A New Direct Biaxial Testing Machine for Anisotropic Materials

by J.P. Boehler, S. Demmerle and S. Koss

Abstract—A screw-driven new biaxial testing machine for the realization of experimental investigations on anisotropic sheet materials, such as composite plates or rolled sheet metals, is presented. The described mechanical concept and servocontrol system allow cruciform specimens to be subjected to large strain biaxial tensile and compressive tests without kinematic incompatibilities. Moreover, for the proper implementation of biaxial tensile tests, the specific problems linked to the anisotropic properties of the investigated materials are taken into account; therefore, for the first time, the biaxial machine is supplied with the original 'off-axes testing device,' consisting of hinged fixtures with knife-edges at each arm of the cruciform specimen. A recently developed optimization method for the optimal design of flat tensile cruciform specimens is shortly reviewed. Numerical simulations illustrate the decisive superiority of the optimized specimen compared with specimen designs proposed in the literature, as well as the necessity to use the 'off-axes' testing technique in biaxial tests on anisotropic materials.

Introduction

With the increasing importance of lighter-weight structures in any industrial branch (e.g. aeronautic or automotive industries), biaxial testing of sheet materials has become of great practice interest. The classical tests under uniaxial stress have turned out to be insufficient to provide a reliable mechanical characterization which is essential to the efficient use of the materials. The biaxial testing enables better predictions to be made on the mechanical properties of new materials (e.g. composites), as well as on the mechanical behavior of, for example, rolled sheet metals during further manufacturing processes. Prior to the realization of a biaxial testing facility, three major problems have to be solved:

- —the applied technique should allow the existence of an area in the specimen subjected to well-defined biaxial states of stress and strain;
- —especially for the testing of rolled sheet metals, in order to optimize metal forming processes for example, the machine should enable large deformations to be subjected to the specimen;
- —the testing system has to take into account the anisotropic properties of the materials; this is valid for both composites and rolled sheet metals.

In order to discuss the first requirement, the four most important techniques, with different testing systems and specimens used to produce states of biaxial stress and strain, are reviewed in the following.

Bending of beams and thin plates can be used to create a biaxial stress state as reported, for example, by Hazell and Marin[1] who tested an aluminium alloy by employing rhomboidal shaped composite plates consisting of a fiberglass honeycombed structure sandwiched between two thin sheets of aluminium. This test allows the material to be analyzed in the second and fourth quadrants of the two-dimensional stress space, however, each principal stress ratio requires a specific shape of the plates. Another disadvantage of this technique consists in the nonuniform stress distribution (in the plane of the specimen, as well as over the specimen thickness), especially after the occurrence of the first plastic deformations, so that the real stresses cannot be determined from the measured forces.

Another technique to be mentioned is the hydraulic bulge test, as reported by Bird and Duncan,[2] or Lukyanov *et al.*,[3] where a pressure is applied to the surface of a round or elliptical flat sheet specimen. On the one hand, it is evident that there appears a stress gradient in the thickness of the specimen. On the other hand, Dudderar *et al.*[4] proved the nonhomogeneous character of the developing stress fields, due to the gripping of the specimen edges. Like the bending method, this technique requires a different specimen's shape for each ratio of the principal stresses ('length of the major axis'/'length of the minor axis'), and this does not allow a variable loading ratio to be applied during the test.

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The most popular technique employed by, for example, Havard and Topper,[5] Andrews and Ellison[6] or Lefebvre *et al.*,[7] use thin-walled tubes subjected to tensile or compressive loads combined with torsional loads or internal-external pressure, in order to create biaxial states of stress. This technique seems to be very versatile, because it allows tests with any constant stress or strain ratio to be performed; but it presents some inconveniences:

—due to the radial stress gradient the stress state is not exactly biaxial;

—for a given orientation θ of the principal-stress directions, the stress ratio is not variable, i.e. it is not possible to realize any stress or strain path;

—the anisotropic properties of tubes and sheet materials are not comparable.

The most realistic experimental technique to create a biaxial stress state in sheet materials consists of applying in-plane loads along two perpendicular directions to the arms of a cruciform specimen. Different types of load application systems have been proposed in the literature.

Hayhurst[8] developed a biaxial creep-rupture tensile testing machine, where a deadweight lever system is used, and the loads are applied by means of wire ropes which pass over pulleys. Due to a relatively high level of friction in the bearings, the testing was limited to a small lever movement, i.e., small strains. A similar technique was proposed by Bremand and Lagarde,[9] where in each direction deadweight loads are applied to the cruciform specimen (made of urethane) by means of nylon threads and pulleys on roller bearings. In order to avoid difficulties due to the friction in the bearings, in the creep testing machine of Morrison[10] and Kelly,[11] the cross-shaped specimen is subjected to deadweight loads by a knife-edge lever system (ratio 20:1) with pull-rods. This technique presents a very good suitability for creep tests, because it enables steady loads to be maintained for long periods of time.

An inexpensive technique was proposed by Ferron and Makinde.[12] It consists of a symmetrical jointed-arm mechanism to apply biaxial, but coupled, in-plane loads to a flat cruciform specimen by using a conventional uniaxial testing machine. The value of the strain ratio can vary between 0 and 1, but it depends on the geometrical configuration of the device, and it is therefore constant during a test.

The use of hydraulic actuators represents a very versatile technique for the application of loads. Besides the device of Shiratori and Ikegami,[13] one of the first machines using this method was devised by Pascoe and de Villiers[14] where the flat cruciform specimen is loaded directly by four double-acting jacks, with a capacity of 200 kN in tension and compression. In order to realize tests at any ratio of the principal strains, Parsons and Pascoe[15] equipped the original device with a closed-loop servocontrol using the measured strains or loads as feedback signals. A similar servohydraulic testing machine was developed by Makinde *et al.*[16] with four pull-push actuators provided with hydraulic wedge grips and a horizontal load frame which is situated out of the specimen plane; this makes it necessary to provide a very stiff construction, in order to minimize the deflection due to high bending moments under load. Another electrohydraulic closed-loop controlled machine, proposed by Chaudonneret *et al.*,[17] has the same type of load frame, but only one actuator per loading direction. Due to this fact, the center of the specimen moves during a test. Thus, in case of large deformations, kinematic incompatibilities cause a side bending of the specimen, even when using flexible links between the specimen's grip heads and the pistons. Additionally, such a system cannot be used for compressive tests. In order to ensure the independence of the loading in the two mutually perpendicular principal directions, Fessler and Musson[18] developed a 300-kN biaxial hydraulic testing machine, where the horizontal arm of the vertical load frame is suspended and balanced independently.

In this paper, a screw-driven electromechanical biaxial testing machine is presented; the machine is conceived to test cross shaped specimens made of anisotropic materials such as flat composite plates or rolled sheet metals. The used specimen design follows from a shape optimization proposed by Demmerle and Boehler[19] which is briefly described in 'The Cruciform Specimen' below.

Mechanical Concept

Compared to the servohydraulic testing facilities, this electromechanical machine represents a low-cost solution for the realization of biaxial tests on sheet materials. It can apply either tensile or compressive quasi-static loads to cruciform flat specimens. For the first time, this machine is supplied with the off-axes testing device (see 'Connection Machine Specimen' below); this enables a nearly perfect implementation of off axes tensile tests on anisotropic materials. At present, the development of a specimen design together with a suitable gripping system to carry out compressive tests is in preparation.

General Features

The presented machine, Fig. 1, consists of four double-acting screw-driven pistons, two in either direction, rigidly supported on an octagonal vertical frame. In each direction, the maximum load is about 100 kN in tension and compression. The unobstructed working area of 900 × 900 mm^2 enables tests on specimens with a size up to 500 × 500 $mm.^2$ The vertical installation of the octagonal frame which surrounds the specimen ensures an excellent accessibility from both sides to the working area; this facilitates the mounting as well as the observation throughout a test, e.g. to take photos, in order to analyze the strain field by applying the stereophotogrammetric method.[20] Furthermore, as the plane of the specimen coincides with the symmetry plane of the frame, the latter is not subjected to high bending moments causing deflections which in turn lead to an in-plane bending of the specimen. On the other hand, due to the vertical installation, it is necessary to compensate the deadweight of each horizontal arm of the specimen together with the gripping system fixed to its end, in order to avoid a side bending of a thin specimen, see (10) in Fig. 2. Note that it is not necessary to employ the same procedure for the lower vertical specimen arm, since the deadweight of the gripping system causes only a negligible prestress in the cross-sectional area of the specimen.

The four acting pistons enable large deformation tests to be performed without kinematic incompatibilities, be-

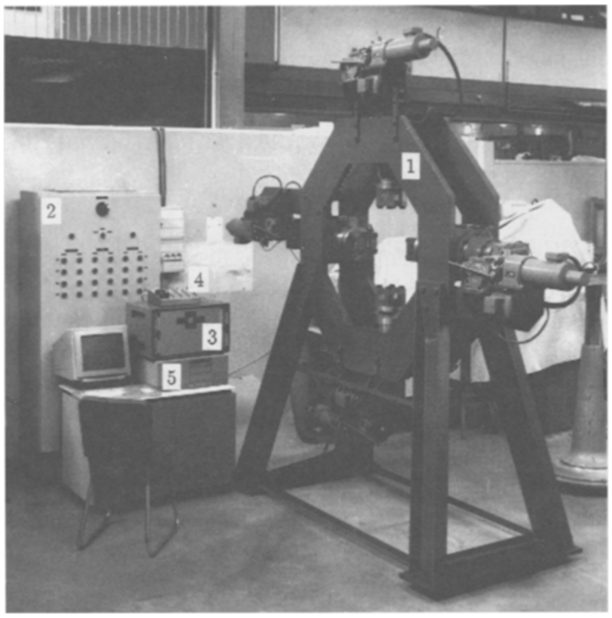


Fig. 1—General view of the testing machine: 1-frame, 2-power unit, 3-analog/logic unit, 4-control panel, 5-servocontrol unit

cause the machine ensures that the center of the specimen remains stationary during the test (see 'Servocontrol System' below); this feature is also important when applying the stereophotogrammetric method. Since the load is applied independently in each of the two principal directions, the loading ratio (F_2/F_1) can be varied infinitely, even during a test.

The machine provides two operation modes: the mode 'test' to carry out the tests and the mode 'fast feed' to position the pistons. In the 'test' mode, the actuators can operate under two control modes, that is, the manual and the numerical control. In the case of the manual control, the desired value signals are obtained from the control panel. In the case of the numerical control, they are obtained from calculations by a microcomputer taking into account the feedback signals of different measurement facilities (e.g. load cells or strain gages); this allows any stress or strain path to be realized. Additionally, three synchronization modes can be chosen: all four pistons can work independently, or the two opposite pistons of each direction are synchronized, or there is a total synchronization of all four pistons. The mode 'fast feed' provides just a manual control without synchronization which is sufficient for the mounting and the dismantling of the specimens.

The Drive Unit

As shown in Fig. 2, each of the four actuators is supplied with two motors: a geared variable-speed dc-motor (1) with a gear ratio of 61.4 which is active in the 'test' mode, and a ac-motor (2) used to realize the high displacement speeds in the 'fast feed' mode. For each pair, a manual clutch (3) enables one of the two motors to be chosen, and a coupled safety device prevents the simultaneous operation of both motors. After another reduction in the two-step gear (4), $r = 309$, the rotation of the shaft is transformed into a translation of the piston (5) by means of an endless screw (lead = 2 mm) situated inside the piston itself. A high precision screw is used, in order to minimize the clearance when reversing the load. In the mode 'test,' the displacement speed of each piston is infinitely variable between 0.003 mm/min and 0.3 mm/min, and controlled by rectifier-governors (type TMD, Leroy-Somer). The synchronization between the motors is realized by means of signals derived from optical incremental encoders (6) which measure the present motor speed (see 'Servocontrol System' below). The four ac-motors work independently from each other and allow the pistons to be moved with 20 mm/min without speed control.

The full stroke of each piston is limited to about 245 mm. In order to prevent damage to the screw-driven pistons, the two extreme positions are detected by microswitches (7) which are connected directly to the control circuit.

Connection Machine Specimen (The Off-axes Testing Device)

One of the most important originalities of the presented machine consists in the realization of the first 'off axes' testing device for biaxial tensile tests on anisotropic materials, cf. Demmerle and Boehler.[19] In the so called 'off-axes' test, the principal directions of the applied loads do not coincide with the privileged axes of the material symmetries; it results that in the domain of elastic behavior, as well as in the domain of plastic behavior, the principal directions of the stress state, in general, do not coincide with that of the strain state, Boehler.[21] Boehler *et al.*[22] showed that in off-axes tests on anisotropic materials this noncoincidence initiates the development of strongly nonhomogeneous stress and strain fields in the specimen when using the classical testing method consisting of rigidly clamped heads. Moreover, since the stress and strain states do not correspond to the intended ones, erroneous experimental data are obtained which cannot be used for a proper identification of constitutive laws for anisotropic materials. The solution to these problems was found by

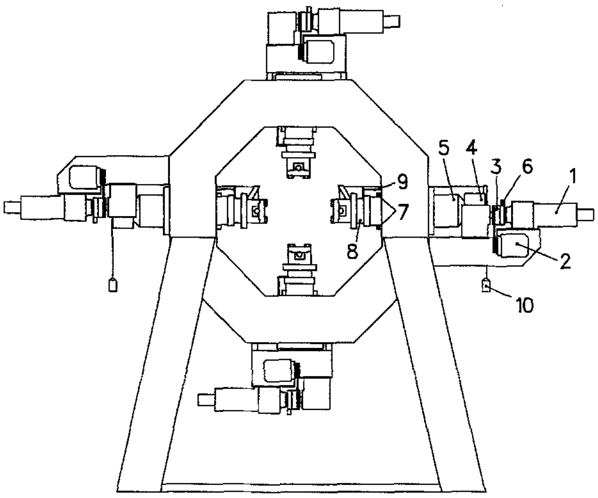


Fig. 2—Components of the direct biaxial testing machine

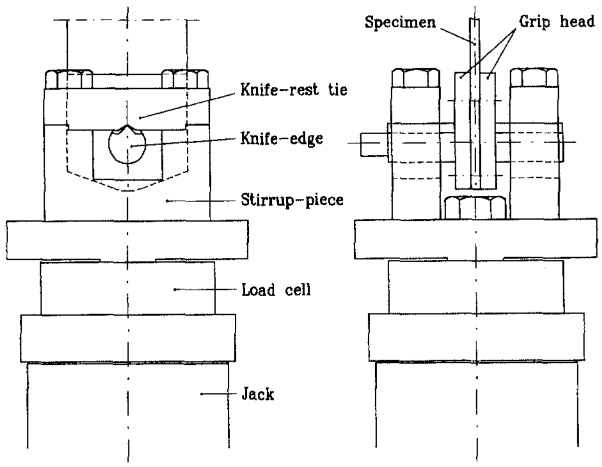


Fig. 3—Off-axes testing device for biaxial tensile cruciform specimens

Boehler[21, 23, 24] and Boehler *et al.*[25,22] who devised a specific off-axes testing device for uniaxial tensile specimens, consisting of hinged fixtures with knife edges. This technique allows the development of an almost nonconstrained strain state in the test region, and it has proved its efficiency to produce homogeneous stress and strain fields in various tests on uniaxial tensile specimens made of orthotropic rolled sheet steel, cf. Boehler.[23]

The above described technique constitutes the basis of the herein developed connection between the machine and the arms of the cruciform flat specimen, Fig. 3. On each of the four jacks of the biaxial testing machine, two stirrup-pieces are fixed. The knife-rest ties of each couple support one knife-edge passing through the grip head and the specimen itself at the ends of the specimen arms. The grip head consists of two rigid steel plates with a central hole (allowing the knife edge to pass through) fixed on both sides to the end of each specimen arm by means of four screws. Thus, the load is applied through the stirrup-pieces and the knife-rest ties to the knife-edge, which allows a nonconstrained rotation of the grip head fixed to the specimen. As shown in the case of uniaxial tensile specimens, this rotation is indispensable, in order to obtain homogeneous stress and strain fields in the central test region. Nevertheless, due to the complexity of the biaxial specimen geometry, this testing device will not be able to effect an absolutely perfect homogeneity of the stress and strain states compared to its former application to uniaxial tensile specimens, but the results presented in 'The Cruciform Specimen' make clear the absolute necessity of the proposed biaxial off-axes testing device for anisotropic materials.

Measurement and Control

The operation of the machine requires two personal computers. The first one (CPU Intel 80386 SX, 16 MHz, 1 MB RAM) is integrated in a 40-channel digital data-acquisition system Vishay (MicroMeasurements) System 4000, which processes all data concerning the measurements of loads, strains or displacements and, if required, that of temperature. The second one (CPU Intel 80386 DX, 16 MHz, 4 MB RAM) represents, together with the developed particular software (in BASIC), the core of the servocontrol system.

Load and Strain-measurement Equipment

In contrast to most biaxial testing machines, each piston is supplied with a load cell, see (8) in Fig. 2, i.e., two cells in either loading direction. Although the second load cell per axis is not absolutely necessary for the proper operation of the machine, it is very helpful, for instance, to check the symmetry of the testing device, and to supervise the symmetry of the loading conditions during the test, in order to avoid the application of shear loads to the specimen's test region. The used load cells of the type StrainSert FL25UM(C) have a capacity of 25 000 lb (≈ 111 kN) and ensure a precision of 0.1 percent by employing a full strain-gage-bridge configuration.

In order to carry out strain measurement on the specimen, two different methods are applied, as reported by Koss.[26] In the domain of small deformations, strain gages, MicroMeasurement type EA, are used. With the so-obtained data the elastic coefficients of the material are determined. Since the measurement of large deformations on thin structures by means of strain gages is disturbed by a strengthening of the assembly 'structure + glue + gage,' cf. Koss and Boehler,[27] high-strain extensometers Schenck (types DSA25/20 and DSA10/20) are used for this purpose. They have a gage length of 25 mm and 10 mm, respectively; strains up to 40 percent can be measured accurately. The original straight knife-edges were replaced by one-point contact knife-edges, which must be used when performing off-axes tests on anisotropic materials, because in such tests the principal directions of the strain tensor are unknown *a priori*, so that the extensometers cannot be placed in these directions; thus the knife-edges mounted on the extensometer arms must be able to rotate without sliding.

Additionally, each actuator is supplied with two displacement transducers (LVDT), cf. (9) in Fig. 2. The first one (Schaevitz 12F100), with a stroke of ±100 mm and a precision better than 0.25 percent, is used to monitor the displacement of the piston in the operation mode 'fast feed' while mounting the specimen; this is very important, in order to place the specimen exactly in the center of the testing device. The other one (Schaevitz 12F20), with a stroke of ±20 mm and a precision better than 0.10 percent, provides the measurement of the global displacement of the piston during the test.

Note that the signals of each measurement facility described above can be used as feedback signals in order to control the machine.

Servocontrol System

The servocontrol system consists of the control panel, the servocontrol unit and the analog/logic unit, Fig. 4. In the following, these three components are described together with a more detailed survey of the operation modes.

THE CONTROL PANEL

Together with the servocontrol unit, the control panel is responsible for the communication between the user and the system. It is equipped with several switches to select the operation mode under manual or numerical control, with or without synchronization. Furthermore, it provides for each of the four dc-motors two switches (start/stop and forward/backward) and a potentiometer, which supplies the TMD-governor of the respective motor with the desired value input for the motor speed. Each of the ac-motors can be manipulated by two further switches (start fwd/stop fwd and start bwd/stop bwd). The present configuration of all switches is only transmitted to the analog/logic unit, and thus to the computer, when pressing the 'validation' push-button on the panel; this allows the operation of the motors to be changed simultaneously, e.g. to start or stop the motors, and to reverse their direction of rotation. Note that the outputs of the four potentiometers are transmitted immediately to the analog/logic unit, i.e., no validation is necessary, so that an 'on line' variation of the motor speed can be performed.

THE SERVOCONTROL UNIT

The servocontrol unit is linked by an interface card to the analog/logic unit; thus it has access to all information which is necessary to control the machine. At the core of the servocontrol system, it has to accomplish three main tasks.

(1) Synchronization of the pistons. As stated earlier, in order to carry out realistic tests under large deformations, it is indispensable for the homogeneity of the kinematic fields in the specimen test area that the center of the specimen does not move during the test, i.e., in either direction a perfect synchronization of opposite pistons has to be guaranteed. For this purpose, although the TMD-governors enable constant speeds of the dc-motors to be maintained, a closed-loop control system was realized by the use of optical incremental encoders. The encoders provide a TTL quadrature output, and using the highest precision one pulse corresponds, theoretically, to a linear displacement of the piston of about $1.6 \cdot 10^{-6}$ mm, which constitutes an excellent basis for the required synchronization quality. In the following, the synchronization process is briefly described. According to the chosen mode, one or two pistons are considered as 'independent', the others play the role of the 'dependent' pistons. Now, in order to carry out the synchronization, the microcomputer counts in permanence the pulses from the four optical incremental encoders and computes, by means of a program realizing a PID-controller, the respective corrected signal to be sent to the TMD-governor of each 'dependent' piston. The speeds of the 'independent' pistons are supplied by the respective potentiometers, and are always considered as references; they are not manipulated by the synchronization algorithm. In general, the synchronization signals are updated every 0.3 s-0.4 s.

(2) Numerical control of the machine. Besides the manual control from the control panel, the machine is also provided with a numerical control by the microcomputer. In contrast to the manual control, the desired value inputs for the displacement speeds of the pistons are not supplied by the potentiometers on the control panel, but by the computer itself. In order to calculate the desired value inputs, specific software can be written taking into account the data from any measuring device on the tested specimen, as well as signals from other external facilities. Therefore, the microcomputer has a permanent access to the analog signals of the measurement facilities and to other external analog inputs by means of the 16-channel ad-converter of the analog/logic unit. The analog signals may characterize the measured forces applied to the specimen arms, the strains at the specimen center, the displacements of the pistons, or signals derived from other measurement or control devices. The calculation of the updated desired value inputs is carried out between the synchronization cycles.

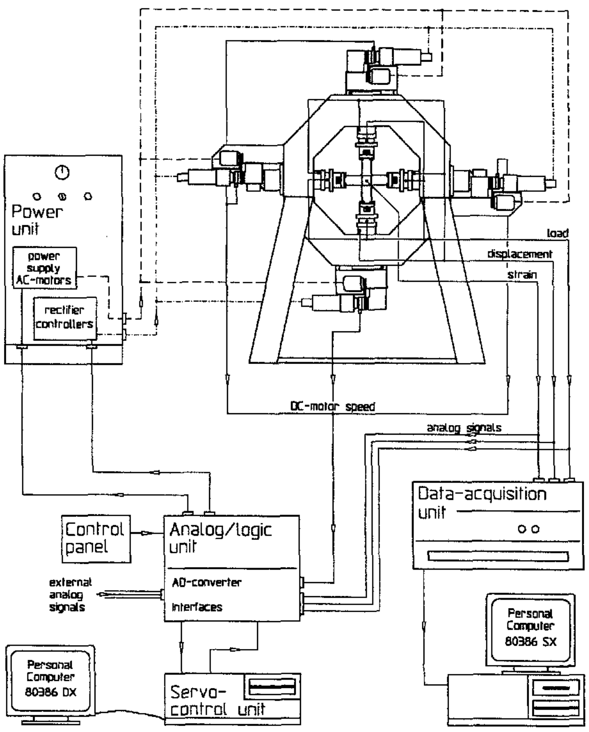


Fig. 4—Schematic structure of the measurement and control facilities

(3) Communication with the user. At the beginning of the test, the computer reads the present configuration of the control panel (after validation) and checks possible incompatibilities in the chosen options. If the mode 'test' is selected, supplementary informations on the control characteristics are demanded (e.g. maximum motor speed, precision of the optical incremental encoders, definition of channels for external- or feedback-inputs, desired force-, strain- or displacement-ratios); if the 'fast feed' mode is chosen, no further input is required and the ac-motors can be activated from the control panel. During the test, in order to monitor the course of the testing procedure and the quality of the synchronization, the counted pulses of the optical incremental encoders are used to compute the motor speeds, as well as the speeds and the displacements of the pistons; all of this information is displayed on the screen. Furthermore, the input signals of every external

channel can be displayed. Thus, the computer enables the user to act immediately upon the control characteristics.

THE ANALOG/LOGIC UNIT

The third component of the servocontrol system, the analog/logic unit, constitutes the interface between the servocontrol unit, the control panel, and the power supply of the eight motors. It is equipped with a 16-channel ad-converter that enables a permanent access to any analog feedback signal.

Besides the described interface function, this unit supervises two safety devices: (1) the 'end of stroke' device, which prevents damage to the screw system by detecting the two extreme positions of each piston; and (2) the second device, which prevents simultaneous operation of both motor types (ac- and dc-motors), which could damage the optical incremental encoders and dc-motors.

OPERATION MODES

In the 'test' mode, where the variable speed motors are active, the following options are available.

(1) All four pistons work independently. Controlled by the TMD-governor, each motor follows the desired value input of its own potentiometer. Thus, it is easily possible to vary the speed, even during the test.

(2) Synchronization of the two pistons in either direction. In each direction X and Y, only one potentiometer is active (X1 and Y1), i.e., X1 and Y1 are the 'independent' pistons, X2 and Y2 the 'dependent' ones.

(3) Synchronization of all four pistons. This option enables equibiaxial tests to be realized. Only the potentiometer X1 is active imposing the speed to the three 'dependent' pistons.

(4) Closed-loop numerical control by means of the servocontrol unit. In this case, all potentiometers are inactive and the desired value inputs are computed by the microcomputer, taking into account the feedback signals. The machine can operate under displacement, force or strain control. Note that this option includes the synchronization of opposite pistons. These features offer a lot of possibilities to perform tests and to realize any loading path or to automate cyclic tests.

In the mode 'fast feed', the ac-motors work independently.

The Cruciform Specimen

As already discussed in the introduction, the machine is conceived to perform tests on flat cruciform specimens of isotropic and anisotropic materials. The problem arising now is to find a specimen design that enables the proper realization of biaxial tests under large deformations, taking into account the phenomena linked to the initial (e.g. composite materials, rolled metals) and strain-induced anisotropy. In the literature, all the used specimen designs have been obtained empirically, for example by means of photoelastic or numerical analysis on given prototypes. In order to obtain adequate specimen designs in a rational manner, Demmerle and Boehler[19] developed a new design criterion and applied engineering-optimization techniques to optimize the geometry of any biaxial flat specimen. The present paragraph gives a short survey of this work.

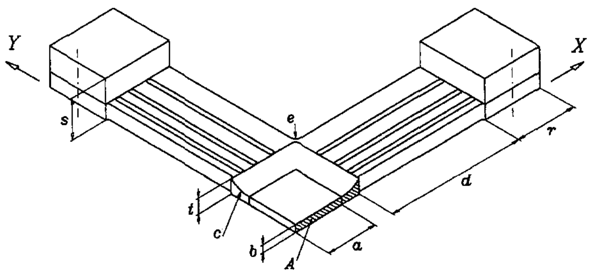


Fig. 5—Geometrical parameters of the numerical model

Criterion for Specimen Optimization

Generally, when employing an optimization method to solve a problem, it is necessary to have at our disposal a well-defined criterion. In the case of cruciform specimens' optimization, the criterion for the assessment of the specimen design must take into consideration the following three requirements:

- —there exists homogeneous stress and strain distributions in the test section;
- —the stress values in the test region are identical to the nominal stress values, which are obtained by dividing each of the two applied loads through the corresponding cross- sectional area *A*, Fig. 5. This is very important for the identification of constitutive laws;
- —the highest stress level can be observed in the test section and it is there, where initial yielding occurs; no stress concentration appears outside this area.

Since the optimization should be performed on a computer by means of a finite-element software, the criterion must represent an objective and mathematically well-defined measure to evaluate to which extent the three requirements are satisfied. The empirical methods consisting of 'looking' at isostress or isodisplacement lines do not constitute an adequate way to solve this problem.

The basic idea of the proposed criterion consists in the statistical description of the stress fields in the test region of the specimen. For the values of the stresses in this area, the standard deviations, as well as the deviations from the corresponding nominal values, are computed. These deviations represent mathematically well-defined measures to evaluate the homogeneity of the considered stresses, as well as the compatibility between actual and nominal stresses, cf. Demmerle and Boehler.[19]

For each considered load case, the criterion is composed of three parts. The first part evaluates the homogeneity of the stress distribution in the test section. In the second one, the compatibility of the actual stresses in the central test region with the corresponding nominal stresses is investigated. The last part considers the location and the importance of stress concentrations in the entire specimen. The weighted sum of the three parts constitutes the 'load-case specific criterion.'

According to the application of the investigated specimen to biaxial tensile tests in the entire first quadrant of the two-dimensional stress space (σ_x, σ_y), the total criterion takes into account the following three load cases, in which the displacements of the specimen's grip heads are imposed:

equibiaxial load, $\Delta y/\Delta x = 1$;
biaxial load, $\Delta y/\Delta x = 0.4$;
uniaxial load, Δy = 'free.'

Finally, the total criterion consists of the sum over these three load-case specific criteria; it enables a general evaluation of any biaxial plane specimen to be performed. The optimization of the specimen design is carried out by minimizing the value of the proposed criterion, with the help of standard numerical methods.

The Optimized Specimen Design

The optimization process is carried out in two steps. In the first step, the criterion is used to assess different basic concepts for the shape of a biaxial flat specimen. In the second step, the best basic design, i.e., that with the lowest criterion value is employed to create a finite-element model, where 12 parameters realize the complete description of the specimen's geometry, Fig. 5. The value of the criterion is then minimized by varying these geometric parameters.

Concerning the basic specimen design, the best results have been obtained for that proposed by several investigators like Hayhurst[8] or Kelly.[11] The design is based on a concept of Mönch and Galster,[28] and presents a series of limbs, separated by slots, extending from each edge of a uniformly thinned square-shaped central region. This geometry enables the deformations to be concentrated on the thinned gage area, whereas the slotted arms reduce the lateral stiffness of the specimen; this is essential for the proper realization of biaxial tests on flat cruciform specimens. The numerical model based on this specimen design is presented in Fig. 5, in order to illustrate the 12 geometrical parameters. The model consists of only a one-eighth part of the specimen, i.e., $x \leq 0$, $y \leq 0$, $z \leq 0$. As most important originalities, the numerical model takes into consideration the off-axes testing technique described in 'Connection Machine Specimen' above, and it allows nonuniform slot- and limb-width distributions within each specimen arm.

The 12 parameters describing entirely the specimen geometry are:

—the half-length *a* of the thinned central test section;
—the half-thickness *b* of the thinned central test section;
—the transition radius *c* between the initial thickness 2*t* of the arms and the thinned region;
—the length *d* of the arms;
—the radius *e* of the corner fillet between two adjacent arms;
—the ratio *f* between the cross sectional area of the slots and that of the limbs;
—the parameter *g* describing the distribution of the slot widths within the arms;
—the parameter *h* describing the distribution of the limb widths within the arms;
—the number *n* of slots within the arm width;
—the length *r* of the grip heads;
—the half-thickness *s* of the grip heads;
—the half-thickness *t* of the plate, of which the specimen is made.

TABLE 1—ORTHOTROPIC ELASTIC COEFFICIENTS OF A COLD-ROLLED CU-ZN ALLOY WITH 12 WT PCT ZN, BASED ON INVESTIGATIONS BY LIU AND ALERS[29]

Young's modulus	Poisson's ratio	Shear modulus
E_1 = 109.9 GPa	ν_{12} = 0.330	G_{12} = 51.80 GPa
E_2 = 142.1 GPa	ν_{23} = 0.280	G_{23} = 55.51 GPa
E_3 = 155.0 GPa	ν_{31} = 0.353	G_{31} = 57.28 GPa

For the automatic shape optimization, two geometrical parameters are maintained constant (initial plate thickness 2*t*, number of slots per edge *n*). The ten others are variable while minimizing the criterion value.

For both steps of the optimization procedure, the material of the specimen was considered to be isotropic elastic. It is characterized by elastic material coefficients corresponding to the Young's modulus in the rolling direction and the Poisson's ratio in the rolling plane of a rolled orthotropic Cu-Zn alloy, (Table 1):

Young's modulus: E = 109.9 GPa,
Poisson's ratio: ν = 0.33.

Later on, the anisotropic behavior of the so-optimized specimen is investigated for rolled Cu-Zn alloy.

The optimized geometry of the specimen with an initial plate thickness of 7 mm and 7 slots within each arm is shown in Fig. 6. It is important to point out that the second step, the automatic optimization, has to be repeated, if a specimen with different material properties or with a different initial plate thickness should be investigated. The number of slots within each arm depends on the desired specimen size.

In the following, the performance of the optimized specimen for anisotropic elastic materials is discussed by the simulation of off axes tests on an orthotropic cold-

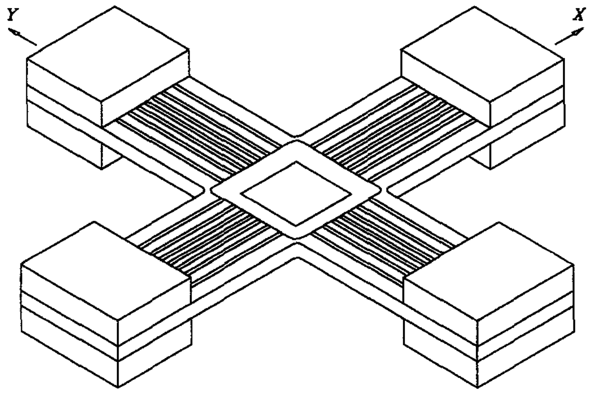
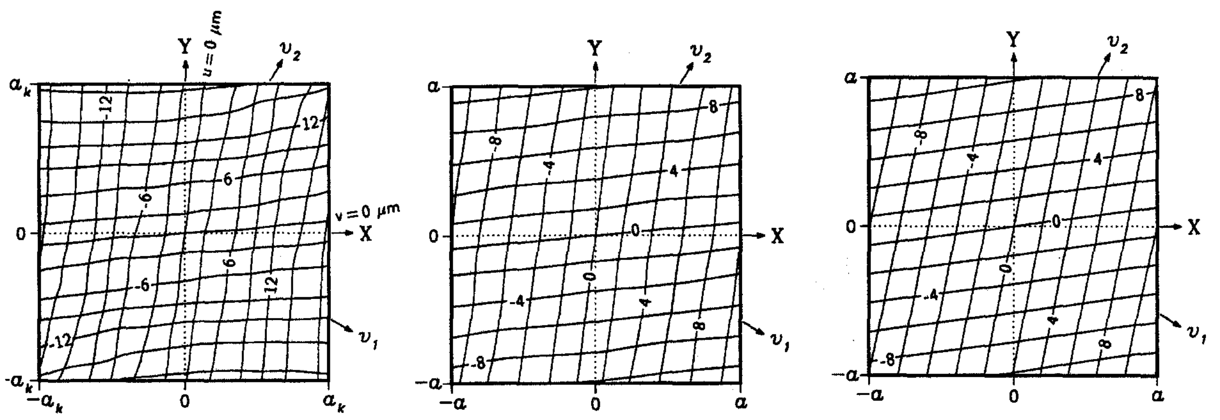


Fig. 6—Geometry of the optimized specimen in the case of isotropic material



(a) Specimen by Kelly (13mm) rigidly clamped

(b) Optimized specimen (7mm) rigidly clamped

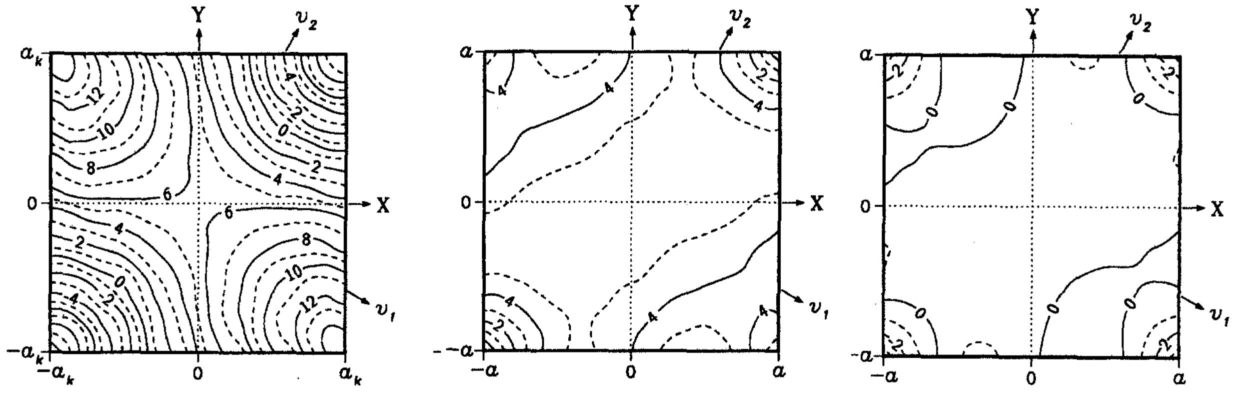
(c) Optimized specimen (7mm) off-axes testing technique

Fig. 7—Isodisplacement lines inside the test section of the specimens; anisotropic elastic material, off axes equibiaxial load; step between two isolines for (a): 3 μm; for (b), (c): 2 μm

rolled Cu-Zn alloy. The nine independent orthotropic elastic constants with respect to the material symmetry axes (v_1, v_2, v_3) are given in Table 1. The initial orientation θ between the principal directions (x, y) of the applied loads and the privileged axes (v_1, v_2) of the material orthotropy is fixed to 30 deg. Three different finite-element analyses have been carried out. First, in order to show the superiority of the optimized specimen design compared with those proposed in the literature, the specimen designed by Kelly[11] with rigidly clamped grip heads is compared with the optimized specimen, mounted with the same gripping system. Then, these results are compared with those obtained by employing the off-axes testing technique together with the optimized specimen, in order to point out the great importance of this technique.

The distinct differences between these three models are visualized by isoline plots of the displacements (u = const, v = const, in the x and y direction, respectively) and of the shear stresses (σ_{xy} = const) in the central test region of the specimen in the case of equibiaxial loading conditions, Figs. 7 and 8. We can observe a quasi-perfect homogeneity of the strain state in the entire test section of the optimized specimen using the off-axes testing technique [Fig. 7(c)]. This is expressed by the isodisplacement lines consisting of two families of equidistant straight lines. They are not parallel to the specimen axes (x, y), which, in turn, do not coincide with the principal directions of the deformations. The isodisplacement plots for the two specimens mounted with rigidly clamped heads show heterogeneousness in the strain fields inside the test section [Figs. 7(a) and (b)]. This is expressed by the isodisplacement lines, which are neither straight nor equidistant. Nevertheless, the results are better in plot 7(b); this underlines the superiority of the optimized specimen even with rigidly clamped heads.

The most distinct difference between the two testing methods is observed for the shear stress field σ_{xy} (Fig. 8).



(a) Specimen by Kelly (13mm) rigidly clamped

(b) Optimized specimen (7mm) rigidly clamped

(c) Optimized specimen (7mm) off-axes testing technique

Fig. 8—Isostress lines for σ_{xy} inside the test section of the specimens; anisotropic elastic material, off axes equibiaxial load; step between two isolines: 1 MPa

In the plot shown in Fig. 8(c), $\sigma_{xy} \approx 0$ is valid in almost the entire test section. Due to this fact, the principal directions of the stress state are well determined, since they coincide with the principal directions of the applied load, which are also the axes of the specimen. In Fig. 7, we have seen that the principal directions of the deformations are different from the specimen axes; thus, they do not coincide with the principal directions of the applied load. In fact, generally, in off-axes tests on anisotropic materials, the principal directions of the deformations and that of the stresses do not coincide. The shear stress fields concerning the two specimens with rigidly clamped grip heads [Figs. 8(a) and (b)] exhibit strong gradients for the specimen (a) and much weaker gradients for the optimized specimen (b); for both specimens the value in the center of the test section is $\sigma_{xy} \approx 5.8$ MPa. Compared to the difference $(\sigma_{yy} - \sigma_{xx}) \approx 22.7$ MPa, this leads to an angle of about 13.6 deg between the principal directions of the stresses and the axes (x, y) of the applied load. Thus, when employing the rigidly clamped heads device in tests on anisotropic materials, the obtained data cannot be useful to identify constitutive laws, because the principal axes of the obtained biaxial stress field cannot be determined.

Conclusions

The described screw-driven new biaxial testing machine together with the use of the specific off-axes testing device and specimens with optimized design allow a nearly perfect implementation of off-axes tensile tests on anisotropic sheet materials. The four double-acting pistons and the control ensuring a very accurate synchronization of each couple of opposite pistons, enable large deformation tests to be realized without kinematic incompatibilities or introduction of parasitic moments into the specimen.

The machine is provided with two control modes: the manual control and the closed-loop numerical control. The feedback signals may characterize the applied forces to the specimen's arms, the strains in the central test region, or the displacements of the pistons. Thus, any stress or strain path can be realized throughout the test.

The optimization of the flat cruciform specimen geometry was performed on isotropic material, but the results show that even for anisotropic materials the optimized specimen design results in considerably more reliable experimental data than the designs already proposed in the literature. Besides the fact that the optimized specimen was used for the numerical simulation, this superiority is also due to the application of the specific off-axes testing technique to the biaxial tensile test.

A direct optimization of a specimen made of anisotropic material requires an extension of the proposed design criterion; this investigation is presently in progress.

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