectively. The standard deviations ranged from 1.57% for $z = 1.2$ cm to a maximum value of 3.65% for $z = 4.7$ cm showing a good agreement between the experimental and the calculated values.

Conclusions

The results of the measurements show that the efficiency of the N-type HPGE detector may be expressed in an analytical form involving the gamma-ray energy E, and the vertical distance from the detector surface, z. The results also showed that a single expression could not be obtained to cover the whole energy range of 59.5–1332.5 keV. Comparison of the calculated efficiencies with the experimental data for the gamma-energy range of $59.5 \leq E \leq 1332.5$ keV and the z range of $1.2 \leq z \leq 7.2$ cm gave a standard deviation between 1.5–3.7%. This shows that the theory agrees very well with the experiment. Thus by means of this analytical Eqs (8) and (9) the efficiency of the detector, at any position within the selected energy, E and the z ranges, may be reliably obtained by calculation without any experimental measurement.

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