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Numerical Prediction of Ocean Waves in the North Atlantic for December, 1959

(With Plate 3 with Figs. 1–10, Plate 4 with Figs. 11–14, Plate 5 with Figs. 15–20, Plates 6 and 7)

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Summary. A severe storm of December 15–18, 1959, over the North Atlantic, covering great areas of ocean with high winds, was responsible for high seas which were measured by a ship-borne wave recorder on the OWS "Weather Reporter", while proceeding from Northern Ireland to Station "J" at lat. 52½°N, long. 20°W. The wave records were calibrated and analyzed for determination of significant wave heights and wave energy spectra. The present paper is an attempt to predict (for comparison) the wave conditions that would have been encountered by the moving ship from analysis of the oceanwide weather records and the use of a high-speed digital computer process for forecasting waves in moving, variable wind systems. The wave prediction technique is dependent on generalizations of empirical laws derived from observed wind-wave relationships. The proverbial non-uniformity of the latter makes possible several versions of supposedly best – fit empirical laws. Trial is made of two different generalizations, of which the second was found to yield predicted significant wave heights in fair agreement with the measurements over a period of several days. Further improvement, however, is possible and the forms of the empirical wind-wave generation laws, likely to be most nearly in agreement with the natural laws, are derived.

Numerische Vorausberechnung von Wellen im Nordatlantik für Dezember 1959 (Zusammenfassung). In der Zeit vom 15. bis 18. Dezember 1959 erzeugte ein schwerer Sturm über dem Nordatlantik, der weite Gebiete mit starken Winden überzog, die hohen Wellen, die vom Wellenschreiber des Wetterschiffes „Weather Reporter" auf seiner Fahrt von Nordirland nach der Station „J" in 52½°N, 20°W gemessen wurden. Die Wellenaufzeichnungen wurden aufbereitet und analysiert, um die maßgeblichen Wellenhöhen und Wellenenergiespektren zu bestimmen. Die vorliegende Arbeit ist ein Versuch, die Wellenverhältnisse (zum Vergleich) vorherzusagen, die von dem fahrenden Schiff nach Analyse der Seewetteraufzeichnungen und der Verwendung von Schnellrechnerdaten zur Vorhersage bei wandernden, veränderlichen Windsystemen angetroffen worden wären. Die Wellenvorausberechnungstechnik ist abhängig von Verallgemeinerungen empirischer Gesetze, die aus den beobachteten Wechselbeziehungen zwischen Wind und Wellen hergeleitet sind. Die sprichwörtliche Uneinheitlichkeit letzterer ermöglicht verschiedene Versionen der mutmaßlich am besten passenden empirischen Gesetze. Zwei verschiedene Verallgemeinerungen werden untersucht, von denen die zweite eine gute Übereinstimmung zwischen vorhergesagten maßgeblichen Wellenhöhen und Messungen über einen Zeitraum von mehreren Tagen erbrachte. Eine weitere Verbesserung ist jedoch möglich, und empirische Wind-Wellen-Gesetze, die den Naturverhältnissen am nächsten kommen, werden abgeleitet.

Prédiction numérique de vagues dans l'Atlantique Nord pour le mois de décembre 1959 (Résumé). Du 15 au 18 décembre 1959 une violente tempête sur l'Atlantique Nord où de vastes étendues de l'océan furent soumises à l'action de vents très forts, souleva de grosses lames qui furent mesurées par un houlographe à bord du navire météorologique «Weather Reporter» se rendant d'Irlande du Nord à la station «J» par 52°30′N–20°00′W. Les enregistrements de vagues furent réduits et analysés pour obtenir les valeurs significatives des hauteurs des lames et du spectre d'énergie. La présente étude est un essai de prédiction (pour comparaisons) des vagues qu'aurait rencontrées sur sa route le navire, au moyen d'une analyse des renseignements météorologiques s'étendant à tout l'océan et effectuée en utilisant un calculateur digital à grande vitesse pour la prédiction des vagues dans des systèmes de vents variables en déplacement. La technique de prédiction des vagues repose sur des généralisations de lois empiriques tirées de l'observation des relations entre vent et vagues. L'instabilité proverbiale de ces relations autorise plusieurs versions des lois empiriques supposées les mieux adaptées. On fait l'essai de deux généralisations différentes et on trouve que la seconde permet de prédire des hauteurs de vagues qui concordent assez bien avec les hauteurs effectivement mesurées pendant plusieurs jours. Une amélioration ultérieure reste cependant possible et on en déduit des lois empiriques qui semblent se rapprocher le plus des lois naturelles.

Introduction. The month of December, 1959, produced some exceptionally stormy weather over the North Atlantic Ocean and several weather ships reported very high seas in the neighborhood of about 100 to 400 miles off the coast of Europe and Ireland. Some of these conditions have already been reported by H. Walden [1961a, 1963] and J. Darbyshire [1961], also by C. L. Bretschneider, H. L. Crutcher, et al. [1963], and by L. Moskowitz

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[1963], W. J. Pierson Jr. and Moskowitz [1963], and attempts to account for the high waves have been made in the papers of Walden and Darbyshire and also by L. Baer [1962a, b] and Pierson (publication pending) by invoking various wave hindcasting procedures.

Darbyshire, in particular, develops a numerical prediction system, which encompasses the North Atlantic as an x-y grid, and uses his formulae for spectral-energy-density of waves in a combination graphical-numerical process which evaluates the period-distribution of energy in each square of the grid at selected times. Darbyshire's investigation covers the period from December 16 to 31, 1959 and compares his predictions with the waves observed by the British weather ship at Station "J", on latitude 52½°N, longitude 20°W.

Walden's study of 1961 treats the earlier period round December 6, 1959, and the complex sea conditions encountered by the weather ship at Station "K" on latitude 45°N, longitude 17°W. His paper of 1963 estimates the wave conditions at Station "J" on December 16, 1959, by methods somewhat equivalent to those of this paper.

Baer [1962a, 1962b] uses a numerical method somewhat similar to Darbyshire's, utilizing the Neumann uni-directional spectrum with adaptations for directional effects based on the project SWOP experiments (J. Chase, et al., [1957]). His analysis covers the period from December 16 to 18, 1959.

It is for the latter period that Pierson, Neumann and Walden in cooperation with Bretschneider, Crutcher and the writer have cooperated in assembling particulars of the sea conditions observed by the British weather ship "Weather Reporter" at Station "J", (Bretschneider, Crutcher, et al., [1963]). It is for this period, too, that the present study attempts a prediction of the wave conditions that the "Weather Reporter" would have encountered, using a method (B. W. Wilson [1961, 1962a]) that is rather radically different from those of Darbyshire and Baer and the still more recent work of Pierson.

II. Meteorological analysis. 1. The weather sequence for December 1959. The important weather system crossing the North Atlantic Ocean and giving rise to the severe sea conditions in the latter half of December, 1959, is illustrated in Fig. 1 (see Plate 3) by the succession of frontal positions assumed by a large cyclone, centered almost north of Newfoundland at 0000 Z on December 15 and showing a track on a north-easterly course towards the Faroe Islands between Scotland and Iceland at about 1200 Z on December 17. The frontal complex appears to have been double-centered at 0000 Z on December 15 but within 24 hours had settled into a pronounced occluding cold front, backed up by high winds over a vast extent of ocean. By the time the front had moved into the North Sea, a new front was beginning to occlude off the east coast of Newfoundland.

Also shown in Fig. 1 are the successive positions at corresponding times occupied by the OWS "Weather Reporter" as she moved out from Northern Ireland towards her station at position "J" on latitude 52½°N, longitude 20°W. It was during the critical 24 hours of December 17 that she encountered the highest seas. By the end of that day the storm front had moved on out of the area of the map, Fig. 1, apparently detached from its low-pressure center which had become retarded over the British Isles.

Some of the features of the critical period are revealed in the sample synoptic maps of Figs. 2 to 9 (see Plate 3). Figs. 2, 4, 6 and 8 are effectively representative of the basic data available in regard to the weather for the period 1800 Z, December 16, to 1200 Z, December 17, and show the pressure patterns of the storm systems. The warm and cold fronts are designated in the standard manner of meteorological maps. The positions of all ships known to have reported weather and sea data, including the weather ship, are shown in these charts. Information given by them regarding atmospheric pressure and sea state are indicated, whenever available, by relevant numerals (see Legend).

In the corresponding sequence of maps (Figs. 3, 5, 7, 9) the wind patterns are given for the same interval, as derived by the particular analysis pursued for this study. We shall anticipate discussion of this analysis for purposes of pointing out certain salient features of the storm. The maps show contours or isotachs of surface wind velocity in knots (full-lines) and indicate ships positions, reported wind directions and wind speeds in the conventional manner. The overall circulation or streamlines of the surface wind flow are shown in dash-line.

A characteristic feature of the storm is the light winds encountered at the low pressure center as the surface flow spirals inward and thence upward to higher levels of the atmosphere. Sharp discontinuities of wind direction are evinced along the occluded portions of the weather fronts; less drastic discontinuities in direction where the fronts are not occluded. High winds encompass the low pressure center in a sort of pseudo-horseshoe isotach-pattern which is not unlike the characteristic wind distribution in a hurricane (cf. C. S. Gilman and V. A. Myers [1961]; B. W. Wilson [1962a]). Unlike a hurricane, however, the highest winds are more to the rear of the pressure center, more diversely spread, and of lesser magnitude in their ultimate extreme. The zone of highest wind, in fact, appears to lie quite close to the center of curvature of the advancing cold front.

At 0000 Z on December 17, near the peak of the storm, the extent of ocean covered by winds in excess of 40 knots was of the order of 375,000 nautical square miles, with some 45,000 n. sq. mi. of this area holding winds in excess of 60 knots. The vast size of this overall area is roughly comparable to the total land areas of the states of Texas, New Mexico and Arizona combined, and is testimony to the enormous magnitude of the disturbance. By 0600 Z of December 17, the area of 40 knot winds had contracted to about 250,000 n. sq. mi., and of 60 knot winds to about 15,000 n. sq. mi., the contraction having proceeded almost exclusively from the rear of the storm.

2. Analysis of the weather data. The approach first adopted was to attempt to compute the surface winds for a number of selected positions occupied by ships and compare the calculated velocities with those reported by the ships. The computational procedure followed was essentially the same as that used fairly successfully in hurricane studies (Wilson [1957, 1962a]). Ultimately the calculation procedure was adjudged too unreliable and cumbersome. It was decided finally to place reliance entirely on ship reports of surface wind velocities and attempt to map their isotachs by regarding the reported winds as reasonably representative of velocities at 10 m height above sea level.

By taking account of isobar distributions and spot wind velocity observations and by checking always the consistency of the contouring by the trends of change of one map to the next at six hour intervals, it was found possible to develop reasonable interpretations of the wind velocity distributions over the entire North Atlantic as shown in the examples of Figs. 3, 5, 7 and 9.

The wind patterns in general showed a unique feature that has now been found in many other cases that have been elaborated, notably for hurricane "Audrey" in the Gulf of Mexico in 1957 (cf. Wilson [1962a]); namely, that wind isotachs show a strong tendency towards approximate parallelism with the land boundaries. This is observable in Fig. 5, for instance, where the contours envelop the Bay of Biscay and are deflected by Iceland and Greenland and moulded by North America. The effect is also apparent in the wind patterns of Figs. 3, 7 and 9.

The consistency of change of the wind patterns is believed to be fairly well exemplified in Figs. 3, 5, 7 and 9. It is clear from Fig. 5 that at 0000 Z, on December 17, 1959, both the main storm and its follower off the east coast of North America developed their full powers and highest winds concurrently. The reason for this is not outwardly apparent from the isobaric distributions alone and must be sought in other factors perhaps not completely evidenced in ordinary synoptic weather maps. One notices, however, from Fig. 1, that there was a startling acceleration of frontal movement between 1200 Z on December 16 and 0000 Z on December 17, in keeping with the intensification and expansion of the isotach patterns.

III. Directions of wave approach to the weathership. 3. Waves encountered by the OWS "Weather Reporter". In Fig. 1 it has already been pointed out that the weather ship "Weather Reporter" was moving towards location at Station "J" (lat. $52\frac{1}{2}^0$N, long. 20^0W) at the very time that she encountered highest seas in the latter part of December 1959. Her progress is shown with greater definition in Fig. 10, from which it is evident that her advance was considerably retarded in the period from 0000 Z, December 17, to 0000 Z, December 18.

The "Weather Reporter" was equipped with a ship-borne wave recorder (M. J. Tucker [1956]) and was thus able to measure the sea conditions objectively by double integration of the output of its accelerometers, in addition to the more subjective visual observations that are taken rountinely. These observations have been discussed and analyzed in the team-paper (Bretschneider, Crutcher, et al., [1963]) and will be referred to again in a later part of this paper.

Of immediate interest now is the matter of how the waves were reaching the weather ship and the problems for wave-forecasting posed by the motion of the observing station itself.

The average path of the ship is found to lie along the great circle route *AA* connecting with Station "J" (lat. 52½°N, long. 20°W) and the ship location at 0600 Z on December 15, off Northern Ireland. Extended, this great circle passes through lat. 40°N, long. 45°W, and appears as a slightly curved track on the conical projection of Fig. 10 (see Plate 3). Close examination of the weather maps showed that in the interval, December 15 to 22, 1959, the winds were predominantly directed towards the north-east, in approximate parallelism to the great circle *AA*. It seemed logical, therefore, in the first instance, to develop a space-time wind-field for this particular direction for application of the numerical procedure (Wilson [1961]), to take cognizance of continuous time changes in wind velocity and fetch. A special advantage of this was that the motion of the ship could also be accommodated at the same time, as will become apparent in later discussion of Fig. 11 (see Plate 4).

It seems appropriate here to point out that, as result of initial wave hindcasts made for the path *AA*, it was recognized that some of the high waves encountered by the weather ship in the interval 1200 Z, December 17, to 0000 Z, December 18, could not be explained. Re-examination of the weather maps then disclosed that the weather ship had not always been bucking head seas, as supposed. In fact, as shown in Fig. 10, the predominant waves had reached her on the port side at 1200 Z on December 16 and from the starboard side in the interval between 1200 Z on December 17 and 0000 Z on December 18 (see also Walden [1963]).

4. Variability of wave direction. Further close examination of the weather maps showed that wave directions reported by ships always agreed closely with the wind directions. This is apparent in the sampling of charts presented in Figs. 2 to 9. Wave directions are recorded at ship positions in Figs. 2, 4, 6 and 8 and suggest that the wave rays are almost identical with the wind streamlines as given in Figs. 3, 5, 7 and 9. The obvious conclusion must be that the predominant waves always veer with the wind and that sufficient space and time is always available for this herding to be accomplished by the wind. The waves then grow in size continuously along a multi-directional (variable) great circle path according to the changes of the wind at the times the waves have reached any particular location.

The curved paths shown in Fig. 10, supposedly followed continuously by waves in this herding action by the wind, have been determined on the assumptions that the predominant waves in existence at any time had a phase velocity, c, equal to 80% of the prevailing wind speed, U, and that their movement was always along a great circle, although a continuously changing one. The justification for the first assumption is to be found in the general observations of G. Schott [1893], V. Cornish [1934], N. F. Barber and F. Ursell [1948], N. N. Djounkovski and P. K. Bojitch [1949], Darbyshire [1952], and G. Neumann [1952b], who independently, and respectively, found ratios for c/U of 0.76, 0.80, 0.75, 0.83, 0.72, 0.80. The actual speed of wave progression in deep water, V – the group velocity – was thus taken as $V = c/2 = 0.4\,U$. The second assumption is considered self-evident as an application of Hamilton's Principle, which requires the most direct path for the waves.

On this basis it appears that the waves which reached the weather ship at 0000 Z on December 17, although coming from the apparent bow-direction, *AA* (Fig. 10), in reality originated in the area of the North Atlantic between Newfoundland and Greenland. Twelve hours later, however, the waves pounding the weather ship on the starboard quarter were those that had originated from the Denmark Straits between Greenland and Iceland. Together with the wind they appear to have deflected the ship off course to the south so that by 1800 Z on December 17, the weather ship had to turn into the wind and waves and make slow progress towards the north-west. At 0000 Z on December 18, the waves were still substantially from the north-west, and the ship was now somewhat off course to the north (Fig. 10). As a compromise wave path, representative of the period 1200 Z, December 17, to 0000 Z, December 18, the line *BB* was selected for analysis.

In the continuing forward progress of the ship after 0000 Z, December 18, the wind and waves began to reach the ship from a direction more in accord with *AA* again (Fig. 10), except that at and around 1800 Z, December 18, they appear to have originated along the path *CC*, from the southern part of the ocean.

In addition, then, to the original hindcast made for the direction *AA*, it was deemed necessary to make supplementary hindcasts for the wave-paths *BB* and *CC*.

IV. Space-time wind-fields. 5. Wind-field for the wave path *AA*. The process of developing a space-time wind field has been outlined in previous publications (Wilson [1955, 1957, 1961, 1962a]) and need only be briefly reviewed here. Components of wave velocity along the great circle path *AA* were extracted from each wind pattern (such as Figs. 3, 5, 7 and 9) at every six hours. Transferred to Fig. 11, these distributions over distance at particular times could be plotted and contoured to give the continuous space-time variation of the wind velocity components directed along *AA* (positive towards Northern Ireland, negative in the direction of motion of the ship).

Since the weather ship was in motion, effectively along the great circle path *AA*, its track is recorded in space-time in Fig. 11 by the chain of circles marking the right hand fringe of the shaded area.

The intersections of the weather fronts with the line *AA* are also revealed in Fig. 11 by the sharp discontinuities of isotachs, along which the heavy lines have been given the standard symbolism for warm, cold and occluded fronts. From a comparison of Figs. 1 and 10, it may be inferred that the movement of a cold or occluded front along the line *AA* would be representative of the approximate speed of advance of the fronts across the ocean. From Fig. 11, then we find that the most severe storm featured in Fig. 1 had an overall average speed of advance of about 42 knots, this being the mean gradient of the cold front on December 16. However, it is apparent that during the latter part of December 16, the front accelerated to an average speed of about 50 knots, a feature already remarked upon in Section II (1).

Figure 11 reveals not only the antecedent storm that preceded the huge cyclone of December 16, but the subsequent storms that followed it on December 18 to 19 and still later on December 21. It is clear that these later fronts had slower speeds of advance (less steep gradients dx/dt in terms of distance x and time t), and that the winds trailing them (contoured zones below the fronts) were much less intense.

One great advantage of a wind-field such as Fig. 11 is that it permits continuous monitoring of wind change in the dimension of time, accomplished by careful contouring of the isotachs on a space-time basis. It is clear, for instance, that velocity gradients with respect to time are not linear and that linear interpolations of change between weather maps at every six hours, in the absence of such a wind-field as Fig. 11, would generally be invalid.

6. Wind-fields for the paths *BB* and *CC*. In the cases of the selected wave paths *BB* and *CC* of Fig. 10, the necessity for deriving wind velocity components fell away, since it was always the full wind velocity that bore along the wave path. Construction of the wind-field was hereby simplified, at the expense, of course, of initial work needed to evolve the curved directions *BB* and *CC* in Fig. 10.

Not all of the areas of the wind-fields for *BB* and *CC*, shown in Fig. 12 (see Plate 4), are applicable to the generation of waves that could influence the ship. It is clear from Fig. 10 that between 0000 Z, December 18, and 1200 Z, December 18, there was a very sudden change in wave direction at the ship. Consequently, any waves that supposedly could arise in the lower left-hand (unshaded) area of wind-field *BB* in Fig. 12, to reach the origin later than about 0300 Z, December 18, would be fictitious and could not qualify for consideration in the final results. The same sort of argument applies to the wind-field *CC*. It was thus really unnecessary to complete the contouring of the lower left-hand areas of the wind-fields in Fig. 12, though this was done here for the sake of appearance.

In both Figs. 11 and 12, the shaded areas define the zones of the wind-fields which featured in the numerical computations. The shape of the shaded zone in Fig. 11 was conditioned by the general slope of wave propagation lines traversing the field. Typical such propagation paths are shown starting from the points $X = 1000$ n. mi., $t = 1300$ Z, December 15, and

$X = 1140$ n. mi., $t = 1100$ Z, December 18, from which it is seen that the upper and lower boundaries of the shaded area have approximate parallelism. The left-hand boundary of the shaded area in Fig. 11 was defined somewhat arbitrarily to give an approximate maximum fetch to the ship of about 800 n. mi. The winds beyond this distance were only moderate or light and incapable of affecting the limiting heights to which waves could be generated.

In the case of wind-fields *BB* and *CC*, shown in Fig. 12, delimitation of the shaded areas was also arrived at from the general gradient of wave propagation lines, such as that shown originating in *BB* at $X = 490$ n. mi., $t = 0800$ Z, December 16. The upper limits of the shaded areas were chosen somewhat arbitrarily where wind velocities tailed out to low values (< 15 knots). In both wind-fields *BB* and *CC*, Fig. 12, the ship is considered to be at $X = 0$ for all time, and thus is reached by the waves at the vertical right-hand extremities of the shaded areas.

V. Aspects of the numerical prediction technique. 7. Fundamental empirical relationships based on observation of waves. The computational procedure followed here is essentially the same as that developed earlier, first graphically (Wilson [1955]), and then numerically for high-speed digital calculation (Wilson [1961]). Essentially, the method relies on two empirical relationships, governing growth of significant wave height H and phase velocity c with distance x, derived as generalizations of the special relationships for observations on the growth characteristics of waves in uniform wind velocity U. These two generalizations are:

$$\frac{dc}{dx} = f_1\left(\frac{c}{U(x)}\right) \tag{1}$$

$$\frac{dH}{dx} = f_2\left(\frac{gH}{[U(x)]^2}\right) \tag{2}$$

where f_1 and f_2 are functions of dimensionless variables and $U(x)$ is here a non-uniform wind velocity, variable with the distance x.

In the previous numerical work already mentioned (Wilson [1961]), Eqs. (1) and (2 were evolved mathematically from two empirical formulas, which we shall designate (I), thought to be the best available fit to wave data existing prior to 1955. These data are reproduced in Fig. 13 (see Plate 4) and are the same as those published by Bretschneider [1952a, b]. The regression lines, there shown in long dash, accord with the Formulas I (Wilson [1955]); namely:

Formulas I:

$$\begin{aligned} &(i) \qquad \frac{c}{U} = 1.40 \tanh\left[0.044\left(\frac{gF}{U^2}\right)^{\frac{1}{3}}\right] \\ &(ii) \qquad \frac{gH}{U^2} = 0.26 \tanh\left[0.01\left(\frac{gF}{U^2}\right)^{\frac{1}{2}}\right] \end{aligned} \tag{3}$$

where F is here used, in lieu of x, to represent the fetch.

Although Eqs. (3) were used quite recently by the writer in analysis of the waves generated by hurricane "Audrey" of June 1957 (Wilson [1962a]), there is good reason to believe that they have a tendency to over-estimate wave height in the general case. Thus both Walden [1953/54, 1961] and T. Saville [1954] have pointed out the remarkable differences that exist in the relationships of significant wave height H to fetch x, and duration t, in a constant wind (such as $U = 40$ knots), as advocated by different observors. Figure 14 (see Plate 4), which shows H as a function of x for $U = 40$ knots, is adapted from Walden [1961b], but includes, additionally, the relationships arrived at by D. A. Molitor [1935], H. V. Sverdrup and W. H. Munk [1947], Bretschneider [1952a, 1959], Walden [1958] and Wilson [1955]. It is clear that the differences are very considerable; in fact, the limits differ by as much as a factor of about 2. Equation (3 ii) of Formulas I, which in Fig. 13, appears to be a plausible fit to the data there available, is found in Fig. 14 to give results near the upper limit of all the relationships that have been proposed by others.

It was this fact that led the writer in 1959 to commence a more careful re-appraisal of basic observational wave data. Pierson [1962] has justly pointed out that the wave height data of Fig. 13 have a scatter which involves as much as a factor of 4. His lament, that so much of the data there plotted is unrecoverable in their original form, was almost an echo of the writer's own frustrating discoveries that a great deal of the original data was unavailable for reexamination. In the end, the writer considered only recently published data on waves that had been measured by calibrated instrumentation together with such of the older, reliable data as could be checked or reanalyzed and may also have been omitted from earlier consideration in Fig. 13.

Data almost completely independent of those in Fig. 13 were in this way assembled in Fig. 15 (see Plate 5). Plotted points to the left-hand side of this diagram invoke largely the model experiments of T. Stanton [1932], J. W. Johnson and E. K. Rice [1952], T. Hamada, et al., [1953] and the wave measurements of H. U. Roll [1951] on the North Sea tidal flats. Because, in all these cases, the wind velocity of reference was at a height of only a few centimeters above the surface, it was considered necessary to determine the corresponding wind velocity at the standard reference level of 10 m ($\simeq$ 30 ft.), a rather important consideration neglected in previous works. The means for doing this had been elaborated earlier (Wilson [1960]) and involved, in effect, an extrapolation of the measured wind velocity to prototype conditions at 10 m height through use of the Karman-Prandtl equation for vertical distribution.

In the central zone of the diagram (Fig. 15) the data has reference mainly to small lakes, reservoirs, ponds or tidal flats. The bulk of the measurements derive from the work of R. W. Burling [1954], Darbyshire [1956], O. Czepa and G. Schellenberger [1959], P. W. Roest [1960] and Schellenberger [1962], respectively for Abbots Lagoon, California; Staines Reservoir, England; Lough Neagh, Ireland and the Irish Sea; Muggelsee, Germany; the Ijsselmeer, Holland, and the Müritzsee, Germany.

The right-hand side of Fig. 15 contains data drawn mainly from observations made in oceans and seas where large wind fetches are possible. The recordings of the USS "Augusta" and HMS "Scylla" in the English Channel in 1944 (C. T. Suthons [1945]) are utilized, along with the extensive measurements documented by Darbyshire [1956, 1959] for the Irish Sea and the North Atlantic Ocean, by Roll [1949] for the North Sea and by L. Rundgren [1958] for the Arabian Sea. Without disparaging the very fine contributions that Neumann [1952 a, b] has made to the observations of ocean waves, it was decided not to use his extensive visual measurements made on board the "Heidberg" in the Atlantic Ocean and Caribbean Sea.

An attempt was made in Fig. 15 to sort from the observations of Burling [1954] and Darbyshire [1959] those that definitely depended on unstable atmospheric conditions, for which the prevailing air-sea temperature difference was negative ($> -2^0$ F). Plotted points for the latter consideration carry a distinctive but related symbolism (see Legend, Fig. 15) to those for stable atmospheric conditions. Burling's results tend to show that the parameters $\frac{gH}{U^2}$ and $\frac{c}{U}$ for unstable conditions definitely overlie those for stable situations. Darbyshire's results on the other hand, fail to reveal any such definitive arrangement.

It is clear from Fig. 15, which reproduces the regression lines of Formulas I for comparison, that the new data in general fall considerably below those assembled in Fig. 13. Regression lines fitted to Fig. 15 seem to accord reasonably well with the short dash straight lines having the equations:

Formulas II:

$$\left.\begin{aligned} &(i) \qquad \frac{c}{U} = 0.057\left(\frac{gF}{U^2}\right)^{0.3} \\ &(ii) \qquad \frac{gH}{U^2} = 0.0025\left(\frac{gF}{U^2}\right)^{0.4} \end{aligned}\right\} \qquad (4)$$

It was decided then at the outset of the present study to utilize Eqs. (4) in a first attempt at predicting the ocean waves likely to have been encountered by the weather ship. As will later be shown the predicted wave heights fell short of the observations, a circumstance that might almost have been anticipated from the fact that Eq. (4 ii) of Formulas II yields a complete lower bound to all the relationships convened in Fig. 14.

In the interim that the writer was compiling Fig. 15 it transpired that R. L. Wiegel [1960, 1961] was also assembling wave data in the same general form. Wiegel used extensive data from Roll [1949], besides much of the original data, reworked or refined, that had gone to make up Fig. 13. His results are shown in Fig. 16 (see Plate 5), which is adapted to a symbolism in some accord with Fig. 15. Reproduced as standards of comparison in Fig. 15 are the regression lines for empirical Formulas I and II.

It is clear that Wiegel's assemblage of data in Fig. 16, although revealing general trends which are unmistakeable, has a frightening degree of scatter, which now involves variations of the parameters $\frac{gH}{U^2}$ by a factor of 10. The preponderance of values for $\frac{gH}{U^2}$ definitely override Eq. (4 ii) of Formulas II – the short dash regression line. To the left hand side of Fig. 16 it is found that Wiegel's plots of Hamada's [1953] experimental data lie higher than the writer's plots of the same data in Fig. 15. The reason for this must be ascribed to the fact the Wiegel failed to correct the model wind velocities to prototype values. To the right-hand side of Fig. 16, however, there is not such ready explanation for the differences between Fig. 15 and 16, unless it be that some of the low lying values of $\frac{gH}{U^2}$ in Fig. 15 reflect the possibility that duration of the wind was finite and affecting the issue.

In respect to the relationships for $\frac{c}{U}$ in Figs. 15 and 16 it is found that the differences are not so great. In both cases the data on the right-hand side of the diagrams tend to under-lie the short dash regression line representing Eq. (4 i) of Formulas II.

In an attempt to compromise on all the results of Figs. 13, 15 and 16, the full-line regression lines (Formulas III) were used in a second prediction trial applied to this study. The empirical equations for these are:

Formulas III:

$$\begin{aligned} &(i) \qquad \frac{c}{U} = 1.80\left[1 - \left\{1 + 0.00622\left(\frac{gF}{U^2}\right)^{\frac{1}{3}}\right\}^{-5}\right] \\ &(ii) \qquad \frac{gH}{U^2} = 0.0024\left(\frac{gF}{U^2}\right)^{0.45} \end{aligned} \tag{5}$$

The effect of Eq. (5 ii) on wave height as a function of fetch for the particular case of $U = 40$ knots is shown by the full-line curve III in Fig. 14. This is rather better centered in the scheme of things in that diagram, though, as will later be discussed, it is still clearly not the best fit to the observational data of Figs. 13, 15 and 16.

8. The differential equations of wave height and period growth. The above discussion has indicated how Formulas II and III came to be selected for trial in the present wave prediction study. To make their application possible, Eqs. (4) and (5) had to be generalized to the forms of Eqs. (1) and (2). The manner of doing this has been discussed at some length in an earlier publication (Wilson [1961]), so that we may content ourselves here by merely stating the form of the resultant differential equations. Thus denoting, for convenience, the dimensionless parameters:

$$\begin{aligned} &(i) \qquad Z = \frac{c}{U(x)} \\ &(ii) \qquad Y = \frac{gH}{[U(x)]^2} \end{aligned} \tag{6}$$

which are the dependent variables of the problem, then the differential changes of c and H with respect to x for variable wind velocity $U(x)$ become in the case of Formulas II:

Formulas II′:

$$\begin{aligned} &(i) \quad \frac{dc}{dx} = 0.9830\, Z^{-\frac{7}{3}} / U(x) \\ &(ii) \quad \frac{dH}{dx} = 7.600 \cdot 10^{-4} Y^{-\frac{3}{2}} \end{aligned} \qquad (7)$$

and in the case of Formulas III:

Formulas III′:

$$\begin{aligned} &(i) \quad \frac{dc}{dx} = 4.9488 \cdot 10^{-2}\, U^{-1} \left(\frac{1.80}{1.80 - Z}\right)^{-\frac{8}{5}} \left[1 - \left(\frac{1.80 - Z}{1.80}\right)^{-\frac{1}{5}}\right]^{-2} \\ &(ii) \quad \frac{dH}{dx} = 4.1251 \cdot 10^{-3} Y^{-\frac{11}{9}} \end{aligned} \qquad (8)$$

where, in both Eqs. (8) and (9), c is measured in knots, x in nautical miles, U in knots and H in feet, with Z dimensionless and Y given in units of (ft – knots^{-2}) as:

$$Y = 11.27\, H/[U(x)]^2 \qquad (9)$$

9. The equations for initiating the numerical prediction. As explained in detail elsewhere (Wilson [1961]), the numerical procedure which links the windfields of Figs. 11 and 12 with the generating equations, (7) or (8) requires the definition of the former in terms of a space-time network. The spacings of this reticulation form the maximum increments of integration λ and τ for evaluating the increments of wave velocity, Δc, and wave height, ΔH, via Eqs. (7) or (8). It should be noted that X is distance measured from right to left in the wind-fields, Figs. 11 and 12, while x is distance measured from left to right from any particular point (X, t). The increment Δx of Eqs. (7) or (8) equals λ, the network spacing, when the wave group-velocity, V ($= c/2$), has reached 10 knots, but is otherwise less than λ while $V < 10$ knots.

It was found expedient to adopt, as before, values for λ and τ of 10 n. mi. and 1 hour respectively. A compilation was therefore made of the values of wind velocity at each grid-point of such a space-time lattice, superimposed on Figs. 11 and 12, these being the necessary input data to be punched on cards and committed to the memory of the computer.

Computational procedure requires that the computer starts from each grid-point of the wind-field lattice and calculate the path of the wave generated in space-time (from left to right in Figs. 11 and 12) by interpolating at each crossing of the network the appropriate wind-velocity defined by the nearest grid-point values, stored in the memory. The computer at the same time calculates the increments Δc and ΔH from Eqs. (7) and (8) and integrates the significant wave height H and period T along the way.

Certain starting formulas have to be fed to the computer for initiating an operation. The basis for these has been explained in the writer's earlier work (Wilson [1961]), so that it is considered necessary here merely to quote the formulas having relevancy to Formulas II′ and III′ of the two prediction systems used. These are:

Formulas II″: (First prediction, this study)

$$\begin{aligned} &(i) \quad \Delta x_1 = 0.3711\, U_1^{\frac{4}{7}} \\ &(ii) \quad c_2 = 1.4277\, U_1^{\frac{4}{10}} \Delta x_1^{\frac{3}{10}} \\ &(iii) \quad H_2 = 1.9070 \cdot 10^{-2}\, U_1^{\frac{6}{5}} \Delta x_1^{\frac{2}{5}} \end{aligned} \qquad (10)$$

Formulas III'': (Second prediction, this study)

$$\begin{aligned} &(i) & \Delta x_1 &= 0.7005 \; U_{11}^{\frac{1}{2}} \\ &(ii) & c_2 &= 2.2910 \, U_1^{\frac{1}{3}} \Delta x_1^{\frac{1}{3}} \\ &(iii) & H_2 &= 3.1941 \cdot 10^{-2} \, U_1^{1.10} \Delta x_1^{0.45} \end{aligned} \tag{11}$$

It is axiomatic that the values of wave velocity, height and period at the start, respectively $c_{,}$, $H_{,}$, and $T_{,}$, are all zero.

10. Features of the computational program. A general flow diagram for the numerical computation is illustrated in Fig. 17 (see Plate 5) as pertaining to the use of Formulas III' and III''. The inset sketch of the lattice of the wind-field and the space-time propagation path of significant waves starting from any particular mesh intersection (X_i, t_j) is a guide to the procedures outlined in the flow diagram. Integration of the wave path and of the wave properties proceeds at the beginning in differential increments of time $\Delta t = \tau$, or 1 hour. As soon as the group-velocity V attains 10 knots, as explained before, the differential increment of integration switches from time to distance and is taken as $\Delta x = \lambda$, or 10 n. mi. This changeover in the program is accomplished by a machine command to interrogate the value of V at each step k in the early phases of generation.

The complete program for the numerical prediction of waves according to this system is given in Fortran language in Appendix A (see Plate 6). The locations of the Formulas III' and III'' are clearly identified therein.

In Appendix B (see Plate 7) a portion of the wind-field AA is printed out to show the form in which the wind velocity data at space-time lattice points were punched on to cards for input to the digital computer. The number of lattice points involved in the shaded area of Fig. 11 for wave path AA was approximately 13,600. For wind-fields BB and CC of Fig. 12 the corresponding numbers were approximately 2700 and 900 respectively.

A CDC No. 1604 high speed digital computer was used to accomplish the computations to the general program of Appendix A. All three wind-fields required an overall input and computing period of 6.33 hours of machine time. Of this about six hours was required for the actual computation which meant that the average time taken for a single wave propagation calculation from a space-time lattice point to the ship was about 1¾ seconds. This is amazingly fast considering the complex calculations that have to be made.

The card output from the No. 1604 computer was recorded on an input tape by an IBM No. 1401 machine. Some 16,000 data cards were processed in this way in only 20 minutes.

In the final phase of the program it was arranged that both the significant wave height and the period of the waves, from each single wave propagation path reaching the ship, be plotted against their times by an automatic machine process. The input tape from the IBM 1401 was therefore fed to an IBM No. 7090 computer and the data transferred to a plot tape. The time required for this was 30 minutes. Finally an SC No. 4020 plotter reproduced the information from the plot tape to visible form in approximately 5½ minutes. The overall machine-time for the complete calculation of three wind-fields, embodying some 17,200 grid-points, was thus about 7.30 hours.

The times quoted above are not inclusive of the key-punch operation of transferring tabulated wind velocity data to punched cards. About 35 to 40 hours of key punch and verification time were involved in assembling some 1500 input data cards.

VI. Results of the numerical predictions. 11. Predicted significant waves occurrent at the weather ship. It has been pointed out in Section 7 how Formulas II and III came to be selected as bases for prediction. The generalizations II' and II'' of Eqs. (7) and (10) were tried first in their application to this study with the results shown in Fig. 18 (see Plate 5) by the distribution of fine points, marking the upper fringe of all grid-point calculations for the wind-field AA. The observed mean level of significant wave height (full-line) obtained from ship-borne wave recordings is included in Fig. 18 for comparison. Except for the six hours between 1200 Z, December 16 and 0000 Z, December 17 and between 1200 Z, December 20 and 0000 Z, December 21, the results are disappointingly low.

This was not held to be a fault of the method of prediction but rather of the adoption of Formulas II as the most appropriate empirical relationships between wind and wave parameters. The search for the cause of low prediction drew the conclusion, already discussed with the aid of Figs. 13 to 16 in Section 7, that Formulas II are obviously not the best selection for encompassing the observational data. It was decided then to adopt the more conservative Formulas III, as a compromise between Formulas I and II.

At the same time it was realized that the wind-field AA alone could not be expected to account for the waves reaching the weather ship between December 15 and 21. The importance of wind-fields BB and CC was now discovered, as detailed earlier in Section 6.

The results of adopting Formulas III and their generalizations III′ and III″, embodied in Eqs. (8) and (11), along with the additional wind-fields BB and CC, are shown in their entirety in Fig. 19, as plotted automatically by the computer. The individual grid-point calculations from the wind-fields yield a pattern of dots which assume a clear-cut upper-bound. Since the prediction procedure relies on the statistical concept of a significant wave and its growth under wind action, it follows that the envelope or maxima of all possible "significant" waves are the only true significant waves encountered at the ship. The envelopes from Fig. 19 (see Plate 5) then are introduced into Fig. 18 as the dash-line curves to reveal what now appears to be rather better general agreement between prediction and observation of significant wave heights.

Over the 48 hour period between 1200 Z, December 17 and 1200 Z, December 19, the agreement of Formula III with observation in Fig. 18 is quite good, the error being not much in excess of 10% of the maximum wave heights recorded. The predictions are less satisfactory outside of this period, although when the confidence limits on the ship-borne wave recorder heights and the visually observed wave heights are taken into account, as in Fig. 20 (see Plate 5), the errors of prediction seem still less consequential. The greatest disparities of height occur near the beginning and end of the period December 16 to December 21 treated in Fig. 20.

It is of interest to note in Fig. 20 that visual observations of high swells from the WSW direction are in very good agreement with the down-sloping envelope of wave-heights along wave path AA (from WSW). It is presumed that temporary lulls between groups of high waves from the direction BB at this time (near 1800 Z on December 17) enabled the WSW swells to assert themselves and catch the eye of visual observors.

Satisfactory comparison of wave periods in Fig. 20 is not possible, since available observations refer only to average wave periods and not significant.

12. Possible improvements in wave prediction. It is clear from Fig. 18 that the forecasting method is obviously very sensitive to the empirical laws, such as Formulas I, II or III, that are prescribed to relate the wind and waves. At the same time, the method affords the means of reaching backwards towards the goal of ever closer approximations to the apparent natural laws.

The results of using Formulas III suggest, if anything, that they are still not the most acceptable compromise to all the scattered data of Figs. 13, 15 and 16. Equation (5 ii) of Formulas III, when plotted in Fig. 14 for the particular case of a 40 knot wind ($U = 40$ knots), is found to yield a curve, which though median among so many other relationships that have been proposed, still shows a tendency to yield rather high waves at long fetches. This is precisely what Fig. 18 shows too. Thus the dash-line curve of significant wave height, prior to 1200 Z on December 17 in Fig. 18, definitely over-estimates the wave heights over the long fetches of wave path AA. Over the much shorter fetches afforded by wind-field BB, the prediction in Fig. 18, from 1200 Z, December 17 to 0300 Z, December 18, has proved to be quite accurate, suggesting that Formula III is fairly precise at short fetch. This indicates then that new Formulas IV need to be devised to yield a result such as that of Curve IV in Fig. 14. We shall return to this point again.

It is instructive, meanwhile, to consider what the effect of the two trial Formulas II and III has been in respect of typical space-time paths followed by generated waves through the wind-fields. For consideration of this aspect we select two cases, chosen at random, for the wind-field *AA* of Fig. 11. The waves, emanating from the two points (X_i = 1000 n. mi.; t_j = 1300 Z, December 15) and (X_i = 1140 n. mi.; t_j = 1200 Z, December 18), yield the tracks shown in long-dash line for Formulas II and in short dash-line for Formulas III.

Considering first of all the starting point (X_i = 1000 n. mi.; t_j = 1300 Z, December 15) it is seen in Fig. 11, that the paths diverge as a result of the differences of Eqs. (7 i) and (8 i). This brings the uppermost (Formulas III) track into zones of higher wind, thereby ensuring higher waves and an earlier arrival time (by 3.3 hours) at the ship. Higher waves are also, of course, ensured by the fact that Eq. (5 ii) of Formulas III yields larger H-values than the corresponding Eq. (4 ii) of Formulas II.

The situation for the second test point (X_i = 1140 n. mi.; t_j = 1200 Z, December 18) shows that the short-dash (Formulas III) track in Fig. 11 over-rides and diverges from the

Table 1

Comparative numerical wave predictions, using Formulas II and III, along direction AA (Fig. 11) **from starting point** (X_i = 1000 n.mi, t_j = 1300 Z, December 15)

Numerical Step No.	Distance from Zero	Formulas II					Formulas III				
		Time (GCT)		Wind Velocity	Wave Height	Wave Period	Time (GCT)		Wind Velocity	Wave Height	Wave Period
k	X_k (n. mi)	t_k (day-Hr.)		U_k (knots)	H_k (ft.)	T_k (secs.)	t_k (day-Hr.)		U_k (knots)	H_k (ft.)	T_k (secs.)
1	1000	15	13.00	35.5	0	0	15	13.00	35.5	0	0
13	900	16	00.47	23.1	10.4	8.4	15	22.69	31.0	16.1	9.6
23	800	16	08.02	26.8	10.8	9.2	16	05.47	22.5	17.7	9.8
33	700	16	14.73	46.5	14.1	10.8	16	11.95	51.1	22.2	11.1
43	600	16	20.50	52.5	18.0	12.3	16	17.49	49.1	28.4	12.7
53	500	17	01.63	53.8	21.5	13.5	16	22.50	52.6	33.0	13.8
63	400	17	06.34	46.4	24.0	14.5	17	03.13	53.4	38.1	14.8
73	300	17	10.79	43.1	25.4	15.2	17	07.51	46.1	41.4	15.4
74	290	17	11.22	43.1	25.5	15.3	17	07.93	45.7	41.7	15.4

Table 2

Comparative numerical wave predictions, using Formulas II and III, along direction AA (Fig. 11) **from starting point** (X_i = 1140 n.mi, t_j = 1200 Z, December 18)

Numerical Step No.	Distance from Zero	Formulas II					Formulas III				
		Time (GCT)		Wind Velocity	Wave Height	Wave Period	Time (GCT)		Wind Velocity	Wave Height	Wave Period
k	X_k (n. mi)	t_k (day-Hr.)		U_k (knots)	H_k (ft.)	T_k (secs.)	t_k (day-Hr.)		U_k (knots)	H_k (ft.)	T_k (secs.)
1	1140	18	12.00	20.4	0	0	18	12.00	20.4	0	0
8	1100	18	19.69	40.6	5.9	5.9	18	18.57	40.9	7.1	6.4
18	1000	19	04.57	38.0	10.8	8.9	19	02.65	38.0	16.1	9.7
28	900	19	11.48	27.5	12.2	10.1	19	09.16	23.6	19.1	10.4
38	800	19	17.75	20.9	13.2	11.0	19	15.42	24.2	20.9	10.7
48	700	19	23.63	34.9	14.1	11.7	19	21.54	32.6	22.1	10.9
58	600	20	05.11	35.0	15.5	12.5	20	03.46	35.3	24.6	11.5
68	500	20	10.25	34.8	17.1	13.3	20	09.07	38.4	27.5	12.2
78	400	20	15.14	19.0	17.8	13.8	20	14.42	20.7	29.1	12.4

long-dash (Formulas II) track, causing the former to gain some advantage from slightly higher winds. In this case, however, the paths converge again as they approach the ship so that the arrival time difference is quite small (0.72 hours).

Comparative details of the numerical calculations in the two cases are evinced in Tables 1 and 2 below. These tables are typical of the kind of information that the numerical computations can be made to yield for this system of wave prediction; information, moreover, that also makes possible the prediction of the energy spectra of the waves (cf. Wilson [1962 b]).

13. Optimum formulas for future prediction. We return to the point made in the last section that more precise prediction formulas can be suggested, on the basis of the present work, to bring greater coherency to the scattered observational data of Figs. 13, 15 and 16 and to coagulate these data, so to speak, in terms of empirical equations that show promise of approach to the probable natural laws of wind-wave relationships.

The fact that such wide scatter of data has been found in Figs. 13, 15 and 16 is really not surprising when it is considered that the significant wave height H is itself only a statistical dimension of the waves, that its corresponding wave velocity c (or period T) is difficult to measure, that the wind velocity U is rarely uniform for any great length of time, that the fetch F is often ill-defined and, particularly in the case of oceanic data, often a subjective estimate of the observor, and that the duration of the wind may not have been indefinite, the stability of the atmosphere constant nor the depth of water effectively infinite, as required. However, because the conglomerations of scattered data, drawn from so many observors, from so many parts of the world, from wave tank to ocean, show such systematic trends in the relationship of the dimensionless parameters $\frac{c}{U}$ and $\frac{gH}{U^2}$ versus $\frac{gF}{U^2}$, it is possible to conceive of a precise significant wave height, H, a precise velocity c (or period T), that would be generated unfailingly by an ideal wind of invariable velocity U acting over a finite fetch F, devoid of boundary effects from shorelines, etc., under conditions in which neither duration of the wind, stability of the atmosphere nor the depth of water could affect the issue. We shall attempt to define such an idealization.

First we note on the basis of Figs. 15 and 16 that the curve of Eq. (5 i), of Formulas III, pertaining to wave velocity c, though an excellent fit to the left of the diagrams, is still somewhat high in respect to the majority of the data at the right-hand side of the diagrams. Because of this it must tend to overpredict wave periods and velocities.

Secondly, it is clear that the curve of Eq. (5 ii), of Formulas III, pertaining to wave height H, must, on the basis of earlier discussion, depart from any straight line form, as in Fig. 15, by deflecting downwards, intermediate between the short-dash curve of Formulas II and the full line of Formulas III.

The required adjustments to meet the deficiencies are achieved in the following proposed final prediction Formulas IV:

Formulas IV:

$$\begin{aligned} &(i) \quad \frac{c}{U} = 1.37\left[1 - \left\{1 + 0.008\left(\frac{gF}{U^2}\right)^{\frac{1}{3}}\right\}^{-5}\right] \\ &(ii) \quad \frac{gH}{U^2} = 0.30\left[1 - \left\{1 + 0.004\left(\frac{gF}{U^2}\right)^{\frac{1}{2}}\right\}^{-2}\right] \end{aligned} \qquad (12)$$

Equations (12) have the properties that when $\frac{gF}{U^2}$ is small the results become asymptotic to

$$\begin{aligned} &(i) \quad \frac{c}{U} = 0.0548\left(\frac{gF}{U^2}\right)^{\frac{1}{3}} \\ &(ii) \quad \frac{gH}{U^2} = 0.0024\left(\frac{gF}{U^2}\right)^{\frac{1}{2}} \end{aligned} \qquad (13)$$

while when $\frac{gF}{U^2}$ is large the asymptotic values are constants:

$$\begin{aligned} &(i) \quad \frac{c}{U} = 1.37 \\ &(ii) \quad \frac{gH}{U^2} = 0.30 \end{aligned} \tag{14}$$

It is of interest to note here that Eq. (13 ii) has theoretical substantiation from O. M. Phillips [1957, 1958] and H. Charnock [1958] and that it is actually in precise agreement with an empirical formula proposed by Bretschneider [1957]. It is possible to show from Charnock's version of Phillip's [1957] result that the theoretical constant in their equation, of the form of Eq. (13 ii), is of a corresponding order of magnitude, though low by a factor of about 0.6. Its estimation was dependent on many uncertainties and thus cannot disparage the observational result of Eq. (13 ii).

The asymptotic value of $\frac{gH}{U^2}$ at large fetch, Eq. (14 ii), is found to be in exact agreement with the result of C. G. Rossby and R. B. Montgomery [1935]. It is also not far removed from the values of 0.33, 0.26 and 0.24 obtained respectively by Suthons [1945], Sverdrup and Munk [1947] and Neumann [1952b]. It may be remarked that it is not difficult to arrive at a pseudo-constant less than 0.30 from Eq. (12 ii) when $\frac{gF}{U^2}$ is still large, (say, in the range 10^5 to 10^6), but not infinite. Thus for the range of $\frac{gF}{U^2}$ from 10^5 to 10^6, beyond which little, if any, ocean wave data exist, Eq. (12 ii) gives values of $\frac{gH}{U^2}$ from 0.242 to 0.288.

The value of the constant 1.37 in Eq. (14 i) is the same as that arrived at by Sverdrup and Munk [1947] and by Neumann [1952b]. The larger value of 1.95 derived more recently by Bretschneider [1959] cannot be accommodated by an equation of the type of Eq. (12i) which now provides a very satisfactory backbone to the uppermost sets of data in Figs. 15 and 16. To avoid confusion Formulas IV have not been plotted in Figs 13, 15 and 16.

When the specific value $U = 40$ knots is fed to Eq. (12 ii) the resulting relationship between wave height, H, and fetch F, assumes the form of the curve IV in Fig. 14. This is considered to occupy a most satisfactory position among all the possibilities that have been proposed, even though at large fetch its indications of wave height are still high in relation to the estimates of the European school. It is possible, of course, that a further test application of Formulas IV to the weather situation of this study could suggest still greater refinement of Eq. (12 ii), though this now seems doubtful in view of the asymptotic agreements already noted.

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List of Symbols

- c velocity in deep-water of significant waves
- F fetch or distance over which wind blows
- f_1, f_2 functions of variables
- g acceleration due to gravity
- H height of the significant wave
- i integer subscript of distance (= 1, 2, 3...)
- j integer subscript of time (= 1, 2, 3...)
- k integer subscript (Fig. 17) (= 1, 2, 3...), defining numerical step

m	integer subscript (Fig. 17), (= 1, 2, 3...)
$\mathrm{MOD}_1(t_k)$	defines the fraction by which the number t_k exceeds its nearest integer
$\mathrm{MOD}_{10}(X_k)$	defines the fraction of 10 by which the number X_k exceeds $10\,p$ in which p is the largest integer for which $10\,p < X_k$
n	integer subscript (Fig. 17), (= 1, 2, 3...)
T	period of significant waves
t	variable time
t_j	particular value of t [$= j\,\tau$]
t_k	value of t elapsed to the kth step from the start of the digital computation (Fig. 17)
t_{k+1}	value of t elapsed to the $(k+1)$th step from the start of the digital computation (Fig. 17)
t_n	particular value of t [$= n\,\tau$]; either $t_n = t_k$ or $t_n = t_k - \mathrm{MOD}_1(t_k)$
t_{n+1}	particular value of t [$= (n+1)\tau$]
Δt	incremental length of time
$(\Delta t)_k$	particular value of Δt [$= t_k + 1 - t_k$]
U	component of surface wind velocity uniform along a line of fetch
U_1	initial value of U at space-time lattice point (X_i, t_j)
U_k	value of U at space-time lattice point (x_k, t_k) or $(X_k, t_j + t_k)$
U_m	value of U at space-time lattice point (X_m, t_n)
U_{m+1}	value of U at space-time lattice point (X_{m+1}, t_n)
U_n	value of U at space-time lattice point (X_m, t_n)
U_{n+1}	value of U at space-time lattice point (X_m, t_{n+1})
$U(x)$	component surface wind velocity along a line of fetch, variable over the fetch (continuous function of x)
V	group-velocity of significant waves in deep water
X	distance from coastal station along given fetch line
X_k	value of X defining the position x_k [$= X_i - x_k$]
X_m	particular value of X [$= m\lambda$]; either $X_m = X_k$ or $X_m = X_k - \mathrm{MOD}_{10}(X_k)$
X_{m+1}	particular value of X [$= (m+1)\lambda$]
x	variable distance or fetch over which the wind blows
x_k	value of x from the start to the k-th step of the digital computation (Fig. 17)
x_{k+1}	value of x from the start to the $(k+1)$-th step of the digital computation (Fig. 17)
Y	dimensionless parameter [$= gH/U^2$]
Y'	differential coefficient of Y with respect to gx/U^2
Z	dimensionless parameter [$= c/U$]
Z'	differential coefficient of Z with respect to gx/U^2
Δx	incremental length of fetch
$(\Delta x)_k$	particular value of Δx [$= x_{k+1} - x_k$]
λ	interval of distance in space-time lattice
π	universal constant (3. 14159...)
τ	interval of time in space-time lattice

References

Baer, L., 1962a: An experiment in numerical forecasting of deep water ocean waves. Lockheed Missile & Space Co., Sunnyvale, Calif. Tech. Rep. LMSC-801296, June.

Baer, L., 1962b: Numerical wind-wave forecasting. Proc. 2nd Interindustrial Oceanogr. Sympos. (Santa Barbara, Dec., 1962), Lockheed Aircraft Corp., May, 1963. 11.

Barber, N. F. and F. Ursell, 1948: The generation and propagation of ocean waves and swell. Philos. Trans. Roy. Soc. London. **240** (A), 527.

Bretschneider, C. L., 1952a: Revised wave forecasting relationships. Proc. 2nd Conf. Coastal Eng., Council Wave Res., Berkeley, Calif. 1.

Bretschneider, C. L., 1952b: The generation and decay of wind waves in deep water. Trans. Am. Geophys. Union. **33**, 381.

Bretschneider, C. L., 1957: Hurricane design wave practices. J. Waterways & Harbors Div., ASCE. Trans. ASCE. **124**, 39.

Bretschneider, C. L., 1959: Wave variability and wave spectra for wind-generated gravity waves. Beach Erosion Board, Corps of Engrs., U.S. Army. Tech. Mem. No. 118. 122pp.

Bretschneider, C. L., W. J. Pierson Jr., H. Walden, and R. Gelci, 1962: (Discussion on) Deep water wave generation by moving wind systems. Proc. ASCE. **88** (WW 1), 153.

Bretschneider, C. L., H. L. Crutcher, J. Darbyshire, G. Neumann, W. J. Pierson, H. Walden, and B. W. Wilson, 1963: Data for high wave conditions observed by the OWS "Weather Reporter" in December 1959. Dt. Hydrogr. Z. **15**, 243.

Burling, R. W., 1954: Surface waves on enclosed bodies of water. Proc. 5th Coastal Eng. Conf. (Grenoble, France, 1954); Council Wave Res. Berkeley, Calif. 1.

Burling, R. W., 1955: Wind generation of waves on water. Ph. D. Thesis, Imperial Coll. Sci. & Tech., Univ. London.

Charnock, H., 1958: A note on empirical wind-wave formulae. Quart. J. Roy. Meteorol. Soc. London. **84**, 443.

Chase, J., et al. 1957: The directional spectrum of a wind generated sea as determined from data obtained by the Stereo Wave Observation Project. Coll. of Eng., New York Univ., Tech. Rep.

Cornish, V., 1934: Ocean waves and kindred geophysical phenomena. Cambridge, London.

Czepa, O. and G. Schellenberger, 1959: Zur Charakteristik winderzeugter Oberflächenwellen von Binnenseen. Gerlands Beitr. Geophys. **68**, 171.

Darbyshire, J., 1952: The generation of waves by wind. Proc. Roy. Soc. London. **215** (A), 299.

Darbyshire, J., 1956: An investigation into the generation of waves when the fetch of the wind is less than 100 miles. Quart. J. Roy. Meteorol. Soc. **82**, 461.

Darbyshire, J., 1959: A further investigation of wind generated waves. Dt. Hydrogr. Z. **12**, 1.

Darbyshire, J., 1961: Prediction of wave characteristics over the North Atlantic. J. Inst. Navig. **14**, 339.

Defant, A., 1961: Physical Oceanography. Oxford [usw.]. **2**. 598pp.

Djounkovski, N. N. and P. K. Bojitch, 1949: La Houle, et son action sur les côtes et les ouvrages côtiers [transl. from the Russ.]. Paris 1959. 95.

Gelci, R., H. Gazale, and J. Vassal, 1957: Prévision de la houle, la méthode de densités spectro-angulaire. Bull. d'Inform., Com. Centr. d'Océanogr. **9**, 416.

Gilman, C. S. and V. A. Myers, 1961: Hurricane winds for design along the New England Coast. Proc. ASCE. 87 (WW 2), 45.

Hamada, T., H. Mitsuyasu, and N. Hose, 1953: An experimental study of wind effect upon water surface. Transportation Tech. Res. Inst., Japan. Rep. No. 8. 22pp.

Johnson, J. W. and E. K. Rice, 1952: A laboratory investigation of wind generated waves. Trans. Am. Geophys. Union. **33**, 845.

Krylov, J. M., 1958: Statistical theory and the computation of ocean wind waves [in Russian]. GOIN, P. 2, No. 42.

Molitor, D. A., 1935: Wave pressures on sea walls and breakwaters. Trans. ASCE. **100**, 984.

Moskowitz, L., 1963: Estimates of the power spectra for fully developed seas for wind speeds of 20 to 40 knots. Dep. of Meteorol. and Oceanogr., New York Univ. Geophys. Sci. Rep. No. 63–11. 41pp.

Neumann, G., 1952a: Über die komplexe Natur des Seeganges. P. 1, 2. Dt. Hydrogr. Z. **5**, 95, 252.

Neumann, G., 1952b: On the complex nature of ocean waves and the growth of the sea under the action of wind. Circular 521: "Gravity Waves", Nat. Bureau Standards (Washington). 61.

Phillips, O. M., 1957: On the generation of waves by turbulent wind. J. Fluid Mech. **2**, 417.

Phillips, O. M., 1958a: Wave generation by turbulent wind over a finite fetch. Proc. 3rd U.S. Nat. Congr. Appl. Mech. ASME. 785.

Phillips, O. M., 1958b: The equilibrium range in the spectrum of wind-generated waves. J. Fluid Mech. **4**, 426.

Pierson, W. J., Jr., G. Neumann, and R. W. James, 1955: Practical methods for observing and forecasting ocean waves by means of wave spectra and statistics. Hydrogr. Off. U.S. Navy Dept., H. O. Publ. No. 603.

Pierson, W. J., Jr., 1962: (Discussion on) Deep water wave generation by moving wind systems. Proc. ASCE. 88 (WW 1), 155.

Pierson, W. J., Jr., 1963: The interpretation of wave spectra in terms of the wind profile instead of the wind measured at a constant height. Dept. of Meteorol. and Oceanogr., New York Univ., Geophys. Sci. Rep. 63–15. 32pp.

Pierson, W. J., Jr. and L. Moskowitz, 1963: A proposed spectral form for fully developed wind seas based on the similarity theory of S. A. Kitaigorodskii. Dep. of Meteorol. and Oceanogr., New York Univ., Geophys. Sci. Rep. 63–12.

Roest, P. W., 1960: Wave recording on the Ijsselmeer. Proc. 7th Conf. Coastal Eng. (Scheveningen, Netherlands, Aug., 1960); Council Wave Res., Berkeley 1961. 53.

Roll, H. U., 1949: Über die Ausbreitung der Meereswellen unter der Wirkung des Windes (auf Grund von Messungen im Wattenmeer). Dt. Hydrogr. Z. **2**, 268.

Roll, H. U., 1951: Neue Messungen zur Entstehung von Wasserwellen durch Wind. Ann. Meteorol. **4**, 269.

Rossby, C. G. and R. B. Montgomery, 1935: The layer of frictional influence in wind and ocean currents. Pap. Phys. Oceanogr. & Meteorol. **3**, 101pp.

Rundgren, L., 1958: Method for calculation of maximum wave dimensions applied to the conditions of Lushington Shoal. Hydraul. Div., Roy. Inst. Tech., Stockholm. Bull. No. 55. 36pp.

Saville, T., Jr., 1954: Wave forecasting. Proc. 1st Conf. Ships & Waves (Hoboken, N.J., Oct., 1954); Council Wave Res. Berkeley, Calif. 1955. 78.

Schellenberger, G., 1962: Untersuchungen über Windwellen auf einem Binnensee. Acta Hydrophysica. **7**, 67.

Schott, G., 1893: Über die Dimensionen der Meereswellen. Petermanns Geogr. Mitt. Erg.H. 109, 82.

Stanton, T., D. Marshall, and R. Houghton, 1932: The growth of waves on water due to the action of wind. Proc. Roy. Soc. London. **137** (A), 283.

Suthons, C. T., 1945: The forecasting of sea and swell waves. British Admir. Naval Weather Service. Mem. No. 135/45. (Rev. April, 1950.) 84pp.

Sverdrup, H. V. and W. H. Munk, 1947: Wind, sea and swell; theory of relations for forecasting. U.S. Navy Dep., Washington. H.O. Publ. No. 601. 44pp.

Titov, L. F., 1955: Wind waves on the oceans and seas [in Russ.]. Leningrad.

Tucker, M. J., 1956: A ship-borne wave recorder. Trans. Inst. Nav. Arch., London. **98**, 236.

Walden, H., 1953/54: Die Wellenhöhe neu angefachter Windsee nach Beobachtungen atlantischer Wetterschiffe und des Fischereischutzbootes „Meerkatze". Ann. Meteorol. **6**, 296.

Walden, H., 1956: Stau der Wellenenergie im wandernden Windfeld. Dt. Hydrogr. Z. **9**, 225, 280.

Walden, H., 1957: Methods of swell forecasting demonstrated with an extraordinarily high swell off the coast of Angola. Proc. Sympos. Behavior of Ships in a Seaway. (Netherlands Ship Model Basin, Wageningen.) **1**, 427.

Walden, H., 1958: Die winderzeugten Meereswellen. T. 1. Dt. Wetterdienst, Seewetteramt. Einzelveröff. No. 18, H. 1 u. 2.

Walden, H., 1961a: Der hohe komplexe Seegang am nordatlantischen Wetterschiff „K" am 6. Dezember 1959. Dt. Hydrogr. Z. **14**, 239.

Walden, H., 1961b: Comparison of one-dimensional wave spectra recorded in the German Bight with various "theoretical" spectra. Proc. Conf. Ocean Wave Spectra, Nat. Acad. Sci. (Prentice-Hall, Inc., N. J.) 67.

Walden, H., 1963: An attempt of hindcasting the high waves observed by the OWS "Weather Reporter" at position "J" on 17 December 1959. Dt. Hydrogr. Z. **16**, 1.

Wiegel, R. L., 1960: Wind wave and swell. Proc. 7th Conf. Coastal Eng., (Scheveningen, Netherlands, Aug., 1960), Council Wave Res., Berkeley 1961. 1.

Wiegel, R. L., 1961: Some engineering aspects of wave spectra. Proc. Conf. Ocean Wave Spectra, Nat. Acad. Sci. (Prentice-Hall, Inc., N. J.) 309.

Wilson, B. W., 1955: Graphical approach to the forecasting of waves in moving fetches. Beach Erosion Board, Corps of Engrs., U.S. Army. Tech. Mem. No. 73. 31pp.

Wilson, B. W., 1957: Hurricane wave statistics for the Gulf of Mexico. Beach Erosion Board, Corps of Engrs., U.S. Army. Tech. Mem. No. 98; also Proc. 6th Coastal Eng. Conf., (Gainsville, Fla., Dec., 1957) Council Wave Res., Berkeley, Calif. 1958. 68.

Wilson, B. W., 1960: Note on surface wind stress over water at low and high wind speeds. J. Geophys. Res. **65**, 3377.

Wilson, B. W., 1961: Deep water wave generation by moving wind systems. Proc. ASCE. 87 (WW 2), 113; also Trans. ASCE **128** (1963), 104.

Wilson, B. W., 1962a: Deep water waves generated by hurricane "Audrey" of 1957. Proc. 8th Coastal Eng. Conf. (Mexico City, Mex., Nov., 1962), Council Wave Res., Berkeley, Calif. April, 1963.

Wilson, B. W., 1962b: Deep water wave generation by moving wind systems (Discussion Closure). Proc. ASCE. 88 (WW 3), 175; also Trans. ASCE **128** (1963), 138.

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GEDENKTAGE

Am 15. Juni 1965 vollendete Professor em. Dr. **Georg Wüst** sein 75. Lebensjahr. Anläßlich seines 70. Geburtstages hat sein Wirken für die Meeresforschung an anderer Stelle von dem inzwischen verstorbenen Mitherausgeber dieser Zeitschrift, Theodor Stocks, eine ausführliche Würdigung erfahren (Th. Stocks: Georg Wüst und seine Stellung in der neueren Ozeanographie. Petermanns Geographische Mitteilungen, 1960, S. 292–295). Dort ist auch die vollständige Liste der wissenschaftlichen Veröffentlichungen von Georg Wüst bis zum Jahre 1960 abgedruckt, die 95 Arbeiten umfaßt. Wenn ihm Schüler, Freunde und Kollegen ihre Glückwünsche zum 75. Geburtstag entgegenbringen, so kann der Jubilar seit der letzten Ehrung befriedigt auf fünf weitere arbeitsreiche Jahre verweisen.

Er nahm 1960 den ehrenvollen Ruf als „Visiting Professor of Oceanography" der Columbia Universität in New York an, wo er vier Jahre bis 1964 gelehrt hat. Er war dort ein geehrtes Fakultätsmitglied, nicht allein deshalb, weil er seit seiner Promotion 1914 als Schüler von Alfred Merz für die Ozeanographie gewirkt hatte, sondern weil er in New York eine Aktivität in der Lehre und bei dem Ausbau des Lamont Geological Observatory entfaltete, die jedem Jüngeren Hochachtung abverlangte. Hinzu kam, daß seine Aktivität sich nicht in Lehre und Organisation erschöpfte. In den letzten fünf Jahren hat Wüst weitere elf wissenschaftliche Veröffentlichungen herausgebracht. Sie galten drei Hauptthemen. Das eine ist die Tiefen-