

Load Actions on the Human Femur in Walking and Some Resultant Stresses

Analysis of ground-to-foot force and leg-segment displacement measurements for the walking human allows calculation of forces in muscles and ligaments at leg joints. Simplified calculations can then be made for the stresses on the femur

by John P. Paul

ABSTRACT—From experimental measurements of ground-to-foot force actions and limb configurations, resultant load actions at junctions of leg segments can be calculated. From a knowledge of the phasic activity of muscles and their anatomical location, the tension in relevant muscles and ligaments may be inferred, and the joint forces obtained. From the measured geometry of the femur, calculations are made of the stresses on the basis of simplifying assumptions of material disposition and behavior.

Introduction

The femur is the longest and strongest bone in the human body, yet the incidence of fractures of the femur in the aged female is such as to constitute a major clinical problem. An analysis of the mechanical factors pertinent to this is, however, particularly complicated, in view of the complex dynamic-force system to which it is subject, the complex anatomical configuration of the member and the variable, anisotropic, rate-dependent mechanical properties of the bone material.

Analysis of the force actions transmitted at the hip joint during walking has been performed by Pauwels (1935)¹⁵ based on the experimental work of Braune and Fischer (1898-1904). This work does not, however, allow the analysis of forces in the "double support" phase of motion. Analysis of the forces in one-legged standing has been performed by many authors, Inman (1947),⁸ Blount (1956),² Strange (1963),¹⁸ etc., assuming a planar-force system. Williams (1968),²⁰ has performed a three-dimensional analysis for this configuration. The only direct determination of hip-joint force is that of Rydell (1966),¹⁶ in which strain gages were fitted to femoral head prostheses implanted in two patients who had suffered fractures of the neck of femur. The results used in this paper correspond to the analysis of normal subjects walking at faster speeds than Rydell's patients. The methods of obtaining these values for hip- and knee-joint forces are described by Paul (1967 a,b)^{13,14} and Morrison (1967).¹²

In brief, Paul and Morrison measured photographically the three-dimensional configuration of

the leg segments during a walking cycle in which the ground-to-foot force actions were measured by a force-plate dynamometer. From these, the resultant forces and moments transmitted between segments were calculated in the same way as by Bresler and Frankel (1950).⁴ The moments are transmitted by tension in muscle or ligament. The complex anatomical system of 22 muscles acting at the hip and 14 muscles and 6 ligaments at the knee are simplified to groups on the basis of their anatomical disposition and, for the muscles, on the basis of their phasic activity as demonstrated by myoelectric signals. The forces in the groups are then calculated from equilibrium equations and, hence, the joint forces are obtained. At the hip, explicit solutions could not be obtained, but limit curves were produced between which the actual value of joint force would lie at any instant. At the knee, explicit solutions were possible for the simplified system. In both cases, the equilibrium equations corresponding to moments about the long axis of the segment were not used. The reason is that these moments correspond to the lateral components of the forces in the longitudinal muscles and could, therefore, be greatly in error due to small displacements of the assumed positions of their lines of action. Williams (1968)²⁰ uses this equilibrium equation and this may be one reason why his value of hip-joint force in standing greatly exceeds other workers' values and, indeed, is in excess of many of the values obtained for walking by Paul and Rydell.

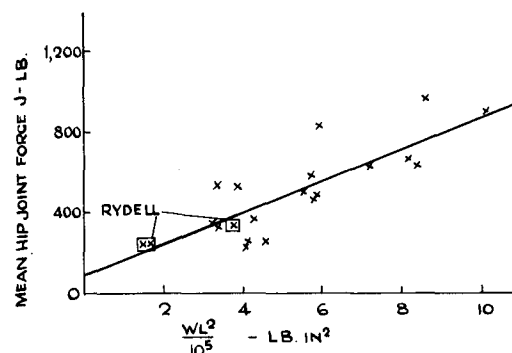


Fig. 1—Relationship between mean joint force J, subject weight W and stride length L

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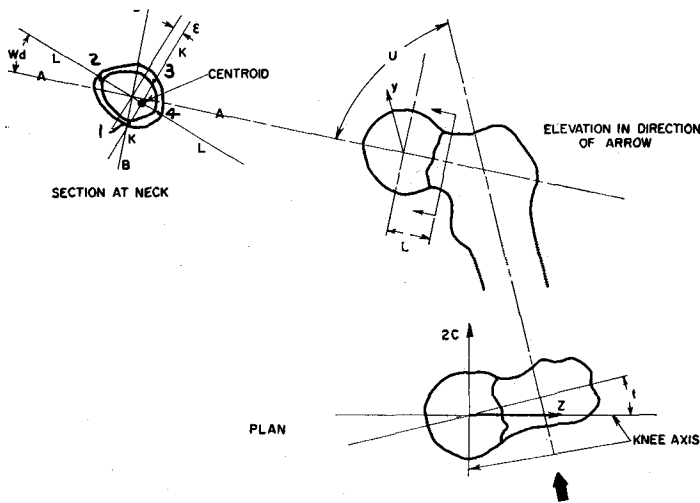


Fig. 2—Reference angles of femoral head (Backman 1957)

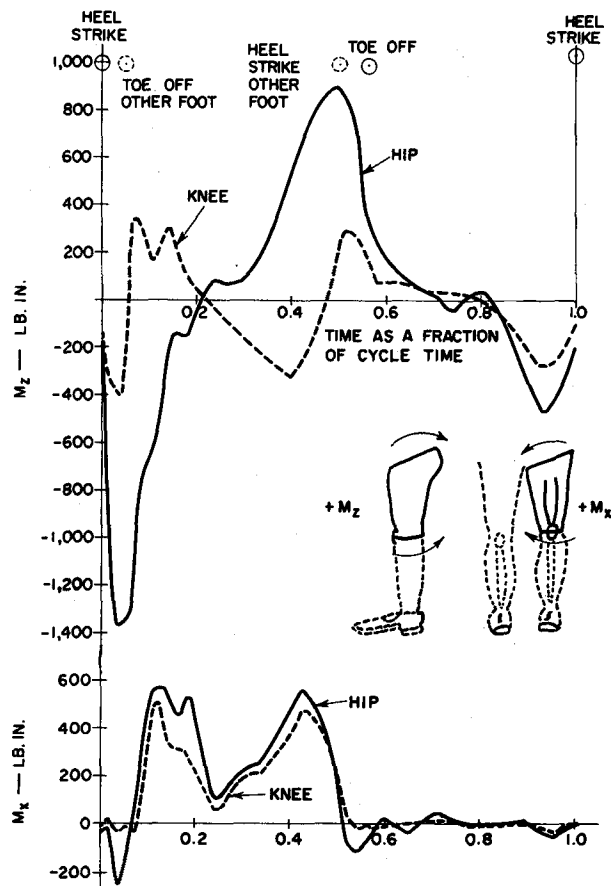


Fig. 3—Moments acting on the thigh

A comparison of Rydell's and Paul's results plotted to a base of body weight, W , x (stride length, L)² is shown in Fig. 1 and correspondence is apparent in the general trend of values.

Calculations of stresses in the femur have been few. Backman (1957)¹ states that Koch (1917)⁹ and Grunewald (1920)⁷ performed calculations of the stresses in the neck of the femur under vertical loading only. Backman presents calculations assuming an idealized eccentric hollow section at the neck and uniform linear elastic behavior of the material. In this analysis, Pauwels' values for the three com-

ponents of hip-joint force are used.

Evans (1957)⁵ summarizes the results of determinations of the strain in femora under vertical loading of the head performed by several experimenters. More recently, Frankel (1960)⁶ reports the results of tests on femora using Stresscoat and resistance strain gages during compressive-loading tests to simulate standing on one leg. Due to uncertainty about the stress-strain relationships, only strain values are reported. Williams (1964)¹⁹ suggests a frozen-stress photoelastic method to analyze the conditions at the neck of the femur under the loading described in Williams (1968).²⁰ An attempt is made to take account of the variation in the elastic properties of the bone across the section.

Determination and comparison of the results of mechanical tests on bone specimens is complicated by the variability of properties between different sites on the same bone, by differences between samples from right and left bone, and by the effect of method of storage and rate of testing. [Evans (1957),⁵ McElhaney et al (1964),¹⁰ McElhaney and Byars (1965)¹¹ Sedlin (1965)].¹⁷ The ultimate strength in compression is found to vary by 24 percent and Young's modulus by 15 percent with strain rates from zero to 1.0/sec. The values for ultimate compressive stress of compact bone in static tests range from 15,000 lb/in.² to 33,000 lb/in.² while those of cancellous bone are in the range of 2100-7200 lb/in.². There is no information on the corresponding values of Young's modulus E , but some studies suggest linear correspondence between E and the ultimate strength.

Analysis

The results presented here are derived from the work of Paul (1967 b)¹⁴ and Morrison (1967).¹² For computational reasons, Paul presented his results for the hip-joint force as orthogonal components relative to the vertical and two horizontal axes through the hip joint. Morrison presented his knee-joint forces as orthogonal components referred to his axes for the tibia. In both cases, the rotation of the segments about their long axes was not measured. Ryker (1952) shows that these angles are small in the stance phase. If AZ and AX are the reference angles measured about the horizontal Z and X axes describing the inclination of the femur, it is shown by Paul that the direction cosines $t_{11} \dots t_{33}$ for the resolution of force components along the femoral axes can be expressed by terms such as:

$$\begin{aligned}
 t_{11} &= 1 - (\tan^2 AZ)(1 - 1/R)/S \\
 t_{12} &= -t_{21} = (\tan AZ)/R \\
 t_{13} &= t_{31} = (\tan AX)(\tan AZ)(1 - 1/R)/S \\
 \text{etc. where } S &= \tan^2 AX + \tan^2 AZ \\
 R &= \sqrt{(1 + S)}
 \end{aligned}$$

The neck of the femur is, however, oblique to the ideal axes as described by Backman (1957)¹ and shown in Fig. 2. The cross section at the neck is unsymmetrical and Backman approximates to it by a hollow eccentric ellipse. For force components X , Y , and Z acting on the head of the femur in the x , y , z directions, it is possible to obtain the resultant force P along the centroidal axis and the moments about the centroidal axes KK , LL as:

$$P = X \cos u \sin t - Y \sin u + Z \cos u \cos t.$$

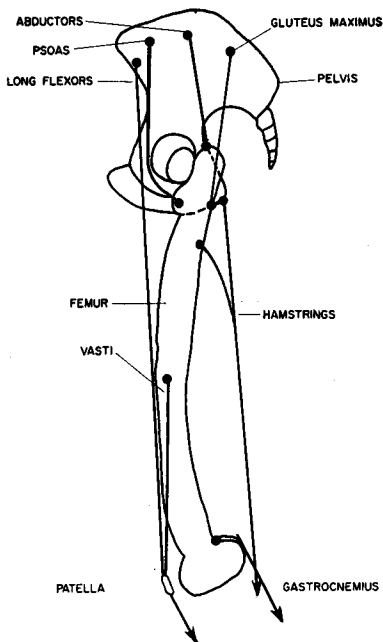


Fig. 4—Lateral view of thigh showing simplified muscle system

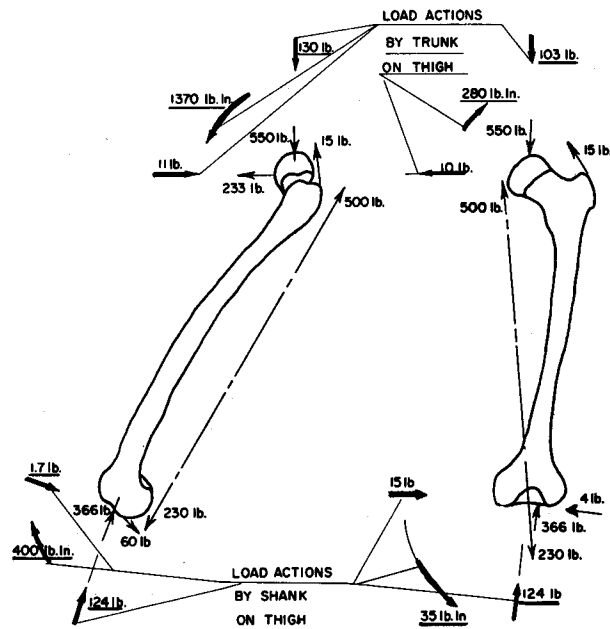


Fig. 5—Force actions on the left thigh, 4 percent of cycle after heel strike

$$M_K = X[L(\cos t \cos w_d - \sin u \sin t \sin w_d) - \epsilon \cos u \sin t] \\ + Y[-L \cos u \sin w_d + \epsilon \sin u] \\ + Z[L(-\sin u \cos t \sin w_d - \sin t \cos w_d) - \epsilon \cos u \cos t] \\ M_L = XL[\cos t \sin w_d + \sin u \sin t \cos w_d] \\ + YL \cos u \cos w_d \\ + ZL[\sin u \cos t \cos w_d - \sin t \sin w_d]$$

Backman quotes the idealized section properties of the neck based on measurements of 29 specimens as:—

$$A = 0.236 \text{ in.}^2 \\ \epsilon = 0.275 \text{ in.} \\ I_K = 0.0452 \text{ in.}^4 \\ I_L = 0.0293 \text{ in.}^4$$

Using Backman's mean values of 13.5 deg for t , 53.7 deg for u , 24.6 deg for w_d and 0.788 in. for L , the longitudinal stresses at points 1-4 in the neck of the femur can be obtained as:—

$$\sigma_1 = 8.1X + 11.5Y + 6.7Z \\ \sigma_2 = 12.8X + 4.1Y - 9.8Z \\ \sigma_3 = -9.3X - 4.7Y - 11.5Z \\ \sigma_4 = -6.8X + 3.1Y + 8.1Z$$

In fact, Backman's neglect of the bending stiffness of the cancellous bone is a pessimistic view. If the section is treated as composite of two materials having elastic moduli in the ratio 5:1, which is approximately the ratio of the ultimate strengths of the materials, the section properties can be obtained as:—

$$A = 0.456 \text{ in.}^2 \quad I_K = 0.089 \text{ in.}^4 \quad I_L = 0.0424 \text{ in.}^4$$

and revised expressions for the stresses can be obtained. Similar expressions can be obtained for corresponding sections in the shaft of the femur.

Experimental Results

Experimental curves of hip and knee moment variation with time are shown in Fig. 3 for a 130-lb male subject, height 6 ft, walking with a stride length of 88.3 in. and a cycle time of 1.08 sec. This subject

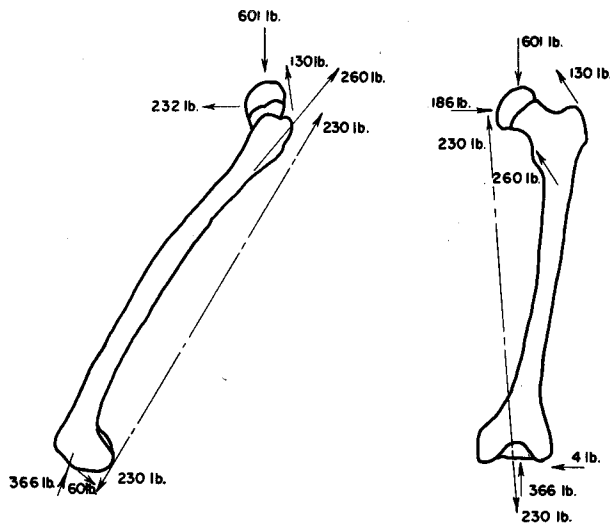


Fig. 6—Force actions on the femur, 4 percent of cycle after heel strike

was chosen as developing an extreme of walking speed in the test range. The moments for hip and knee about the Z axis have no direct relationship and the possible combinations of positive and negative values are analyzed at 4 percent, 9 percent, 41 percent and 52 percent of the cycle time. The major muscular system transmitting these Z-axes moments is shown in Fig. 4 where it can be seen that the muscle groups named "long flexors" and "hamstrings" contribute to the transmission of moments both at the hip and knee joints. They can, therefore, only be economically used if the moments at both axes are of the same sign.

In Fig. 5, the force actions on the complete thigh, at 4-percent cycle time, are shown by heavy lines and the first calculated values for hip and knee muscle and joint forces by lighter lines. It is apparent that the biceps muscle group is unbalanced since it would require to transmit 230 lb at the knee and 500 lb at

the hip. The alternatives are to keep the 500-lb value and maintain knee equilibrium by tensing the vasti, thus further loading the knee joint or to take part of the hip moment on the gluteus maximus muscle and this latter choice is shown in Fig. 6. The hip-joint force is, thereby, increased since the lever arm of the gluteus maximus about the Z axis is less than that of the biceps.

In the situation at 9-percent cycle time illustrated in Fig. 7, the Z-axis moments are opposite in sign at the knee and hip and there is, therefore, no efficient way of using the available two-joint muscles. The knee moment is transmitted by tension in the patellar ligament developed by the vasti muscles. The X-axis moment at the knee is taken to be transmitted by an offset of the resultant force and by tension in the lateral collateral ligament.

At 41 percent of the cycle, as shown in Fig. 8, the Z-axis moments are again opposite in sign but in directions contrary to the previous situation. The sartorius muscle of the thigh is suitably situated to transmit this action but myoelectric records show that it is not then active. The two-joint muscle of the shank, gastrocnemius, is active at this phase and can transmit the necessary moments at the knee and ankle. The hip moment is taken to be transmitted by ilio-psoas which winds round the front of the capsule of the hip joint so that although its resultant effect is a vertically upward pull on the femur, this is partly transmitted by radial pressure on the head and the resultant force causing stress in the neck is correspondingly lower than it would otherwise be.

In Fig. 9, the Z-axis moments at the knee and hip are shown transmitted by the one-joint muscles only. The long flexors of the hip can transmit this load action, but electromyographic evidence shows only one of the muscles in this group to be active, and the solution is, therefore, based on the action of the one-joint muscles.

It will be noticed that, in the four cases, the vertical

component of hip-joint force lies in the range 550 to 850 lb, i.e., 4.2 to 6.5 times body weight. The corresponding knee-joint forces are 1.5 to 3.7 times body weight.

On attempting a stress analysis, it is apparent that the critical sections are at the neck, and on the shaft at a section in the plane which divides the hip to knee axis in the ratio one to four. Using Backman's average measurements and his derived section properties, the longitudinal stresses were calculated ac-

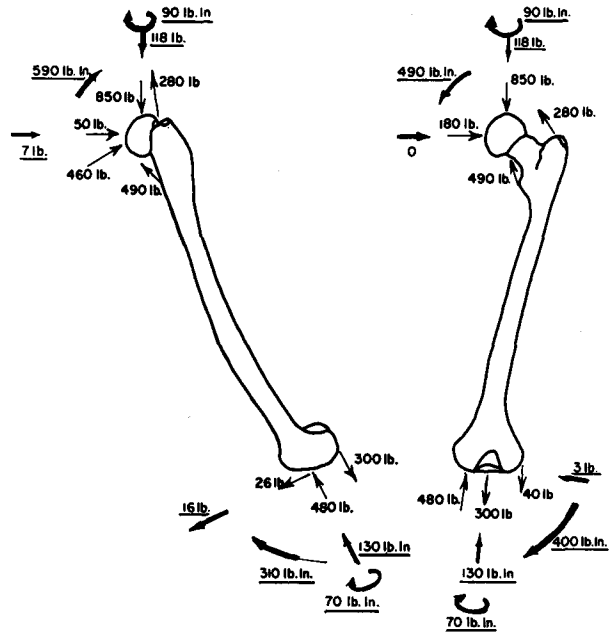


Fig. 8—Force actions on the left thigh, 41 percent of cycle after heel strike

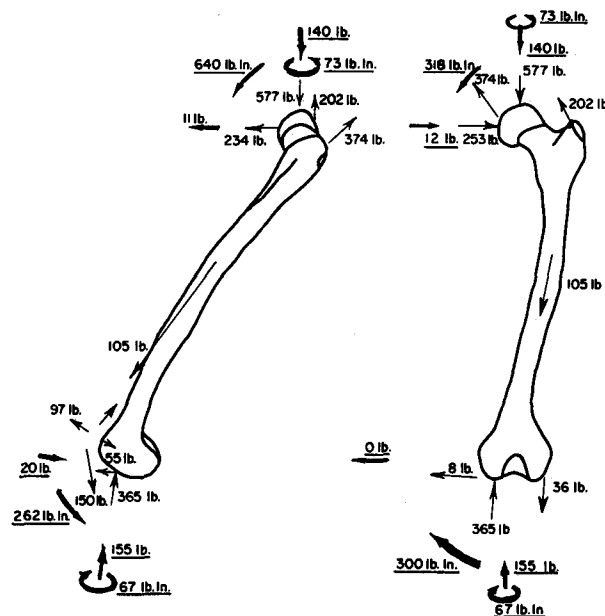


Fig. 7—Force actions on the left thigh, 9 percent of cycle after heel strike

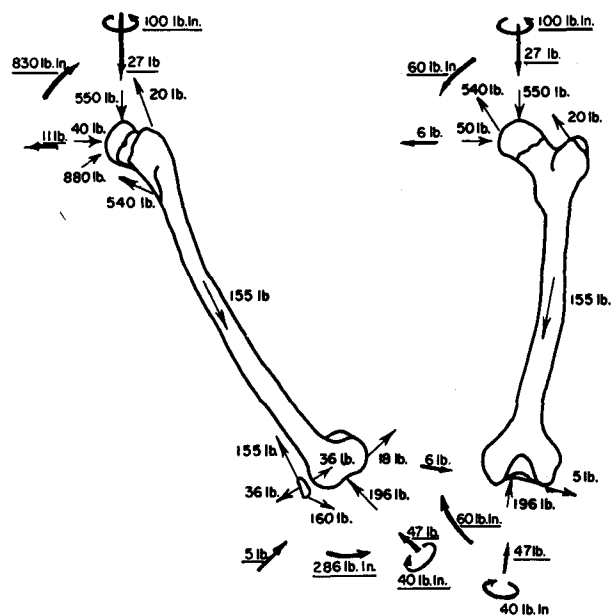


Fig. 9—Force actions on the left thigh, 52 percent of cycle after heel strike

TABLE 1—LONGITUDINAL STRESS σ AND SHEAR STRESS τ DUE TO SHEAR FORCE (LB/IN.² \times 1000) IN THE FEMORAL NECK AT LOCATIONS NUMBERED AS SHOWN IN FIG. 2

Stress	Time from Heel Strike, % of Cycle			
	4	9	41	52
σ_1	-5.6	-5.6	-7.5	-0.7
τ_1	2.2	1.7	3.0	5.1
σ_2	-3.5	-4.4	-3.3	5.2
τ_2	2.0	2.4	2.6	0.9
σ_3	0.68	-0.1	0.9	-3.9
τ_3	2.2	1.7	3.0	5.1
σ_4	-1.0	0.2	-1.7	-5.4
τ_4	0.7	0.7	1.0	0.4

ording to the equations given and the values at the ends of the principal axes are shown in Table 1. This table shows also the shear stresses due to shear force at these points.

Discussion

The values of the hip-joint forces quoted here are greater than those of Backman and Rydell, principally since this test subject is walking at a greater speed.

The stress values calculated using Backman's section properties are, therefore, greater than his values of $\sigma_1 = 2000$, $\sigma_2 = 0$, $\sigma_3 = -2000$, $\sigma_4 = 3400$ lb/in.². $\tau_1 = \tau_3 = 3400$ lb/in.². Backman performs no calculations for τ_2 and τ_4 due to shear force, nor for any shear stresses due to torsion. The table shows values for τ_2 and τ_4 . The shear stress due to torsion was not determined exactly, but its value, calculated ignoring the offset of the shear center from the centroid, was of the order of 700 lb/in.² maximum at position 2 and additive to the tabulated values. It is apparent that the material of the femoral neck is subjected to a complex stress system which may give principal stresses of magnitude 8500 lb/in.² compressive, or 5600 lb/in.² tensile with maximum shear stresses of 4800 lb/in.². The literature reviewed gave no indication of possible failure criteria for bone under two-dimensional stressing. Evans quotes ultimate stresses in pure shear of 11,000 lb/in.², in tension of 16,000 lb/in.². This would indicate a safety factor of only some 2.5 to 3 times for this subject when walking, which seems surprisingly low.

Backman obtained his idealized cross section by replacing the central cancellous bone by a 20-percent increase in cortical thickness. If the elastic modulus of the cancellous bone is 1/5 that of the cortical, calculations of the section properties show Backman's view to be pessimistic. For the test results under discussion, this reduces the maximum compressive stress to 4200 lb/in.² and the tensile stresses to 2700 lb/in.² which seem more reasonable in relation to the ultimate values.

Corresponding values for the maximum tensile and compressive stresses in the shaft were based on an idealized hollow elliptical section of dimensions based on experimental measurements and gave maximum tensile and compressive stresses of 5100 and 5300 lb/in.² respectively. In view of these significantly higher stresses in the shaft, it would appear surprising that the clinical problem is fracture of the

neck. There are other factors relevant, however. The present analysis did not compare the effective cross sections of those femora likely to fracture, i.e. aged females. Also, the force actions considered are for walking. It is suggested that these fractures occur due to a spasm of antagonistic muscle action which could exert much greater forces than the determinate loading system assumed for walking.

All the calculations, up to this point, have been based on linear elastic behavior of the material. Figure 3 shows that M_Z rises from zero to its maximum value in a time space of 0.04 of the cycle, i.e., 0.043 sec. At worst, the stress at this time is -5800 lb/in.². Taking Young's modulus as 3×10^6 lb/in.², this gives a strain rate of 0.045/sec. For this value, the results of McElhaney and Byars suggest an increase in modulus of about 12 percent, and it is not considered that this is a factor to alter the analysis significantly.

The analysis is approximate in that: Paul's and Morrison's results have scope for large experimental error due to the complexity of the calculations and measurements and the simplifying assumptions made; the stresses in an idealized section of short length are calculated using simple elastic theory; no account is taken of the positional and directional variation in mechanical properties of the material.

Summary

Force actions on the femur of a walking subject are shown at four stages in the cycle. Stresses calculated on a simplified basis are found to have maximum of 4200 lb/in.² and 2700 lb/in.² in compression and tension, respectively.

References

1. Backman, S., "The Proximal End of the Femur," *Acta. Radiol. Supp.*, 146 (1957).
2. Blount, W., "Don't Throw Away the Cane," *Jnl. Bone. Jt. Surg.*, 38A, 695 (1956).
3. Braune, W. and Fischer O., "Der Gang des Menschen," *Abh. d. Koenigl. Saechs. Gesellsch. d. Wissensch. Math. Phys.*, 21-28 (1898-1904).
4. Bresler, B. and Frankel, J. P., "The Forces and Moments in the Leg During Walking," *Trans. ASME*, 72, 27 (1950).
5. Evans, F. G., "Stress and Strain in Bones," Thomas, Springfield, Ill. (1957).
6. Frankel, V. H., "The Femoral Neck," *Almqvist & Wiksells, Uppsala, Sweden* (1960).
7. Grunewald, J., "Die Beanspruchung der longen Rohrenknochen des Menschen," *Zt. Orthop. Chir.*, 39, 27, 129 and 257 (1920).
8. Inman, V. T., "Functional Aspects of the Abductor Muscles of the Hip," *Jnl. Bone. Jt. Surg.*, 39 (3), 607 (1947).
9. Koch, J. C., "The Laws of Bone Architecture," *Am. Jnl. Anat.*, 21, 177 (1917).
10. McElhaney, J., Fogle, J., Byars, E. F., and Weaver, G., "Effect of Embalming on the Mechanical Properties of Beef Bone," *Jnl. Appl. Physiol.*, 19 (6), 1234 (1964).
11. McElhaney, J. and Byars, E. F., "Dynamic Response of Biological Materials," ASME Paper No. 65-WA/HUF-9 (1965).
12. Morrison, J. B., "The Forces Transmitted by the Human Knee Joint During Activity," Ph.D. thesis, University of Strathclyde, Glasgow, Scotland (1967).
13. Paul, J. P., "Forces Transmitted by Joints in the Human Body," *Proc. Inst. Mech. Eng.*, 181 (3), 8 (1967 a).
14. Paul, J. P., "Forces at the Human Hip Joint," Ph.D. thesis, University of Glasgow, Scotland (1967 b).
15. Pauwels, F., "Der Schenkelhalsbruch ein mechanisches Problem," Ferdinand Enke, Stuttgart, Germany (1935).
16. Rydell, N. W., "Forces Acting on the Femoral Head Prosthesis," *Acta. Orthop., Scand. Suppl.* 88 (1966).
17. Sedlin, E. D., "A Rheological Model for Cortical Bone," *Acta. Orthop., Scand. Suppl.* 83 (1965).
18. Strange, F. G. St. C., "The Hip," Heinemann, London (1963).
19. Williams, J. F., "A Stress Analysis of the Proximal End of the Femur," M. Eng. Sc. Thesis. University Melbourne, Australia (1964).
20. Williams, J. F., "A Force Analysis of the Hip Joint," *Bio. Med. Eng. Jnl.* 3 (8), 365 (1968).