The Alignment Error of the Hole-drilling Method

The stress–strain relationship for the eccentric hole case has been derived and expressed in terms of the off-center distance (hole center to strain-gage-rosette center) and the polar angle

by Hsin-Pang Wang

ABSTRACT—The hole-drilling method is one technique for measuring residual stresses. All the existing equations for the calculation of residual stresses are based on the assumption that the hole is located at the rosette center. In this paper, the stress–strain relationship for the eccentric hole case has been derived and expressed in terms of the off-center distance and the polar angle. The alignment error is studied and demonstrated by two examples, namely, a uniaxial-stress field and a hydrostatic-stress field. The error analysis yielded the following typical result: ten percent of hole radius off-center will yield about five-percent measurement error for the standard rosette (EA-09-062-RE-120).

Introduction

The hole-drilling method for determining residual stresses has been developed by many researchers[1-5] for several decades. The method is based on the fact that residual stresses can be calculated from the measurement of surface strains which result when stresses are relieved by a hole. There are various procedures to relax the stresses, either the mechanical method, i.e., hole drilling or some modern techniques such as abrasive jet.[6]

Because of possible eccentricities occurring between a hole center and the rosette center, the alignment-error analysis is important to the final stress results.

Theory

Several assumptions should be made before formulation of the problem: (1) the dimensions of the test specimen are large with respect to the diameter of the hole; (2) the surface deformation will not continue after the hole depth reaches one-diam deep,[2-5] (3) the variation of stresses over the thickness is neglected. With these assumptions the problem is simplified to the case of the stresses around a circular hole in an infinite plate.

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By the hole-drilling method, the residual stresses are not fully relieved at the strain gage unless the gage is infinitesimally small and located at the edge of the hole. However, the relief stress field can be derived by superposing two known functions,[2] the stresses on a flat plate and the stresses around a hole. The relief stresses are:

$$\sigma_r = -\frac{\sigma_1+\sigma_2}{2}\frac{a^2}{r^2} + \frac{\sigma_1-\sigma_2}{2}\left(-\frac{4a^2}{r^2} + \frac{3a^4}{r^4}\right)\cos 2\theta$$

$$\sigma_t = \frac{\sigma_1+\sigma_2}{2}\frac{a^2}{r^2} - \frac{\sigma_1-\sigma_2}{2}\frac{3a^4}{r^4}\cos 2\theta \qquad (1)$$

where σ_1, σ_2 = the principle stresses
a = the hole radius
r = the radial distance from the hole center
θ = the angle between σ_r and σ_1.

Concentric Hole

By Hooke's law, the radial strain is

$$\epsilon_r = \frac{1}{E}(\sigma_r - \nu\sigma_t) \qquad (2)$$

where

E = Young's modulus
ν = Poisson's ratio

Substituting eq (1) into eq (2), the measured strain ϵ_g becomes

$$\epsilon_g = \epsilon_r = \frac{1}{E(r_o/a)^2}\left\{\frac{\sigma_1+\sigma_2}{2}(-1-\nu) + \frac{\sigma_1-\sigma_2}{2}\right.$$

$$\left.[3(1+\nu)/(r_o a)^2 - 4]\cos 2\theta\right\} \qquad (3)$$

As shown in Fig. 1, r_0 is the radial distance from the rosette center to the center of each gage. The gage reading ϵ_g expresses the averaged value of the radial strain over the gage area.

For the uniaxial stress field ($\sigma_2 = 0$), assuming the gage direction coincided with the stress direction, the reading ϵ_g is

$$\epsilon_g\,(\theta = 0) = K_1 \frac{\sigma_1}{E} \tag{4}$$

where

$$K_1 = \frac{-5 - \nu + 3(1+\nu)/(r_0/a)^2}{2(r_0/a)^2}$$

and the uniaxial stress σ_1 is

$$\sigma_1 = \frac{E\epsilon_g}{K_1} \tag{5}$$

In a biaxial-stress field with known principal directions, a two-gage system is used for the measurement. The measured strains are

$$\epsilon_{g1}\,(\theta = 0°) = \frac{1}{E}\,(K_1\sigma_1 + K_2\sigma_2) \tag{6}$$

$$\epsilon_{g2}\,(\theta = 90°) = \frac{1}{E}\,(K_2\sigma_1 + K_2\sigma_2) \tag{7}$$

where

$$K_2 = \frac{3 - \nu - 3(1 + \nu)/(r_0/a)^2}{2(r_0/a)^2}$$

and the pricipal stresses are

$$\sigma_1 = E\,\frac{-K_1\epsilon_{g1} + K_2\epsilon_{g2}}{K_2^2 - K_1^2} \tag{8}$$

$$\sigma_2 = E\,\frac{K_2\epsilon_{g1} - K_2\epsilon_{g2}}{K_2^2 - K_1^2} \tag{9}$$

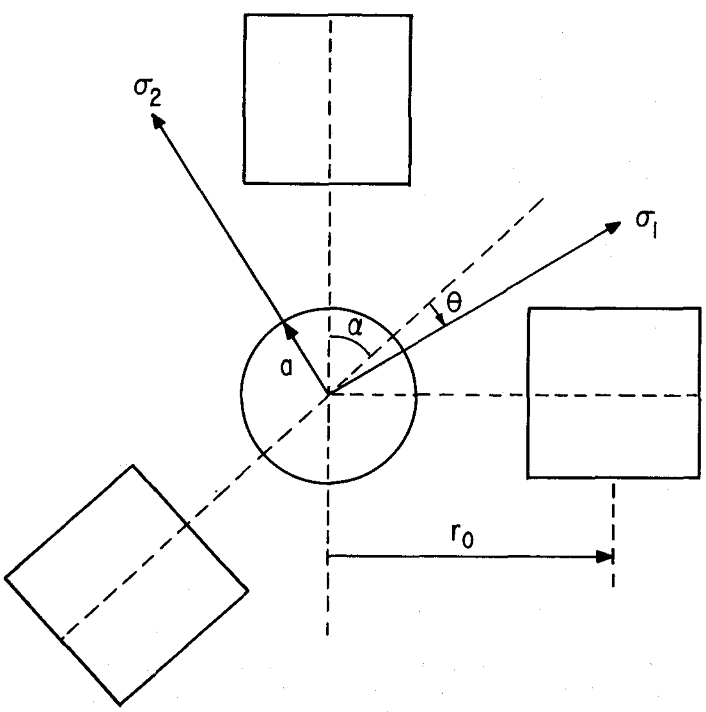

Fig. 1—Dimensions of a strain-gage rosette

If the principal direction is not known, three unknowns σ_1, σ_2 and θ in eq (6) can be solved by three simultaneous equations obtained by putting three gages in the directions θ, $\theta + \alpha$ and $\theta - \alpha$. The general solutions are found to be

$$\theta = ½ \tan^{-1}\left[\left(\frac{1 - \cos 2\alpha}{2\epsilon_\theta - \epsilon_\alpha - \epsilon_{-\alpha}}\right)\left(\frac{\epsilon_{-\alpha} - \epsilon_\alpha}{\sin 2\alpha}\right)\right] \tag{10}$$

$$\sigma_1 = \frac{E}{2}\left\{\frac{\epsilon_\alpha + \epsilon_{-\alpha} - 2\epsilon_\theta \cos 2\alpha}{2A\,(1 - \cos 2\alpha)} + \frac{[(\epsilon_\theta - \epsilon_\alpha)^2 + (\epsilon_\theta - \epsilon_{-\alpha})^2]^{1/2}}{\sqrt{2}\,B \sin 2\alpha}\right\} \tag{11}$$

$$\sigma_2 = \frac{E}{2}\left\{\frac{\epsilon_\alpha + \epsilon_{-\alpha} - 2\epsilon_\theta \cos 2\alpha}{2A\,(1 - \cos 2\alpha)} - \frac{[(\epsilon_\theta - \epsilon_\alpha)^2 + (\epsilon_\theta - \epsilon_{-\alpha})^2]^{1/2}}{\sqrt{2}\,B \sin 2\alpha}\right\} \tag{12}$$

where $A = \dfrac{(1 + \nu)a^2}{2r^2}$; $B = \dfrac{2a^2}{r^2}\left[-1 + \dfrac{3(1 + \nu)a^2}{4r^2}\right]$

Eccentric Hole

Figure 2 shows each parameter in the eccentric-hole analysis, where C_0 is the center of the gage system and C is the hole center. The β angle denotes the gage direction relative to the line connecting the hole center and the center of each gage.

From the basic Mohr's circle, the strains in the θ direction can be found in terms of the principal strains, ϵ_1 and ϵ_2.

$$\epsilon_r = \epsilon_1 \cos^2\theta + \epsilon_2 \sin^2\theta \tag{13}$$

$$\epsilon_t = \epsilon_1 \sin^2\theta + \epsilon_2 \cos^2\theta \tag{14}$$

Similarly, the measured strain ϵ_g can be expressed as

$$\epsilon_g = \epsilon_1 \cos^2(\theta - \beta) + \epsilon_2 \sin^2(\theta - \beta) \tag{15}$$

Solving the simultaneous eqs (13) and (14) for ϵ_1 and ϵ_2, and substituting these two values into eq (15), the measured strain becomes

$$\epsilon_g = \frac{1}{\cos^2\theta - \sin^2\theta}\,[(\epsilon_r \cos^2\theta - \epsilon_t \sin^2\theta)\cos^2(\theta - \beta) + (\epsilon_t \cos^2\theta - \epsilon_r \sin^2\theta)\sin^2(\theta - \beta)] \tag{16}$$

By Hooke's law for plane stress, eq (16) yields

$$\epsilon_g = \frac{1}{E}\,(L_1\sigma_r + L_2\sigma_t) \tag{17}$$

where

$$L_1 = [(\cos^2\theta + \nu \sin^2\theta)\cos^2(\theta - \beta) - (\sin^2\theta + \nu \cos^2\theta)\sin^2(\theta - \beta)]/(\cos^2\theta - \sin^2\theta)$$

$$L_2 = [(\cos^2\theta + \nu \sin^2\theta)\sin^2(\theta - \beta) - (\sin^2\theta + \nu \cos^2\theta)\cos^2(\theta - \beta)]/(\cos^2\theta - \sin^2\theta)$$

In the hole-drilling process, the measured strain is caused by the relief stresses. Therefore, the stress function in the eq (17) should be represented by eq (1). Finally, the measured strain becomes

$$\epsilon_g = \frac{1}{E}\Big\{(-L_1+L_2)\frac{\sigma_1+\sigma_2}{2}\frac{a^2}{r^2} + \Big[L_1\Big(-\frac{4a^2}{r^2} + \frac{3a^4}{r^4}\Big) - L_2\frac{3a^4}{r^4}\Big]\frac{\sigma_1-\sigma_2}{2}\cos 2\theta\Big\} \qquad (18)$$

Using the law of cosine (see Fig. 2) the dimensionless distance r/a is

$$r/a = [(R/a)^2 + (r_0/a)^2 - 2(R/a)(r_0/a)\cos\gamma]^{1/2} \qquad (19)$$

where off-center distance R and γ angle are shown in Fig. 2. By the law of sine, the $\sin\beta$ becomes

$$\sin\beta = \frac{R/a\,\sin\gamma}{[(R/a)^2 + (r_0/a)^2 - 2(R/a)(r_0/a)\cos\gamma]^{1/2}} \qquad (20)$$

For the uniaxial stress case, the gage is assumed to be parallel to the stress field. The strain is

$$\epsilon_g(\theta = -\beta) = K_{11}(r/a,\beta)\frac{\sigma_1}{E} \qquad (21)$$

where

$$K_{11}(r/a,\beta) = \Big\{\Big[-\frac{1}{2} + \Big(\frac{3}{2(r/a)^2} - 2\Big)(1 - 2\sin^2\beta)\Big]\Big[1 - (1-\nu)\sin^2\beta\Big] + \Big[\frac{1}{2} - \frac{3}{2(r/a)^2}(1 - 2\sin^2\beta)\Big]\Big[-\nu - (1-\nu)\sin^2\beta\Big]\Big\} \Big/ \Big[(r/a)^2(1 - 2\sin^2\beta)\Big]$$

$K_{11}(r/a,\beta)$ can be written in terms of R/a, γ and r_0/a by eqs (19) and (20). The stress is then

$$\sigma_1 = \frac{E}{K_{11}(r/a,\beta)}\epsilon_g = \frac{E}{K_{11}(R/a,\gamma,r_0/a)}\epsilon_g \qquad (22)$$

As shown in Fig. 3, a two-gage system is used for the biaxial-stress case. Let $\theta = \theta_1$; $\beta = \beta_1$ for gage one and $\theta = \theta_2$; $\beta = \beta_2$ for gage two. Since the direction of the gage system is consistent with the principal axes, then $\theta_1 = -\beta_1$; $\theta_2 = 90° + \beta_2$. The strains ϵ_{g1} and ϵ_{g2} become

$$\epsilon_{g1}(\theta = -\beta_1, r = r_1) = \frac{1}{E}[K_{11}(r_1/a,\beta_1)\sigma_1 + K_{12}(r_1/a,\beta_1)\sigma_2] \qquad (23)$$

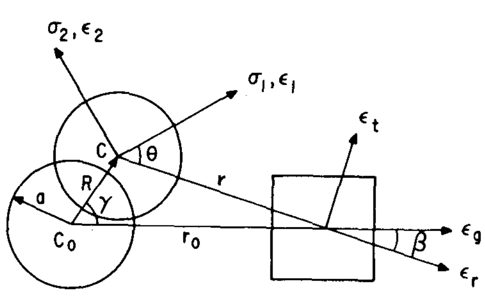

Fig. 2—Schematic showing geometrical parameters and stress components

$$\epsilon_{g2}(\theta = 90° + \beta_2, r = r_2) = \frac{1}{E}[K_{21}(r_2/a,\beta_2)\sigma_1 + K_{22}(r_2/a,\beta_2)\sigma_2] \qquad (24)$$

where K_{11}, K_{12}, K_{21}, and K_{22} can be obtained by eqs (18), (19) and (20) and are listed in the Appendix. Now, the principal stresses σ_1 and σ_2 are

$$\sigma_1 = \frac{E}{K_{11}K_{22} - K_{21}K_{12}}(K_{22}\epsilon_{g1} - K_{12}\epsilon_{g2}) \qquad (25)$$

$$\sigma_2 = \frac{E}{K_{11}K_{22} - K_{21}K_{12}}(-K_{21}\epsilon_{g1} + K_{11}\epsilon_{g2}) \qquad (26)$$

The alignment error analysis will be demonstrated by two extreme cases, the uniaxial-stress field and the hydrostatic-stress field. The general solutions of σ_1, σ_2 and θ for the eccentric case, have not been solved yet due to complexity of eq (18).

Alignment Error

First, the uniaxial case will be discussed. Let σ be the stress solution of the eccentric hole and σ^* be the stress solution of the concentric hole. From eqs (5) and (22), the error is

$$\frac{\sigma^* - \sigma}{\sigma}\,\% = \frac{K_{11}(R/a,\gamma,r_0/a)}{K_1(r_0/a)} \qquad (27)$$

Figure 4(a) shows the alignment error vs. γ angle for the case $R/a = 0.1$. It is seen that the error is higher at the axis of the gage ($\gamma = 0, \pi$) and increases with the hole size. The alignment error vs. the off-center distance R/a is shown in Fig. 4(b). It is obvious that the error increases as the R/a increases.

For the hydrostatic case, the alignment error is dependent upon the number of the gage being used. The single-gage case will be discussed first. Substituting $\sigma_1 = \sigma_2$ into eq (23) yields

$$\sigma = \frac{E\epsilon_g}{K_{11} + K_{12}} = \frac{(r_1/a)^2(1 - 2\sin^2\beta_1)}{-(1+\upsilon)}E\epsilon_g \qquad (28)$$

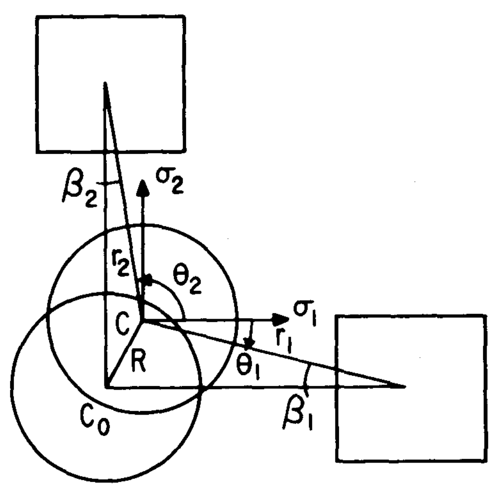

Fig. 3—Schematic of the two-gage rosette

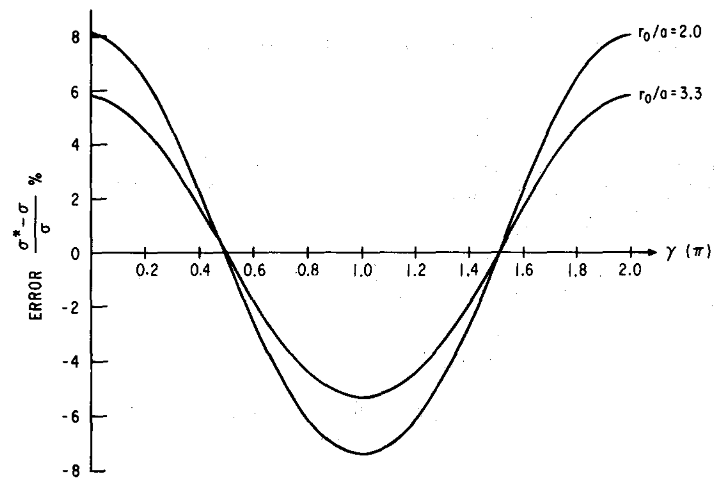

Fig. 4(a)—Alignment-error analysis for the uniaxial-stress field (R/a = 0.1; ν = 0.3)

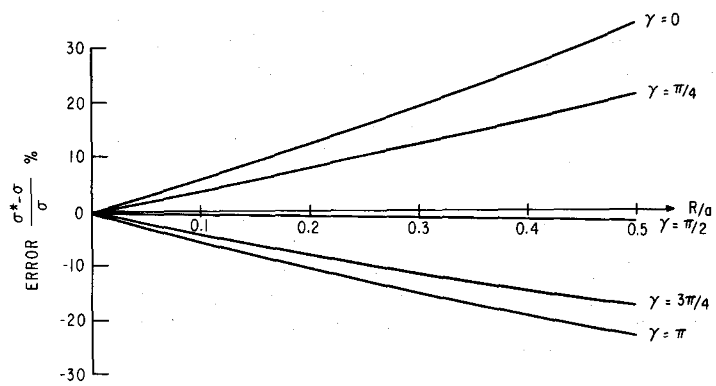

Fig. 4(b)—Alignment-error analysis for the uniaxial-stress field ($\gamma = 0, \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}, \pi$; ($r_0/a$ = 3.3; ν = 0.3)

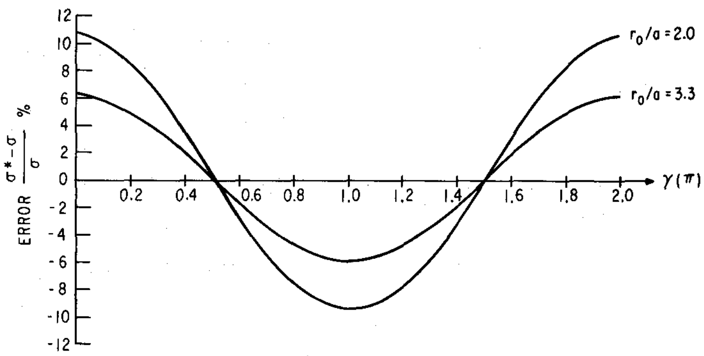

Fig. 5(a)—Alignment-error analysis for the hydrostatic-stress field by single gage (R/a = 0.1; ν = 0.3)

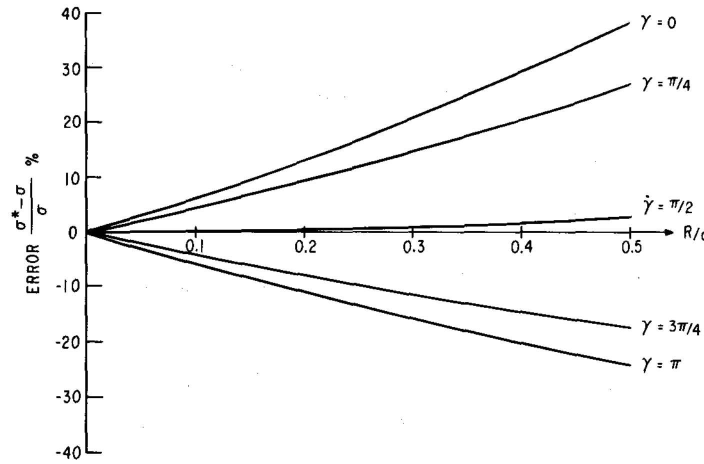

Fig. 5(b)—Alignment-error analysis for the hydrostatic-stress field by single gage ($\gamma = 0, \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}, \pi$; r_0/a = 3.3; ν = 0.3)

Similarly, eq (6) gives the stress σ^* of the concentric hole

$$\sigma^* = \frac{E\epsilon_g}{K_1 + K_2} = -\frac{(r_0/a)^2}{1+\nu} E\epsilon_g \tag{29}$$

Hence, the error of the single-gage is

$$\frac{\sigma^*-\sigma}{\sigma}\ \% = \frac{K_{11}+K_{12}}{K_1+K_2} - 1 = \frac{(r_0/a)^2}{(r_1/a)^2(1-2\sin^2\beta_1)} - 1 \tag{30}$$

By employing eqs (19) and (20), the error becomes,

$$\frac{\sigma^*-\sigma}{\sigma}\ \% = \frac{(r_0/a)^2}{(R/a)^2+(r_0/a)^2-2(R/a)(r_0/a)\cos\gamma-2(R/a)^2\sin^2\gamma} - 1 \tag{31}$$

Figures 5(a) and 5(b) show the alignment error of the hydrostatic case by a single-gage measurement. It is noted that $(r_0/a)^2$ is dominant in the eq (31).

Unlike the single-gage case, the errors for the two-gage system are in two directions.

$$\frac{\sigma_1^*-\sigma}{\sigma}\ \% = \frac{-K_1\epsilon_{g1}+K_2\epsilon_{g2}}{K_{22}\epsilon_{g1}-K_{12}\epsilon_{g2}}\ \frac{K_{11}K_{22}-K_{21}K_{12}}{K_2^2-K_1^2} - 1 \tag{32}$$

$$\frac{\sigma_2^*-\sigma}{\sigma}\ \% = \frac{K_2\epsilon_{g1}-K_1\epsilon_{g2}}{-K_{21}\epsilon_{g1}+K_{11}\epsilon_{g2}}\ \frac{K_{11}K_{22}-K_{21}K_{12}}{K_2^2-K_1^2} - 1 \tag{33}$$

where σ_1^* and σ_2^* are calculated by substituting ϵ_{g1} and ϵ_{g2} into eqs (8) and (9) for the concentric hole.

For the hydrostatic condition, ϵ_{g1} in eqs (25) and (26) can be expressed

$$\epsilon_{g1} = N\epsilon_{g2} \tag{34}$$

where $N = (K_{11} + K_{12})/(K_{22} + K_{21})$.

Now the errors become

$$\frac{\sigma_1^*-\sigma}{\sigma}\ \% = \frac{-K_1N+K_2}{K_{22}N-K_{12}}\ \frac{K_{11}K_{22}-K_{21}K_{12}}{K_2^2-K_1^2} - 1 \tag{35}$$

$$\frac{\sigma_2^*-\sigma}{\sigma}\ \% = \frac{K_2N-K_1}{-K_{21}N+K_{11}}\ \frac{K_{11}K_{22}-K_{21}K_{12}}{K_2^2-K_1^2} - 1 \tag{36}$$

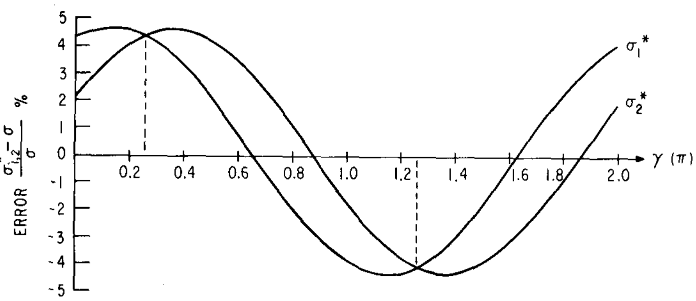

Fig. 6(a)—Alignment-error analysis for the hydrostatic-stress field by two-gage rosette (R/a = 0.1; r_0/a = 3.3; ν = 0.3)

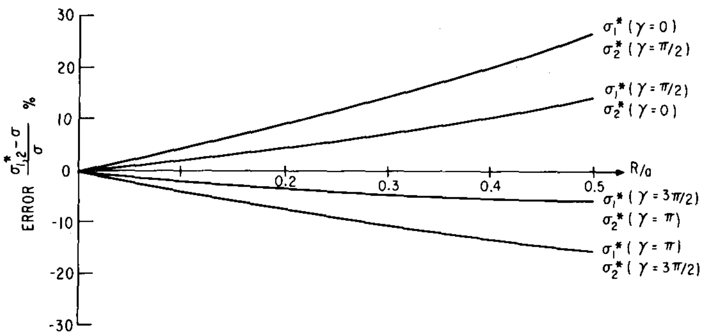

Fig. 6(b)—Alignment-error analysis for the hydrostatic-stress field by two-gage rosette ($\gamma = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}$; r_0/a = 3.3; ν = 0.3)

Where γ is equal to 45 deg or 225 deg, the hole is a symmetric position and the strains from both gages are equal. The errors for these two specific angles are

$$\frac{\sigma_1{}^*-\sigma}{\sigma}\,\% = \frac{\sigma_2{}^*-\sigma}{\sigma}\,\% = \frac{-K_1+K_2}{K_{22}-K_{12}}\;\frac{K_{11}K_{22}-K_{21}K_{12}}{K_2{}^2-K_1{}^2} - 1 \qquad (37)$$

Figures 6(a) and 6(b) show the alignment error of the hydrostatic stress field for two gages and the symmetric characteristic along the ¼ π and 1¼ π directions.

Conclusion

From Table 1 of computed results, it is noted that the alignment error increases when the r_0/a ratio decreases at a constant value of R/a. For a standard rosette (EA-09-062-RE-120) the alignment error is about five percent in the stress solution for the case with 3 mils out of the rosette center in which R/a is equal to 0.1.

It is recommended that the general solution for the eccentric hole case be derived. Then the principal direction and the principal stresses can accurately be calculated by substituting the measured values of R and γ. However, with the present formulation the principal direction can only be solved iteratively.

TABLE 1—COMPUTED RESULTS OF MAXIMUM ALIGNMENT ERROR FOR R/a = 0.1

		r_0/a = 3.3* %	r_0/a = 2.0 %
Uniaxial		5.8	8.1
	One Gage	6.3	10.8
Hydrostatic	Two Gages	4.7	8.3

* The standard rosette (EA-09-062-RE-120).

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APPENDIX

The general expression of coefficients K_{11}, K_{12}, K_{21} and K_{22} in terms of (r_1/a), (β_1), (r_2/a) and (β_2) can be written as:

$$K_{\substack{11\\12}}(r_1/a,\beta_1) = \{[-\tfrac{1}{2} \pm (\frac{3}{2(r_1/a)^2}) - 2)(1-2\sin^2\beta_1)]\,[1-(1-\nu)\sin^2\beta_1] + [\tfrac{1}{2} \mp \frac{3}{2(r_1/a)^2}(1-2\sin^2\beta_1)]\,[-\nu - (1-\nu)\sin^2\beta_1]\} \,/\, [(r_1/a)^2(1-2\sin^2\beta_1)]$$

$$K_{\substack{21\\22}}(r_2/a,\beta_2) = \{[-\tfrac{1}{2} \mp (\frac{3}{2(r_2/a)^2} - 2)(1-2\sin^2\beta_2][1-(1-\nu)\sin^2\beta_2] + [\tfrac{1}{2} \pm \frac{3}{2(r_2/a)^2}(1-2\sin^2\beta_2)][-\nu-(1-\nu)\sin^2\beta_2)\} \,/\, [(r_2/a)^2(1-2\sin^2\beta_2)]$$

where r_1/a, β_1, r_2/a and β_2 are also functions of R/a, γ, and r_0/a based on the following equations:

$$(r_1/a)^2 = (R/a)^2 + (r_0/a)^2 - 2(R/a)(r_0/a)\cos\gamma$$

$$\sin^2\beta_1 = \frac{(R/a)^2\sin^2\gamma}{(R/a)^2+(r_0/a)^2-2(R/a)(r_0/a)\cos\gamma}$$

$$(r_2/a)^2 = (R/a)^2 + (r_0/a)^2 - 2(R/a)(r_0/a)\cos(90°-\gamma)$$

$$\sin^2\beta_2 = \frac{(R/a)^2\sin^2(90°-\gamma)}{(R/a)^2+(r_0/a)^2-2(R/a)(r_0/a)\cos(90°-\gamma)}$$