Simple Optical Processing of Moiré-grating Photographs

Paper explains and illustrates how a few basic optical concepts can be employed in simple ways to improve sensitivity and quality of moire measurements

by Gary L. Cloud

ABSTRACT—The three fundamental optical phenomena of diffraction, two-beam interference, and transformation by a lens form the basis of modern moiré strain-measurement techniques. The improved understanding of diffraction by superimposed gratings and optical spatial filtering leads to a general gain of freedom in designing moiré experiments. Benefits characteristic of refined but very simple optical-data-processing techniques include simpler apparatus, less-demanding procedure, possible large gains in sensitivity, and the ability to choose certain moiré parameters, such as sensitivity, *after* an experiment is concluded and the raw data stored. Sample results from a study of strains near cold-worked holes demonstrate that acceptable results can be had with elementary apparatus and systematic exploitation of optical-data processing.

Introduction

The use of various techniques of optical-data processing in moiré measurement of deformation and strain is neither new nor profound. It seems, however, that benefits derivable from these methods, as well as their basic concepts, are not well understood by many practicioners of experimental mechanics, whose training and experience are likely in mechanics and engineering instead of in physics. Physicists might share some blame for this condition, because their presentation of the subject often is obtusely mathematical and devoid of examples of practical usage of physical phenomena. A result is that experimental methods having much power are not exploited to any substantial degree; and one still hears talk of basic sensitivity limitations of moiré methods and of requirements for refined and expensive apparatus, including sharp gratings of high spatial frequency. In fact, these limitations are no longer very important. One is largely free to choose and design a moiré procedure to fit a given problem and laboratory situation. The continuing development of sophisticated optical devices combines with our increasing maturity in understanding optical phenomena to yield further gains of freedom and power in measurement technique.

This paper has the purpose of drawing together and explaining without mathematics the physical phenomena which are important in optical-data processing of moirégrating photographs to obtain fringe patterns having sensitivity multiplication, pitch mismatch, and, more important, the possibility of choosing certain moiré parameters after an experiment is finished and the data are stored. An example of such a measurement exercise is included.

Classification of Moiré Techniques

The several approaches to creating useful moiré fringes may be divided for convenience of thinking into the following categories:

(1) Direct superimposition of master and specimen gratings by placing them into physical contact for sub-sequent observation or storage of the fringe pattern.

(2) Using optical imaging to indirectly superimpose gratings on a ground glass, in a camera, on a film, or on a partial mirror for real-time observation or storage, such as by double-exposure photography, of the fringe patterns.

(3) Direct superimposition of stored replicas (photographic or transfer grids) of the specimen and master gratings with storage or observation of the fringes.

(4) Superimposition of stored replica master and specimen gratings with optical-data processing and optical imaging to create a fringe pattern for observation or storage.

(5) Certain combinations of the above.

Although useful moiré-fringe patterns can be obtained by the direct superimposition methods, such simple procedures do not yield the best results. It is in the approaches which are conceptually simplest that the most highly refined apparatus is needed to procure an acceptable product; only a small portion of the information which might be contained in a grating photograph is used; and the classical limitations on moiré sensitivity become important.

Increased exploitation of available stored data, better sensitivity, and more control of the measurement process can be had by utilizing only a few basic concepts and simple procedures in optical-data processing.

Three Fundamental Concepts

There are just three phenomena which form the basis

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for optical manipulation of grating replicas to obtain moiré interference-fringe maps. These concepts are described below.

Two-beam Interference

Two-component beams from a single source such as a laser or single-point source can traverse different paths and then be recombined to produce interference fringes. Figure 1 shows this effect for two plane waves. The fringe spacing on the screen is directly related to the angle between the beam axes, the tilt of the screen, and the nature of the two beams. The situation of greatest interest is when two coherent beams intersect at a small angle. It is easy to show that two intersecting beams will produce a pattern of interference fringes in space which are made visible by inserting a screen or optical system into the area of intersection.

Diffraction by Grating

Part of the light passing through or reflected from a grating will be deviated from its original path by an amount dependent on the grating spatial frequency, the wavelength of light and the incidence angle. Figure 2(a) illustrates this behavior for a simple sine grating, where a normally incident beam is divided into three parts. More complex gratings, such as the bar and space type shown in Fig. 2(b), can be thought of as the sum of several sine gratings. A pair of diffracted beams will be produced for each of the component higher-frequency sine gratings. It appears, and correctly, that the diffraction pattern gives a direct indication of the Fourier components of the diffraction grating.

The Lens as a Fourier Transformer

A simple lens acts as a whole-field optical Fourier transforming device. If light with sufficient coherence



Fig. 1—Interference of two coherent collimated intersecting beams

potential (and usually collimated) is passed through some spatial signal (transpareñcy) and then through a lens (order of elements can be reversed), a light distribution which is directly and simply related to the Fourier transform of the input signal will be observed in the back focal plane of the lens. This plane is found most easily by locating the spot where all the light is focussed when the input transparency is not in place. Figure 3 shows a typical arrangement. It is important to keep in mind that distance in the transform plane corresponds to spatial frequency in the input plane. The light distribution gives a visual picture of the spatial-frequency content of the input. That is, a lens is a spatial-frequency analyzer.

Purists might hasten to point out that the two previous concepts—"two-beam interference" and "diffraction by grating"—are really only slightly different manifestations



Fig. 2—Diffraction of narrow light beam by: (a) sine grating; (b) bar-space grating



* ACTUALLY DISTANCE TO BACK FOCAL PLANE FOR THE GIVEN LIGHT BEAM = FOCAL LENGTH FOR COLLIMATED BEAM

of a single physical phenomenon—that being diffraction at an aperture. Such thinking is correct, but it is not especially useful. Separating the concepts allows one to look at the optical processing of grating photographs from two different points of view.

Diffraction by Superimposed Gratings

The theory of moiré-fringe formation by superimposing two diffraction gratings of nearly equal spatial frequencies and orientations has been presented in elegant detail by Guild.¹ His ideas were extended, refined, and demonstrated within the context of moiré strain analysis in a series of definitive papers by Post²⁻⁴ and by Post and McLaughlin⁵ as well as by Holister.⁶ The exposition below draws heavily from Post's fine explanations.

It is assumed that the experimenter has created by some process, probably photographic, two grating transparencies. One of these might be a replica of a master grating, and the other is probably a recording of a distorted specimen grating. The quantity of interest is, of course, the degree and direction of distortion of the second grating. In moiré analysis, the procedure is to superimpose these gratings somehow to produce a high-contrast pattern of interference fringes which is easily observed and photographed and which has optimum properties for subsequent analysis in order to obtain displacement and strain maps.



Fig. 4—Formation of two-beam interference fringes by light diffracted through two sine gratings having slightly different spatial frequencies

INTERFERENCE FRINGES CAN BE VIEWED IN SMALL SCALE ON SCREEN OR CAN BE IMAGED IN LARGE SCALE BY CAMERA The formation and meaning of the fringes produced by passing light through the two diffraction gratings can be understood by calling upon two of the basic concepts outlined above. If a single narrow beam of light is made to pass normally (normal incidence chosen for convenience) through (or reflect from) a sinusoidal amplitude or phase grating, the beam will be divided into three parts at the grating, as was illustrated in Fig. 1. The first part, called the zero order, is the undisturbed portion of the beam which passes directly through the grating. The other two parts, called first orders, deviate symmetrically from the zero order.

Now, consider what happens when a narrow collimated beam passes through two sinusoidal gratings of slightly different spatial frequencies, as illustrated in Fig. 4. Five distinct beam groups will appear in this case. The center group is an attenuated version of the incident beam. The extreme orders each contain only a single component which has been diffracted by each of the two gratings in succession. The intermediate beam groups numbered +1and -1 are the ones of interest. They each contain two beams, the first has been diffracted at the first grating only, and the second has been diffracted at the second grating. These two beams in the group are nearly parallel because the spatial frequencies of the two gratings are nearly equal. Now, if the two beams can be made to overlap (often by inserting a lens or imaging system) and if they both come from a single light source which has a coherence capability great enough for interference to be possible, then the two beams forming a group will interfere with one another (see Fig. 4). The interference seen is that of two coherent beams impinging on a surface with a slight difference of incidence angles. It is a classic example of two-beam interferometry. For a given wavelength of light, the small angular difference between the two beams is a measure of the spatial-frequency difference between the two gratings. The interference-fringe pattern is a function of this angular difference. The result is an interference pattern indicative of pitch and orientation differences of the two diffraction gratings. In short, it is the moiré pattern of the two gratings for the area subtended by the incident beam.

Whole-field Analysis

For moiré strain measurement, it is convenient to illuminate the whole field of the two gratings by coherent collimated (usually) light. In this case, there will be a whole field of beams being diffracted by the first grating, and a second field diffracted by the second grating, as pictured in Fig. 5. A field lens is placed in the diffracted beams to decollimate them and to converge them to a focus. In general, the components diffracted at the first grating will focus at a point slightly displaced from the focus of those diffracted at the second grating. If they are close enough to overlap, then an interference pattern is produced. A more useful procedure is to use another lens and screen (that is, a camera) to construct images of the two grating fields with the light contained in the beam groups. Essentially, the camera forms two images which lie on top of one another. Since the image-forming beams are coherent, the two images interfere with one another. The degree of interference depends mainly upon the relative displacement of the two focal spots which, it must be recalled, depends upon the relative inclinations of the sets of beams coming from the diffraction gratings. The image in the camera displays, then, a pattern of interference fringes which are indicative of the local spatial frequency and orientation differences between the two gratings.

Only minor extensions of these basic ideas suffice to explain the use of moiré gratings in practical measurement situations.

The first complication is that the gratings tend to vary in pitch and orientation from point to point in any strain field of practical interest. One need only apply the reasoning outlined above to each elemental area of the whole field. The result is a set of fringes which vary in direction and spacing from point to point in the field.

The second complication is more difficult to analyze. In general, it is neither wise nor possible to work with



Fig. 5—Diffraction of wide collimated beam by two sine gratings to form whole-field interference pattern sinusoidal gratings. There will exist, therefore, higherorder diffractions at each of the two gratings. The number of orders produced from a single beam by each grating depends mainly upon the sharpness of the grating, that is, the degree to which it approaches a rectangular-wave periodic structure. One finds in such a situation that each group of near-parallel beams consists of several individual beams corresponding to different orders of diffraction at each grating. Figure 6 illustrates this behavior. Guild and Post, in the references cited above, considered these more complex cases in considerable detail. For this work, it is sufficient to observe that the basic diffraction and interference concepts still apply. In general, the interference at the image will involve more than two component images or beams. In practice, the higher-order diffractions can be attenuated to the point where only the basic two beams in each group are of consequence.

Sensitivity Multiplication

There is one important related fact which holds true if the two gratings are of nearly the same spatial frequency. Each higher-order ray group corresponds to a grating frequency which is a multiple of the basic grating frequency. The image formed by any beam group will form a moiré pattern corresponding to grating frequencies



Fig. 6—Diffraction of narrow beam by two barspace gratings to form ray groups containing higher diffraction orders

Note the following: (1) For simplicity, spatial frequencies of grating C and F are nearly equal; (2) diffraction orders beyond ± 2 are not included; and (3) numbers on diffracted rays indicate orders at first and second gratings: C +2, F - 1 means diffraction into +2 order at coarse grating and into -1 order at fine grating



Note the following: (1) Orders beyond ± 2 are not included; (2) diffraction angles shown exaggerated; and (3) numbers on diffracted rays indicate orders at first and second grating F + 1, C - 2 means diffraction into + 1 order at fine grating and then into - 2 order at coarse grating





Fig. 8—Optical system for spatial filtering in Fourier-transform plane and creation of inverse transform of modified image

equal to the diffraction order (or group number) times the fundamental specimen-grating frequency. This concept offers a way of multiplying the moiré sensitivity when the two gratings must be of the same base pitch. All one need do is use the light in a higher-order group to form the image and its fringe pattern. Measurement sensitivity is increased by a factor equal to the ray-group number chosen.

A third and very important extension of the basic concepts arises when the gratings are grossly different in spatial frequency; that is, when one grating frequency is a multiple of the other plus a small additional bit which might be imposed deliberately and/or be the quantity which is to be determined. In such a situation, the diffractions are somewhat more complicated, as is the makeup of each of the diffracted-beam groups. Figure 7 illustrates what happens where the second grating frequency is three times the frequency of the first. The basic idea of forming an interference pattern with the rays of a given group still applies; the question arises as to what such an interference pattern means in terms of the frequency and orientation differences between the two gratings. A general interpretation can be very complicated. An important simplification is that, by design and because of the natural attenuation of high diffraction orders, only two of the component rays in any useful ray group will interact to form a visible fringe pattern. Examination of the two main components in, for example, beam-group number 3 in the case pictured in Fig. 7 produces an answer to the interpretation question. These two beams correspond to the first diffraction order at the fine grating and the third order at the coarse grating. The image formed with these two groups will be the same as that which would be produced by two gratings having nearly equal frequencies at three times the fundamental frequency of the coarse grating. The moiré interference fringes in the image will correspond to those which would be produced by two fine gratings. This conclusion is easily supported by careful theoretical analysis. The effects of the remaining beams will be to increase the background noise in the fringe pattern, perhaps to the point of obscuring the moiré fringes.

A striking feature of the situation just discussed is that the moiré-fringe patterns in the camera image are identical, except for background noise and overall brightness, no matter which ray group is used to form the image. It is possible, and good practice, to utilize whichever group gives the best fringe visibility. Stated another way, the sensitivity is not increased by going to a higher diffraction order, unlike the case when two similar gratings are superimposed.

It is this case where one grating frequency is an integral multiple of the other that has such importance for practical moiré measurement. It allows the use of a coarse specimen grating which is easily applied and photographed. When the grating photograph is superimposed with a finer grating, there appears a moiré-fringe pattern which is the same as that which would be created by two fine gratings. That is, a coarse specimen grating gives a measurement sensitivity which is equivalent to that of a much finer grating. Post³ and others have obtained sensitivity multiplications of 20 and 30 in this way. Multiplications of 3 and 4 are easily had with gratings and apparatus of marginal quality.

Optical Transforms and Spatial Filtering

Another approach to understanding the creation of moiré fringes by superimposing specimen and master gratings in a coherent optical system is based on the fact that a simple lens acts as a Fourier transforming device. A very elementary discussion of the concepts behind this technique and examples of its application are contained in a paper by Cloud.⁷ An elegant, readable and comprehensive review of the subject has been written by Vander Lugt.⁸ Works by Clark, Durelli and Parks,⁹ by Nagae, Iwata and Nagata¹⁰ and by Chiang¹¹ are representative of the fine papers which have been written about using this concept in strain analysis.

Consider again the situation pictured in Fig. 3, where the light passes through a transparency having a transmission which is a function of the space coordinates. The modulated light beam then passes through a simple lens. There will be produced at the back focal plane of the lens (the focus for the undisturbed light beam) a diffraction pattern which is in essence the square of the amplitude of the Fourier transform of the input signal. If the input is a sinusoidal grating, for example, the transform plane will exhibit three bright patches. The central dot corresponds to the uniform field or 'd-c' component of the input. The other two patches indicate the spatial-frequency content of the input, with radial distance in the transform plane representing spatial frequency on the input plane. If the input signal is a 'square wave' bar and space grating, there will be in the transform plane a row of dots whose positions and brightness indicate the presence and importance of various harmonics of the fundamental space frequency at the input. A two-dimensional grid input will



Fig. 9—Example of spatial filtering to create bar grating from a grid

generate a Fourier spectrum at the transform plane which is a two-dimensional array of dots corresponding to the two-dimensional Fourier transform.

Now, if another lens is placed at or near the transform plane, the image of the original input may be cast on a screen. Such a system is shown in Fig. 8. The second lens forms the inverse transform to recover the input. It is possible and often useful, however, to modify the frequency content of the optical image at the Fouriertransform plane before completing the inverse transform. This task can be accomplished by blocking or otherwise changing some portion of the light distribution at the transform plane. Such a procedure is called spatial filtering, coherent optical-data processing, or optical Fourier processing.

A fundamental example of optical Fourier processing is shown in Fig. 9. Here, the input signal is a two-dimensional grid of crossed lines which produces a two-dimensional array of dots at the transform plane. All the dots, except the central vertical row, are blocked by a suitable screen with a slit, which is placed in the transform plane. The inverse transform created by the second lens is found to be a simple grating of vertical parallel lines. The horizontal family of lines is suppressed by the optical filter which has removed all the light required to image the horizontal lines. The potential usefulness of such a process is very great.

In the moiré situation under study here, two superimposed gratings are placed in an optical-data-processing system. A Fourier spectrum of the gratings is created at the transform plane. All but one of the bright patches (actually two or more bright dots close together) are eliminated by a dark mask containing a small hole. The light in this one patch is used by the second lens to form an image on a screen. This lens and screen combination can be an ordinary camera. The image is constructed, then, of light which carries with it information about the periodic structure of the two gratings for whatever fundamental space frequency has been chosen by the placement of the hole. The only rays which get through the hole are those which have been modulated by the gratings at a single space frequency which may be the fundamental grating frequency or one of its harmonics. The image will exhibit moiré fringes which correspond to this space frequency.

A distinctive feature of this approach is that the output image of the gratings is considered to consist of a desirable signal plus a great deal of other information. The important signal is made visible by sifting it from all the extra information. One has a certain latitude in selecting the information that is most useful.

Rationalization of Two Approaches

Having two explanatory models of one process raises

the question of which one is correct—or are both faulty? Actually, the two explanations are not different in basic concept; the difference is one of emphasis. In the diffraction model, we look upon the diversion of portions of the incident beam of light as the important feature. With the Fourier-processing approach, we are concerned with the transfer characteristics of an aperture, which happens to have a lens in it, given an optical signal which is already generated by passing light through a transparency. Of course, the lens would not work correctly if the transparency did not redirect portions of the incident beam by diffraction. This combining and rationalizing of the two approaches could be pursued to a final consistent model. The price for this nicety is a small increase of complexity. Further study of the problem would not contribute to the goals of this paper, so it is abandoned with one final observation. As so often is the case with optical processes, the uniting physical phenomenon is that of interference. This property of light is what makes visible for study those minute differences of propagation direction, path length, or wavefront shape which are the physical manifestations of important processes such as diffraction and double refraction.

A Practical Example

An illustration of some of the benefits derivable from simple optical processing of moiré-grating photographs may be drawn from a study of residual-strain fields around coldworked holes by $Cloud^{12,13}$. Only the bare essentials of a typical result can be offered here.

It is in a study of this sort which requires measurement of a broad range of elastic and plastic strain in the presence of out-of-plane displacements that the flexibility of the optical-data-processing procedure becomes useful. The baseline (zero strain) and deformed grating data are permanently stored on glass photographic plates. It is possible to superimpose these plates with each other or with different submaster gratings in order to gain maximum useful sensitivity multiplication and to improve subsequent fringe reading and data analysis by optimizing the spatialfrequency mismatch of the superimposed gratings.

The cold-working strains were measured by printing line and dot gratings of 1000 lpi (39 lines/mm) space frequency onto the specimen surface with Shipley photoresist. The specimen grating was photographed using high-resolution technique before and after the cold work was imposed. These grating photos were then superimposed in turn with higher-frequency submaster gratings in an optical processor in order to obtain moiré fringes.

The specimen-grating photographs had a spatial frequency of 762 lpi (30 lines/mm) which results with a specimen grating of 1000 lpi magnified 1.3 times. These



Fig. 10-Typical fringe patterns illustrating some improvements possible with simple optical processing:

(a) From double-exposure grating photo and simple spatial filtering of first order

(b) From specimen-grating photo superimposed with submaster grating of slightly different

frequency in optical processor (c) From specimen-grating photo superimposed with submaster having spatial frequency three

times specimen grating plus pitch mismatch

plates could be superimposed with submasters of around 2200 lpi to get a sensitivity multiplication of 3, or of 1542 lpi for a multiplication of 2. An advanced technique, which is mentioned below, gave a multiplication of 4. The various pitch mismatches were chosen to yield the closest fringe spacing obtainable with good fringe visibility. Most grating photoplates were processed with at least three mismatch levels, and sometimes with more than one sensitivity multiplication factor for checking purposes and because it was not possible to assess the quality of a dense fringe pattern through the camera viewfinder, which was used without a magnifier.

To be specific, for most of this study, each baseline photoplate and each data photoplate was superimposed in the processor with submaster grating plates of 2200, 2225 and 2256 lpi. For specimen gratings of poor quality, a sensitivity multiplication of 3 was not practicable and submasters of 1535 and 1492 lpi were used. Some fringe patterns were made for checking purposes with submasters of 763 lpi. For several of the specimens, all or most of these several moire patterns were analyzed to gain redundancy of data. This extra information was useful in gaining an appreciation of probable errors and in studying the sensitivity of the data quality to variations in the optical processing.

It was also possible, at this stage, to select submasters which had density and diffraction characteristics which balanced with the properties of the specimen grating replica to produce the best fringe patterns. Also, the ray group which gave best fringe visibility could always be selected.

The 35-mm negatives of the fringe patterns were enlarged and printed in large size for numerical fringe analysis. The ratio of printed image size to specimen size (not negative size) was approximately 7. This magnification plus the moiré sensitivity multiplication and pitch mismatch gave overall sensitivities which were appropriate to the problem.

Some fringe patterns are reproduced in Fig. 10.

Optical Process to Improve Grating Photography

The discussion so far has centered on the treatment of grating transparencies. It is possible to exploit optical processing at the grating photography stage to further simplify procedures and improve results. A particular example is that a slotted aperture mask can be used to tune a photographic system to an integral multiple (including unity) of a grating frequency. Such a procedure is useful for multiplying sensitivity or for obtaining improved rendering of gratings and fringe patterns. Depth of field is increased, and a camera lens of poorer quality than is normally required for high-resolution photography of moiré gratings is adequate. Discussion and examples of such a procedure have been published by Cloud,¹⁴ and details are not repeated here.

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