Anisotropy of Young's modulus of human tibial cortical bone

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Abstract—*The anisotropy of Young's modulus in human cortical bone was determined for all spatial directions by performing coordinate rotations of a 6 by 6 elastic stiffness matrix. The elastic stiffness coefficients were determined experimentally from ultrasonic velocity measurements on 96 samples of normal cortical bone removed from the right tibia of eight human cadavers. The following measured values were used for our analysis:* $c_{11} = 19.5$ GPa, $c_{22} = 20.1$ GPa, $c_{33} = 30.9$ GPa, $c_{44} = 5.72$ GPa, $c_{55} = 5.17$ GPa, $c_{66} = 4.05$ GPa, $c_{23} = 12.5$ GPa. *The remaining coefficients were determined by assuming that the specimens possessed at least an orthorhombic elastic symmetry, and further assuming that* $c_{13} = c_{23}$ $c_{12} = c_{11} - 2c_{66}$. *Our analysis revealed a substantial anisotropy in Young's modulus in the plane containing the long axis of the tibia, with maxima of 20.9 GPa parallel to the long axis, and minima of 11.8 GPa perpendicular to this axis. A less pronounced anisotropy was observed in the plane perpendicular to the long axis of the tibia. To display our results for the full three-dimensional anisotropy of cortical bone, a closed surface was used to represent Young's modulus in all spatial directions.*

Keywords—*Biomechanics, Cortical bone, Young's modulus, Anisotropy, Elasticity, Ultrasonics*

Med. Biol. Eng. Comput., 2000, **38**, 333–338

1 Introduction

ULTRASONIC TECHNIQUES have proven valuable to the study of the elastic properties of materials. Beginning in the late 1940s such techniques were used to investigate the elastic properties of single crystals (EROS and REITZ, 1958; HUNTINGTON, 1947; MCSKIMIN, 1950, 1953, 1955, 1958; MUSGRAVE, 1954; PRICE and HUNTINGTON, 1950; TRUELL *et al.*, 1969). More recently, ultrasound has been used to study the elastic properties of various types of biologic materials including bone and soft tissues (BONFIELD and GRYNPAS, 1977; HOFFMEISTER *et al.*, 1996*b*; HUNT *et al.*, 1998; KATZ and MENIER, 1987; KNETS, 1978; NATALI and MEROI, 1989; RHO, 1996, 1998; VIANO *et al.*, 1976; YOON and KATZ, 1976*a*,*b*). In many ways, ultrasonic techniques are well suited for the investigation of biologic media. Ultrasonic techniques are non-destructive and potentially non-invasive, making *in-vivo* testing of the elastic properties of tissues a possibility. For *in-vitro* measurements such as those considered in the present study, ultrasonic testing can simplify sample preparation and facilitate the investigation of smaller samples than is possible using conventional mechanical testing techniques (RHO, 1996).

In the specific case of bone, the subject of this study, ultrasonic and conventional mechanical testing techniques may yield different results for the same elastic moduli. For example, WILLIAMS and JOHNSON (1989) found that the ultrasonically determined elastic modulus was about twice as great for measurements of composites formed from polymethylmethacrylate (PMMA) cement and bovine tibial cancellous bone. This difference was attributed to the comparatively high strain rates that ultrasonic waves produce in a material. However, ultrasonic and mechanical testing techniques have been shown to correlate well with one another (ASHMAN *et al.*, 1984, 1987, 1989). The good correlation between moduli measured with ultrasonic techniques and conventional mechanical testing techniques suggest that either technique can provide valuable information about the anisotropic elastic properties of bone.

In this study we report the elastic stiffness coefficients of human cortical bone measured in an earlier ultrasonic study of the human tibia (RHO, 1996). These elastic stiffness coefficients are used to determine Young's modulus in all spatial directions by performing coordinate rotations of a 6×6 elastic stiffness matrix. A detailed analysis such as this produces an extremely large amount of data. We therefore employ a means of compactly displaying these data as a three-dimensional closed surface. We conclude by comparing our results for cortical bone to other materials exhibiting similar elastic anisotropies, and by examining the effects of measurement uncertainties on our analysis.

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First received 14 July 1999 and in final form 4 February 2000

2 Materials and methods

Samples of cortical bone analysed for this study were prepared from the tibia of the right side of eight frozen unembalmed human cadavers (seven men and one woman ranging in age from 45 to 68 years with no history of bone disease). As described previously, samples were obtained from three transverse sections located at 30%, 50% and 70% of the total bone length from the end of the bone (RHO, 1996). Four roughly cubic samples were prepared from four quadrants (anterior, medial, posterior and lateral) of each transverse section. This resulted in a total of 96 samples of cortical bone from the eight tibiae.

For our analysis we defined the 3-axis of our co-ordinate system to be parallel to the long axis of the tibia. The 1 and 2 axes were mutually perpendicular, defining a plane perpendicular to the 3-axis. These axes may be visualised by considering the transverse section of the tibia as possessing an approximately circular geometry. Then the 1-axis is oriented in the radial direction and the 2-axis in the circumferential direction.

The elastic stiffness coefficients used in our analysis were measured ultrasonically (RHO, 1996). Briefly, the procedure involved measuring the velocity of transverse and longitudinal mode ultrasonic pulses propagated along specific directions through the samples. No significant variation was observed in the elastic properties of the specimens obtained from different locations along the length and around the circumference of the cortical bone ($p > 0.1$). We therefore used the mean value of each elastic stiffness coefficient computed from all 96 specimens. Table 1 reports the mean value and standard deviations of these measurements.

We assumed that the specimens of bone analysed in this study possessed at least an orthorhombic elastic symmetry. An orthorhombic system has three mutually orthogonal axes each possessing a two-fold symmetry. The corresponding elastic stiffness matrix requires nine independent elastic stiffness coefficients, and has the form:

$$[c] = \begin{bmatrix} c_{11} & c_{12} & c_{13} & 0 & 0 & 0 \\ c_{12} & c_{22} & c_{23} & 0 & 0 & 0 \\ c_{13} & c_{23} & c_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & c_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & c_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & c_{66} \end{bmatrix} \quad (1)$$

Due to the size and shape of specimens that could be prepared from human cortical bone, two of the nine coefficients (C_{12} and c_{13}) could not be determined experimentally. Therefore the following relationships were assumed:

$$c_{13} = c_{23} \quad \text{and} \quad c_{12} = c_{11} - 2c_{66} \quad (2)$$

reducing the actual number of independent coefficients to seven.

To determine Young's modulus we computed the inverse of the elastic stiffness matrix to obtain the elastic compliance matrix:

$$[s] = [c]^{-1} \quad (3)$$

Table 1 Elastic stiffness coefficients for human tibial cortical bone measured ultrasonically in 96 specimens of bone.

Coefficient	Mean, GPa	Standard deviation, GPa	Relative uncertainty (Std Dev./Mean) * 100%
c_{11}	19.5	2.0	10.2
c_{22}	20.1	1.9	9.5
c_{33}	30.9	2.1	6.8
c_{44}	5.72	0.49	8.6
c_{55}	5.17	0.57	11.0
c_{66}	4.05	0.54	13.3
c_{23}	12.5	1.2	9.6

For the specimens of bone analysed in this study, the elastic compliance matrix had the form

$$[s] = \begin{bmatrix} s_{11} & s_{12} & s_{13} & 0 & 0 & 0 \\ s_{12} & s_{22} & s_{13} & 0 & 0 & 0 \\ s_{13} & s_{13} & s_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & s_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & s_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & s_{66} \end{bmatrix} \quad (4)$$

where $s_{12} = s_{11} - \frac{1}{2}s_{66}$.

The three diagonal elements of the upper left quadrant of the compliance matrix are directly related to Young's moduli measured along the 1, 2 and 3 axes:

$$E_1 = \frac{1}{s_{11}} \qquad E_2 = \frac{1}{s_{22}} \qquad E_3 = \frac{1}{s_{33}} \quad (5)$$

Young's modulus for other angles was obtained by rotating the co-ordinate system to orient one of the axes in the direction of interest. Rotation from an original rectangular co-ordinate system, x_i, to a new co-ordinate system x'_i, may be accomplished using a 3×3 matrix defined such that its components are the direction cosines

$$a_{ij} = \cos(x'_i, x_j) \quad (6)$$

As discussed by AULD (1990) the following transformation can be obtained for the elastic stiffness coefficients for a given coordinate rotation:

$$[c'] = [M][c][M]^T \quad (7)$$

where

$$[M] = \begin{bmatrix} a_{11}^2 & a_{12}^2 & a_{13}^2 & 2a_{12}a_{13} & 2a_{13}a_{11} & 2a_{11}a_{12} \\ a_{21}^2 & a_{22}^2 & a_{23}^2 & 2a_{22}a_{23} & 2a_{23}a_{21} & 2a_{21}a_{22} \\ a_{31}^2 & a_{32}^2 & a_{33}^2 & 2a_{32}a_{33} & 2a_{33}a_{31} & 2a_{31}a_{32} \\ a_{21}a_{31} & a_{22}a_{32} & a_{23}a_{33} & a_{22}a_{33}+a_{23}a_{32} & a_{21}a_{33}+a_{23}a_{31} & a_{22}a_{31}+a_{21}a_{32} \\ a_{31}a_{11} & a_{32}a_{12} & a_{33}a_{13} & a_{12}a_{33}+a_{13}a_{32} & a_{13}a_{31}+a_{11}a_{33} & a_{11}a_{32}+a_{12}a_{31} \\ a_{11}a_{21} & a_{12}a_{22} & a_{13}a_{23} & a_{12}a_{23}+a_{13}a_{22} & a_{13}a_{21}+a_{11}a_{23} & a_{11}a_{22}+a_{12}a_{21} \end{bmatrix} \quad (8)$$

Eqn 7 is referred to as a Bond rotation in recognition of W.L. Bond who originally developed this efficient technique for performing co-ordinate rotations of the elastic stiffness matrix (BOND, 1943).

In the present study, Young's modulus was calculated as a function of angle (of the stress axis) in 2° increments for all spatial directions using custom software to perform a Bond rotation to position the 3-axis along each direction of interest. To our knowledge, this is the first direct application of Bond rotations to analyse the anisotropy of Young's modulus of human bone. The following elastic stiffness matrix (expressed in units of GPa) served as the input for our analysis:

$$[c] = \begin{bmatrix} 19.5 & 11.4 & 12.5 & 0 & 0 & 0 \\ 11.4 & 20.1 & 12.5 & 0 & 0 & 0 \\ 12.5 & 12.5 & 30.9 & 0 & 0 & 0 \\ 0 & 0 & 0 & 5.72 & 0 & 0 \\ 0 & 0 & 0 & 0 & 5.17 & 0 \\ 0 & 0 & 0 & 0 & 0 & 4.05 \end{bmatrix} \quad (9)$$

For each direction of interest, the elastic compliance matrix $[s']$ was obtained by numerically computing the matrix inverse of the rotated elastic stiffness matrix $[c']$. The value for Young's modulus along that direction was obtained from the reciprocal of the elastic compliance coefficient s'_{33}.

3 Results

Fig. 1 illustrates the results of our analysis for human tibial cortical bone. The upper panel displays a polar plot representing the angular dependence of Young's modulus in the meridian plane. The meridian plane is defined as the 1–3 plane before any rotations are applied to the co-ordinate system. This plot reveals substantial anisotropy in Young's modulus with maxima (20.9 GPa) occurring for directions parallel to the long axis of the tibia (the 3-direction), and minima (11.8 GPa) perpendicular to this axis (the 1-direction). The lower panel of Fig. 1 illustrates results for the transverse plane. The transverse plane is defined by the 1–2 plane before rotation.

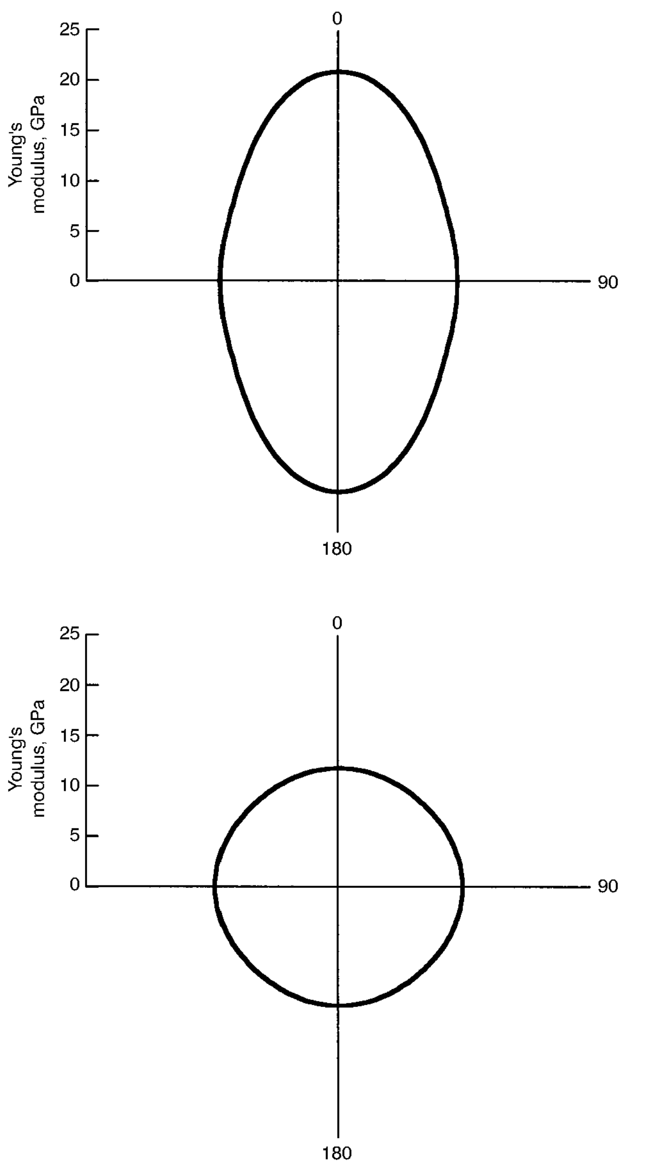

Fig. 1 *Polar plot representations of the angular dependence of Young's modulus for human tibial cortical bone in the meridian plane (upper panel) and the transverse plane (lower panel)*

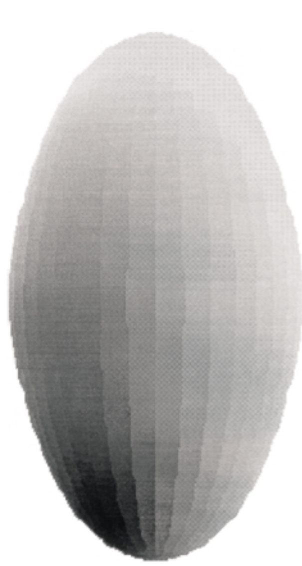

Fig. 2 *Three-dimensional representation of the anisotropy of Young's modulus for human tibial cortical bone*

Young's modulus is very nearly isotropic in this plane. Maxima (12.3 GPa) occur at 90° and 270°, and four equal minima (11.7 GPa) occur at 33°, 147°, 213° and 327°.

The polar plots illustrated in Fig. 1 may be interpreted as the intersections of two orthogonal planes with a closed three-dimensional surface representing Young's modulus in all spatial directions. We obtained this surface directly by computing Young's modulus in 2° increments for all directions. The result is illustrated in Fig. 2. To interpret this result, it is helpful to imagine an observer located at the centre inside the surface. The vector defined by where the observer's line of sight intercepts the surface defines the magnitude of Young's modulus in that direction. The long axis of the surface coincides with the long axis of the tibia. In software this surface may be rotated to any desired viewing angle. This technique of representing Young's modulus as a closed surface is an efficient means of visualising the three-dimensional anisotropic elastic properties of a material.

4 Discussion

It is useful to compare the anisotropic elastic properties of cortical bone to other materials with a similar elastic symmetry. The specimens considered in this study are nearly isotropic in the transverse plane, characteristic of a system with hexagonal symmetry. A hexagonal system has five independent coefficients arranged in an elastic matrix of the form:

$$[c] = \begin{bmatrix} c_{11} & c_{12} & c_{13} & 0 & 0 & 0 \\ c_{12} & c_{11} & c_{13} & 0 & 0 & 0 \\ c_{13} & c_{13} & c_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & c_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & c_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2}(c_{11} - c_{12}) \end{bmatrix} \tag{10}$$

Inspection of Table 1 shows that the measured value for c_{11} is not significantly different from c_{22}. The same is true for c_{44} and c_{55}. If we approximate

$$c_{22} \cong c_{11} \text{ and } c_{55} \cong c_{44} \tag{11}$$

then the elastic stiffness matrix for cortical bone given in eqn 9 has the same form as the matrix for a hexagonal system given in eqn 10. Said another way, the measured values of the elastic

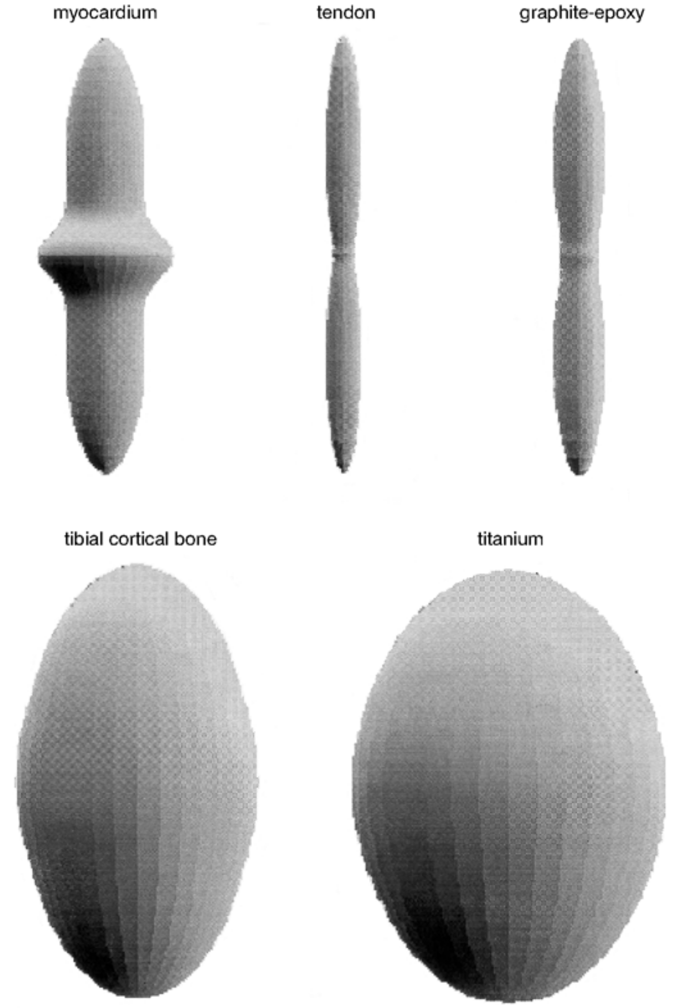

Fig. 3 *Three-dimensional representations of the anisotropy of Young's modulus of human tibial cortical bone and other materials with similar elastic symmetries. Note that these surfaces are not drawn to scale. Table 2 reports the maximum values of Young's modulus for each material*

Table 2 Comparison of the maximum value of Young's modulus of cortical bone to other elastically anisotropic materials. Each surface illustrated in Fig. 3 was individually normalised to the corresponding maximum value listed in this table

Material	Maximum value of Young's modulus, GPa
Formalin-fixed myocardium	0.101
Formalin-fixed tendon	1.37
Unidirectional graphite–epoxy composite	119
Human tibial cortical bone	20.9
Titanium	144

stiffness coefficients suggest that cortical bone is well approximated as a system with hexagonal symmetry.

Many materials possess a hexagonal elastic symmetry including certain crystals such as quartz, and fibre reinforced composite materials possessing a unidirectional arrangement of fibres with a random transverse distribution. For our comparison we chose the following combination of biologic and non-biologic materials: tendon, myocardium, unidirectional graphite epoxy composite, and crystalline titanium. The elastic coefficients of tendon and myocardium were measured ultrasonically using formalin fixed specimens of both types of tissues prepared in such a way that the fibre architecture of these tissues was approximately unidirectional (HOFFMEISTER *et al.*, 1994, 1995, 1996*a*). The elastic coefficients for the graphite epoxy composite also were measured ultrasonically (HANDLEY *et al.*, 1990) and the elastic coefficients for titanium were obtained from the literature (AULD, 1990).

Fig. 3 illustrates the three-dimensional surfaces representing Young's modulus for these materials. The surfaces are oriented such that the direction of the maximum value of Young's modulus is oriented vertically. For myocardium, tendon and graphite-epoxy, this direction corresponds to the fibre axis of these materials. Table 2 reports the maximum values of Young's modulus for each material. Note that the surfaces illustrated in Fig. 3 are *not* drawn to the same scale. For example, the maximum value of Young's modulus for titanium is more than 1400 times greater than the corresponding maximum value for myocardium.

A comparison of these figures suggests that soft tissues such as tendon and myocardium may be modelled reasonably well as fibre reinforced composite materials. This appears to be less true for cortical bone. In the meridian plane, Young's modulus exhibits a less pronounced anisotropy, and lacks the interesting bulge near the equator that seems to be characteristic of fibre reinforced materials. Qualitatively, the anisotropic features of cortical bone appear more similar to certain hexagonal crystalline materials such as titanium.

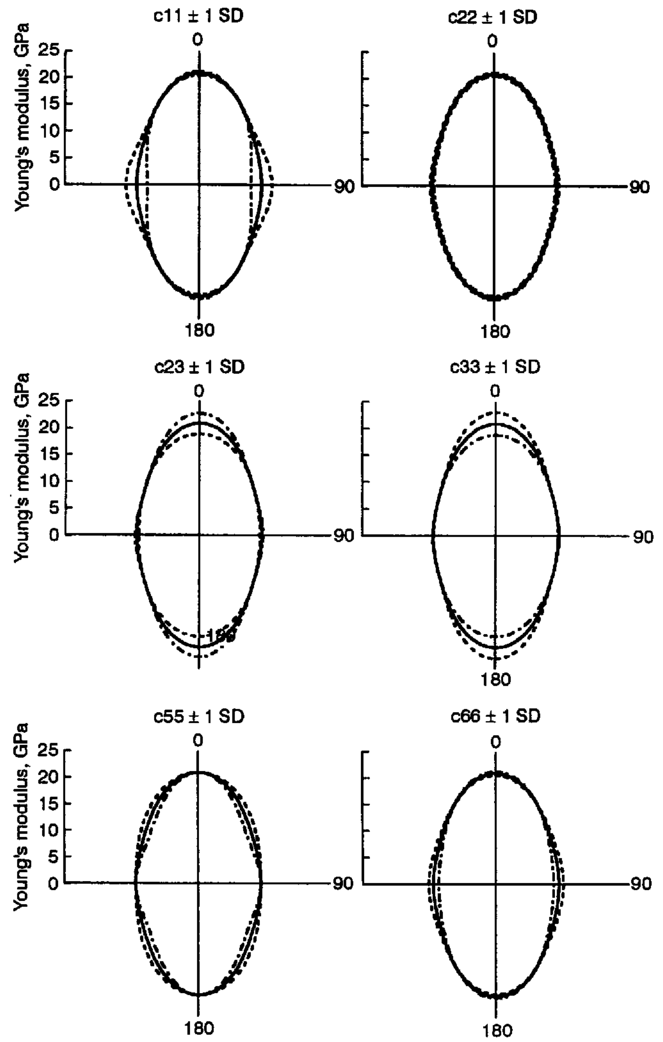

Fig. 4 *Sensitivity of results obtained for human tibial cortical bone on uncertainties in the measured values of the elastic stiffness coefficients. The results are displayed for the meridian plane. Effects of varying each indicated coefficient by (- - - -) plus or minus (- · · · -) one standard deviation*

As part of our analysis, we sought to assess the sensitivity of our results on the uncertainties in the measured values of the seven independent elastic stiffness coefficients c_{11}, c_{22}, c_{23}, c_{33}, c_{44}, c_{55} and c_{66}. Closed form expressions may be obtained for the dependence of Young's modulus on the elastic stiffness coefficients for any direction of interest. However, these expressions can be complicated and not very enlightening. We therefore chose a more graphical means of analysing the sensitivity of Young's modulus on the reported uncertainties in these coefficients. Figs 4 and 5 illustrate our results. Each panel of these figures displays three polar plots. The solid line reproduces the results illustrated in Fig. 1. The dashed lines represent the results that are obtained when the indicated elastic stiffness coefficient is increased by one standard deviation (see Table 1), and the dot-dashed lines indicate the result that is obtained when the coefficient is decreased by one standard deviation.

Fig. 4 presents the results of this analysis for the meridian plane. Young's modulus is most sensitive to uncertainties in c_{11}, c_{23} and c_{33}. Uncertainties in c_{23} and c_{33} produce larger effects parallel to the long axis of the bone, whereas uncertainties in c_{11} produce larger relative effects perpendicular to this direction. Young's modulus is somewhat less sensitive to uncertainties in c_{55} and c_{66}, relatively insensitive to the uncertainty in c_{22}, and completely insensitive to c_{44} (not shown).

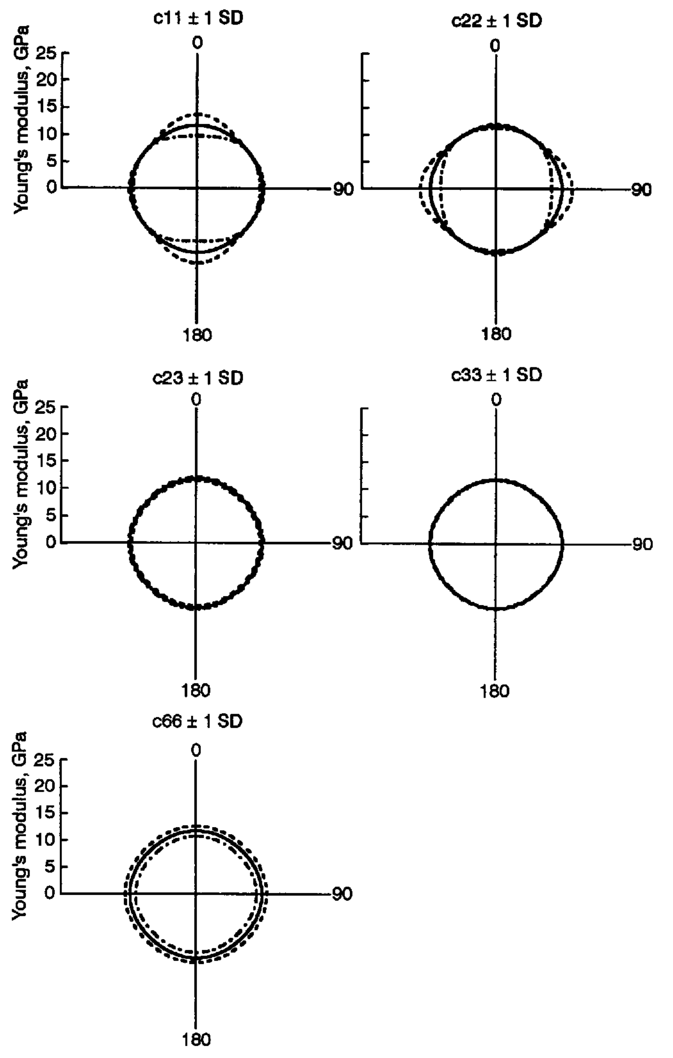

Fig. 5 *Sensitivity of results obtained for human tibial cortical bone on uncertainties in the measured values of the elastic stiffness coefficients. The results are displayed for the transverse plane. Effects of varying each indicated coefficient by (----) plus or minus (-·-·-) one standard deviation*

In the transverse plane Young's modulus is most sensitive to uncertainties in c_{11}, c_{22} and c_{66}, and much less sensitive to uncertainties in c_{23} and c_{33}. Young's modulus is completely insensitive to uncertainties in c_{44} and c_{55} (not shown). Variations in c_{11} produce the largest effect along the 1-axis (the radial direction relative to the tibia), whereas variations in c_{22} produce the largest effect along the 2-axis (the circumferential direction). Variations in c_{66} produce approximately (but not exactly) the same effect for all angles.

Taken together Figs 4 and 5 provide a good indication of the effects of the uncertainties of the measured values of the elastic stiffness coefficients on our results for the anisotropy of Young's modulus in human tibial cortical bone. To a certain extent, this analysis also provides some insight to the sensitivity of Young's modulus on these coefficients. As indicated in Table 1 the relative uncertainties in all seven measured coefficients are, to a rough approximation, the same. It seems fair to conclude that for analysis in the meridian plane Young's modulus is most sensitive to c_{11}, c_{23}, c_{33}, c_{55} and c_{66}. For analysis in the transverse plane Young's modulus is most sensitive to c_{11}, c_{22} and c_{66}.

One limitation of this study is the assumption that the relationships given in eqn 2 are valid for cortical bone. This was necessitated by the fact that c_{12} and c_{13} could not be measured directly. To determine the range over which this assumption was valid, we varied c_{12} and c_{13} individually by known percentages of their assumed values. Fig. 6 shows the result of varying these coefficients by 10%. The dashed lines illustrate the effect of increasing the indicated elastic stiffness coefficient by 10%, and the dot-dashed lines illustrate the effect of decreasing the coefficient by 10%. A comparison of Fig. 6 with Figs 4 and 5 shows that the magnitude of the effect is approximately the same as that produced by measurement uncertainties in the seven directly measured coefficients. Based on this analysis we conclude that if the assumed relationships given in eqn 2 are valid to within 10%, the effect on our analysis of Young's modulus is no greater than that due to the measurement uncertainties in the other coefficients.

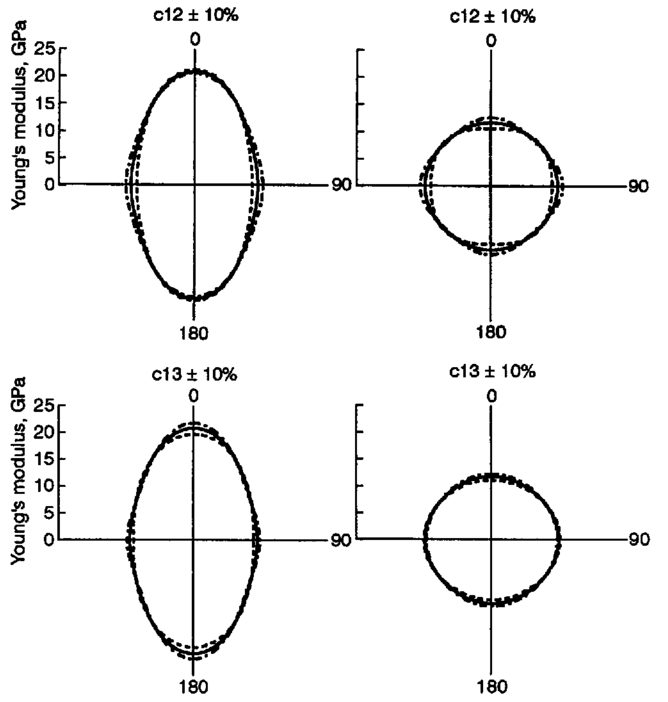

Fig. 6 *Effect of 10% variations in c_{12} and c_{13} on the anisotropy of Young's modulus in the meridian (left) and transverse (right) planes. Effects of varying each indicated coefficient by (----) plus or (-·-·-) minus 10%*

5 Summary

Ultrasonic techniques are well suited for determining the elastic properties of cortical bone. When these techniques are used to obtain the elastic stiffness matrix, a large amount of information is gained about the linear elastic properties of this material. For example, Young's modulus along each of the three principal axes of the material may be obtained by simply computing the inverse of the elastic stiffness matrix. More sophisticated techniques involving rotation transformations of the elastic stiffness matrix may be used to determine Young's modulus in any spatial direction.

The present study reports, for the first time, the elastic stiffness matrix for human tibial cortical bone obtained through the ultrasonic velocity measurements described previously. In addition, this study describes how to apply Bond rotations to perform co-ordinate transformations of the elastic stiffness matrix. This technique is used in a novel fashion to obtain the detailed three-dimensional anisotropy of Young's modulus in human cortical bone. The analysis techniques described in this study may be applied to a variety of materials, and are not limited to cortical bone. In future studies it may be possible to extend these techniques to include other mechanical moduli such as the shear modulus.

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JAE-YOUNG RHO is an assistant professor of biomedical engineering at the University of Memphis. He received his PhD from the University of Texas Southwestern Medical Center at Dallas in 1991. His current research interest is skeletal tissue mechanics, especially the measurement of mechanical properties of bone at various levels of the hierarchical structural organisation using nanoindentation and ultrasound techniques.