Fatigue-life Prediction Using Local Stress-Strain Concepts

The primary emphasis of this paper is on the mechanics of a computer algorithm for cumulative fatigue damage

by Darrell F. Socie

ABSTRACT—A cumulative-damage approach for predicting fatigue-crack initiation in engineering structures subjected to random loading is outlined. This procedure is based on the assumption that if the stresses and strains at the critical location in a structure can be related to the cyclic stress–strain properties of smooth laboratory specimens, the crack-initiation life in the structure will be the same as the specimen. A flow diagram, indicating the steps required for implementing this procedure on a high-speed digital computer, is discussed in detail.

List of Symbols

E = elastic modulus
K' = cyclic-strength coefficient
K_f = fatigue-notch factor
$2N_f$ = reversals to failure
ΔS = nominal stress range
b = fatigue-strength exponent
c = fatigue-ductility exponent
Δe = nominal strain range
n' = cyclic strain-hardening exponent
$\Delta\epsilon$ = local strain range
ϵ_f' = fatigue-ductility coefficient
$\Delta\sigma$ = local stress range
σ_o = mean stress
σ_f' = fatigue-strength coefficient

Introduction

The basic hypothesis of cumulative fatigue-damage analysis, employing materials data obtained from smooth laboratory specimens, is that *if* the local stresses and strains at the critical location of the component are known, the crack-initiation life of the structure can be related to the life of the specimen.

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Paper was presented at 1975 SESA Spring Meeting held in Chicago, IL on May 11-16.

Cumulative-damage analysis reduces the complex problem of fatigue into one of determining the *local* stresses and strains from operating data, usually in the form of nominal strains or loads, and the proper relationship between stresses, strains and fatigue life. Analytical techniques for this procedure require a great deal of repetitive calculations and bookkeeping, which makes it ideally suited for a digital computer. The primary emphasis of this paper is on the mechanics of a computer algorithm for cumulative fatigue damage. Also included is a brief review of the theoretical aspects of the analysis. Finally, the application of this procedure to the results of the SAE Fatigue Design and Evaluation Committee, Cumulative Damage Division test program[1] are discussed.

Strain–Life Properties

Fatigue resistance of metals can be characterized by a strain–life curve as shown in Fig. 1. These curves are determined from polished laboratory specimens which are tested under completely reversed strain control. The relationship between total-strain amplitude, $\Delta\epsilon/2$, and reversals to failure, $2N_f$, can be expressed in the following form:

$$\frac{\Delta\epsilon}{2} = \epsilon_f' \,(2\,N_f)^c + \frac{\sigma_f'}{E}\,(2\,N_f)^b \quad (1)$$

where:

σ_f' = fatigue-strength coefficient
b = fatigue-strength exponent
ϵ_f' = fatigue-ductility coefficient
c = fatigue-ductility exponent
E = elastic modulus

Morrow,[2] Tucker,[3] *et al.* provide definitions of these fatigue properties and tabulate values for a number of metals. Morrow[2] suggested that the strain–life equation could be modified to account for mean stress, σ_o, by reducing the fatigue-strength coefficient by an amount equal to the mean stress.

$$\frac{\Delta\epsilon}{2} = \epsilon_f' (2\,N_f)^c + \frac{\sigma_f' \pm \sigma_o}{E} (2\,N_f)^b$$

— tensile mean
+ compressive mean (2)

When the fatigue properties for a given metal are known and the service environment is defined, the complex problem of fatigue-life prediction becomes one of determining the local-strain amplitude and mean stress for each reversal so that eq (2) can be solved for life.

Fatigue-analysis Procedure

Basic aspects of the overall fatigue-analysis procedure are schematically shown in Fig. 2. In actual practice, the inputs to the analysis are a series of field loads or nominal strains that have been digitized into a sequence of peaks and valleys, the five fatigue properties previously discussed, and the geometric parameters describing the local stress and strain response of the component.

Load–Strain Conversion

In order to convert a load–time history to a strain–time history, a cyclic load–strain curve is used. This curve is analogous to a stress–strain curve. Barron,[4] Fig. 3, shows excellent correlation between a cyclic load–strain curve, obtained from a NASTRAN finite-element model, and experimental results for the SAE keyhole specimen. Since the load–strain curve of the component can be obtained analytically, the fatigue resistance of a component can be evaluated while it is at the 'drawing board' stage of development. Load–strain hysteresis loops behave qualitatively in the same manner as stress–strain hysteresis loops so that the memory features of behavior are preserved (i.e. the strain for a current reversal depends on prior deformation). A technique developed by Wetzel[5] for stress–strain response using an 'availability matrix' is utilized to convert load to strain, because it accurately describes the material memory effect.

The load–strain curve is divided into a number of small load and strain increments as shown in Fig. 4. A large number of increments (50-100) is used because each element is employed to the greatest extent (i.e., there is no interpolation to obtain a partial element). The following rules govern the way in which the elements are used.

1. Start with the largest load value in the spectrum (positive or negative) and determine the corresponding strain.
2. Set the availability coefficient matrix to plus one (+) if the peak is a maximum, and to minus one (−) if the peak is a minimum.
3. Double the load and strain elements using Masing's[6] hypothesis, which has been shown to be valid for aluminum and steels, that the cyclic-loading curve is similar to the initial-loading curve but magnified by a factor of two.
4. Elements are then assembled for each load reversal, using only those elements that have an availability coefficient of the opposite sign as the load reversal, until the control load is reached.

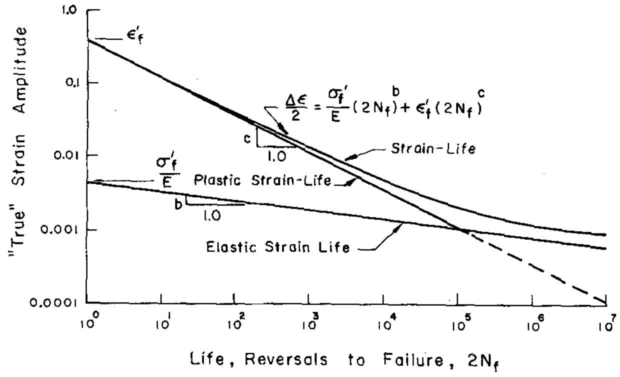


Fig. 1—Typical strain–life curve for mild steel

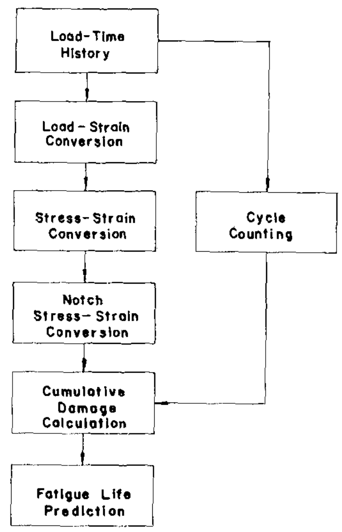


Fig. 2—Overall cumulative-fatigue-damage analysis procedure

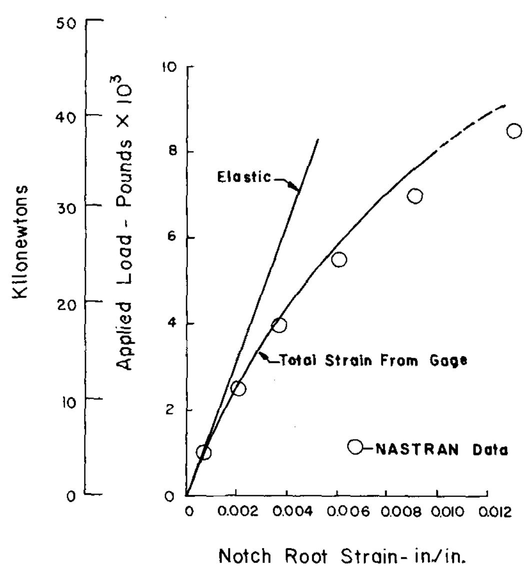


Fig. 3—Load vs. cyclic notch-root strain for the SAE keyhole specimen

A tensile load reversal (+) can only use elements that have an availability of minus one (−).

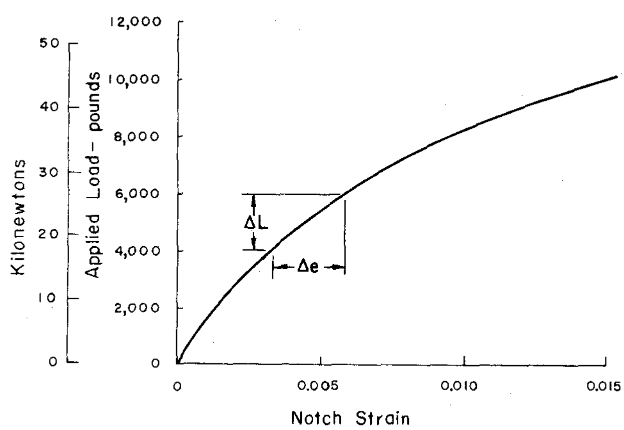
5. After an element has been used in tension, its availability coefficient is set to plus one. In a compressive reversal, the availability coefficient is set to minus one.

To illustrate the conversion procedure, consider the example shown in Fig. 5. The load, strain and availability coefficients, are initialized at 10,000 lb (4.45×10^4N), 0.015 and +1, respectively. Reversal AB uses four elements in compression. After this reversal, the strain is −0.005 and the availability coefficient for the first four elements is set to −1. In reversal BC, three tensile elements are used resulting in a strain of +0.002. Element 4 is skipped in reversal CD, because its availability sign is already negative with the resulting strain of −0.015. This is the same strain that would occur from a reversal going directly from A to D. All of the elements are now available for tensile deformation. Four elements are used in reversal DE and one element in reversal EF. Reversal FA uses two elements, 1 and 5. A closed path is followed in going from A to A, so the starting and ending strains are equal. In a real spectrum there will be round off errors because only full elements are used; however, these errors are at most one percent of full scale if 100 elements are used.

Stress–Strain Conversion

Strains are converted to stresses in exactly the same manner as loads were converted to strains. The cyclic stress–strain curve can be described by the following relation:

$$\frac{\Delta\epsilon}{2} = \frac{\Delta\sigma}{2E} + \left(\frac{\Delta\sigma}{2K'}\right)^{1/n'} \tag{3}$$



Element Matrix

Element	Load	Strain	ΔL	Δe
1	0	0	2,000	0.0015
2	2,000	0.0015	2,000	0.0020
3	4,000	0.0035	2,000	0.0025
4	6,000	0.0060	2,000	0.0040
5	8,000	0.0100	2,000	0.0050
6	10,000	0.0150		

Fig. 4—Load-strain curve and element matrix

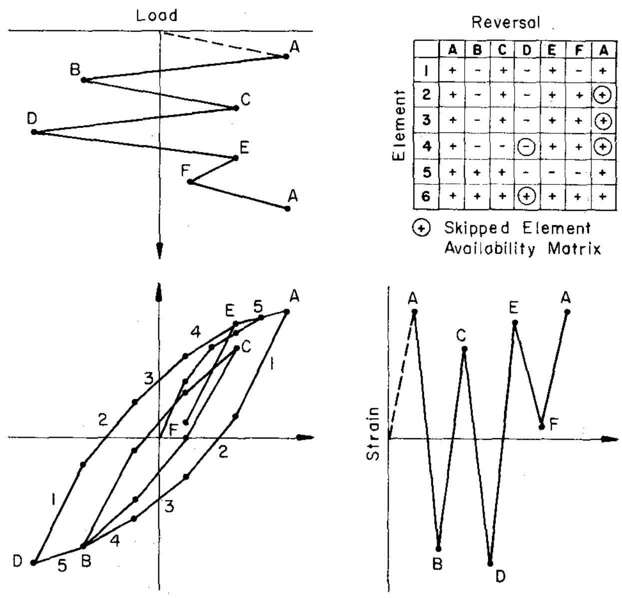


Fig. 5—Load–strain conversion procedure

where:

K' = cyclic-strength coefficient

n' = cyclic-strain hardening exponent

It appears that two additional material properties are introduced into the analysis; however, the following relationships exist between cyclic stress–strain and strain-life properties:

$$K' = \frac{\sigma_f'}{\epsilon_f^{n'}} \qquad n' = \frac{b}{c} \tag{4}$$

Equation (3) is used to calculate the stress that corresponds to each strain element that was previously determined. Since there is no explicit solution, the Newton-Raphson iteration technique is used to solve this expression.

Each time a strain element is used in assembling the load–strain curve, the corresponding stress element is used to assemble the load–stress curve. In the example problem, the stress for reversal CD would consist of elements 1, 2, 3 and 5 with element 4 being skipped. The time history is only processed once to obtain both the stress and strain. Considerable computer time is therefore conserved when processing long histories.

Notch Stress–Strain Conversion

Neuber's rule as modified by Topper,[7] has been shown to be an effective method for relating nominal stresses and strains to the local stresses and strains at the root of a notch. It can be mathematically expressed in the following form:

$$K_f(\Delta S\, \Delta e)^{1/2} = (\Delta\sigma\, \Delta\epsilon)^{1/2} \tag{5}$$

where:

ΔS, Δe = nominal stress and strain range

TABLE 1—COMPLETED ELEMENT MATRIX FOR EXAMPLE PROBLEM

Element	Load	Nominal Strain	Nominal Stress	Notch K_f = 1.1 Strain	Notch K_f = 1.1 Stress
0	0	0	0	0	0
1	2000	.0015	34,800	.0017	37,100
2	4000	.0035	49,000	.0340	51,300
3	6000	.0060	57,400	.0070	59,800
4	8000	.0100	65,500	.0117	68,000
5	10000	.0150	72,100	.0175	74,700

Stress in psi
1 psi = 6.895 × 10³ Pa

$\Delta\sigma$, $\Delta\epsilon$ = notch stress and strain range
K_f = fatigue-notch factor

There is a one-to-one correspondence between nominal and notched closed hysteresis loops (i.e., for each closed loop of amplitude, ΔS and Δe, there is one and only one closed hysteresis loop of amplitude, $\Delta\sigma$ and $\Delta\epsilon$, at the root of the notch). The element matrix already contains ΔS and Δe, so that $\Delta\sigma$ and $\Delta\epsilon$ can be obtained by solving a combination of eqs (3) and (5). A completed element matrix for load–strain, stress–strain and nominal to notch stress–strain conversion is shown in Table 1. The elements in Table 1 represent the total stress or strain (i.e., element 5 is the total stress or strain for elements 1 through 5). Incremental elements, such as the ones used in the example, are obtained by subtracting adjacent elements in the matrix.

One of the difficulties in using the Neuber–Topper rule has been the constant accumulation of error which results from defining a new origin at each reversal point. Equation (5) is of the form of a rectangular hyperbola so that the associative laws of addition do not apply. In Fig. 5, ΔS and Δe from A to B plus ΔS and Δe from B to D is not equal to ΔS and Δe from A to D if a new origin was defined at each reversal. The element-matrix type of approach does not have this type of error because Neuber–Topper rule is only used to scale closed hysteresis loops and the mean stress is referenced to the applied load rather than the previous reversal.

Cycle Counting

Common cycle counting techniques in use today are peak, range, range-pair and rainflow. Of these various methods, rainflow has been shown to be superior and yields the best fatigue-life estimates.[8] The apparent reason for this is that rainflow counting defines a series of closed stress–strain hysteresis loops when the spectrum starts and ends with the largest value and the constitutive relations between stress and strain are defined as was done above. Any counting technique that counts closed hysteresis loops is equivalent to rainflow. Referring again to Fig. 5, one finds three closed hysteresis loops, AD, BC and EF. Each time a hysteresis loop is closed, an element in the availability matrix is skipped. For example, in reversal CD element 4 was skipped, which indicates that a closed loop was formed consisting of elements 1, 2 and 3. In reversal FA elements 2, 3 and 4 were skipped, indicating a closed loop consisting only of element 1. The rule for cycle counting is that a closed hysteresis loop is formed whenever one or more elements in the availability matrix is skipped. There are no unused elements in a closed hysteresis loop, so that the stress and strain for closed loop BC correspond to the stress and strain for element 3 in the element matrix. Mean stress is obtained by keeping a running total of the stress after each reversal and adding or subtracting one-half of the closed-loop stress, depending on whether the loop was closed on a tensile or compressive reversal.

In the computer program, load can be directly converted to notch stresses and strains because of the structure of the element matrix. Another advantage of this technique is that several sets of fatigue properties and notch factors can be evaluated by a single processing of the time histories.

Damage Calculation

In previous sections, techniques for determining the stress and strain of each reversal have been discussed. The two variables, strain amplitude and mean stress, are used in eq (2) to solve for reversals to failure. An explicit solution to this equation does not exist due to the exponents. However, the Newton–Raphson iteration technique has fairly rapid convergence. Damage for each reversal is the reciprocal of the reversals to failure

$$\text{Damage} = \frac{1}{2\,N_f} \tag{6}$$

A closed hysteresis loop contains two reversals, so the damage is twice the damage of a single reversal. Miner's linear-damage rule[9] is used to predict failure. Total damage per data block is the linear sum of the damage for each reversal. Blocks to failure is the reciprocal of the total damage. Appendix A contains a Fortran IV algorithm for this damage analysis.

Applications

This procedure is used to evaluate the results of the SAE Cumulative Damage test program. Three different types of load histories, Fig. 6, are applied to the test specimen shown in Fig. 7. Two steels are used, U.S. Steel's Man-Ten and Bethlehem's RQC-100.

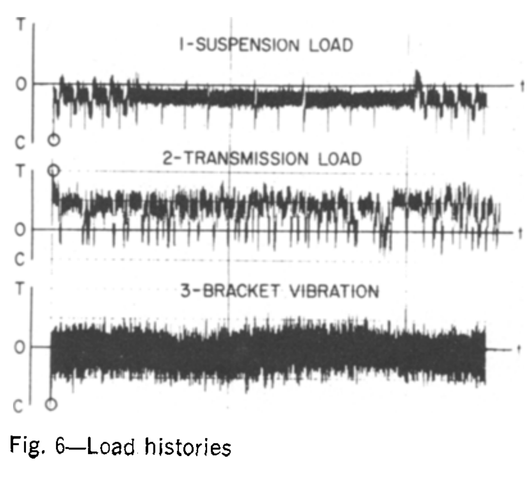


Fig. 6—Load histories

Tests are conducted at several load levels for each spectrum which result in fatigue lives that range from 10^4 to 10^9 reversals. A complete description of the test program is found in Ref. 1.

A summary of the predicted and actual crack-initiation lives is shown in Fig. 8. These predictions are made using the cyclic load–strain curve, shown in Fig. 4, for Man-Ten and a similar curve for RQC-100. No notch factors are used. Materials properties, shown in Table 2, from Ref. 1 are employed. For perfect correlation, all the data points should lie along the 45-deg line. All but four of the predicted lives are within a factor of three of the actual data. This agreement is quite good considering there are two steels, three types of load histories and, at least, three different load levels.

To gain insight into the potential variability of the prediction technique, the effect of material properties, mean stress and fatigue-notch factor are investigated. Three different sets of published material properties for Man-Ten and RQC-100, shown in Table 2, are used for life predictions. Predicted and actual fatigue lives for the RQC-100 steel, subjected to the transmission spectrum loads, are shown in Fig. 9. The vertical scale is in pounds and represents the maximum load in the spectrum. All other loads are scaled to this maximum. The horizontal scale is in terms of data blocks. For the transmission history, one data block is equivalent to 1708 reversals. Similar results were also obtained for Man-Ten steel and the other two load histories. At longer lives, material variability can account for a factor of five in fatigue life for the material properties and spectra used in this analysis. Material properties are, therefore, an extremely important part of the analysis and care must be exercised when obtaining properties from a handbook or data bank.

When the effects of mean stress are neglected, the analysis reduces to a strain-based approach. Equation (1) is used to calculate damage when mean stress is not considered. Mean stress can account for a factor of three at longer lives in this type of analysis for these spectra as shown in Fig. 10. At shorter lives, there are minor mean-stress effects because of cyclic plasticity. The effect of mean stress is not as important, at least for these spectra, as the difference in material properties.

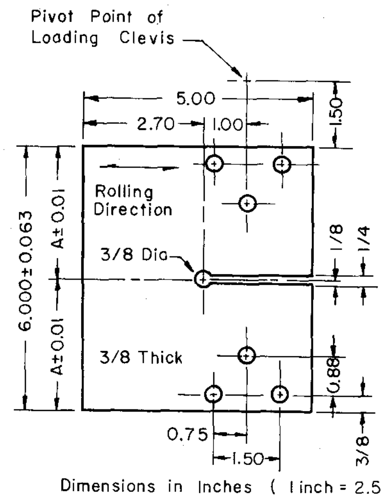


Fig. 7—Sketch of SAE keyhole specimen

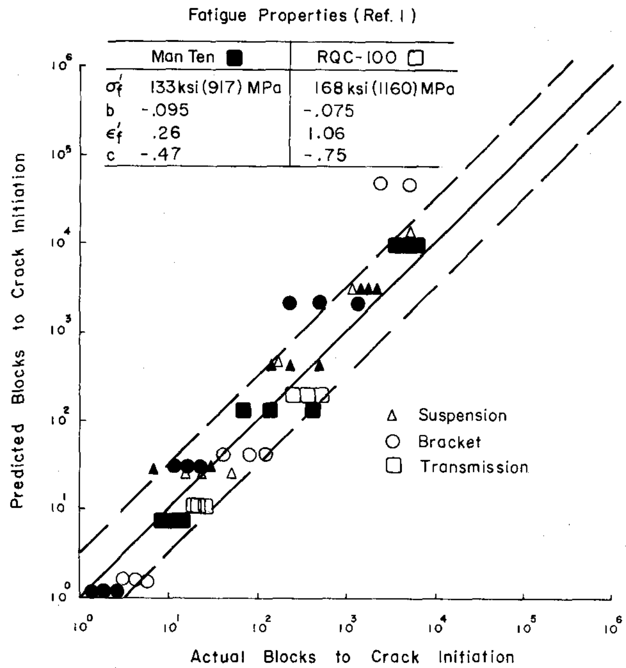


Fig. 8—Actual and predicted blocks to failure for SAE test program

TABLE 2—FATIGUE PROPERTIES USED IN THIS ANALYSIS

Man-Ten			
Reference	1	11	10
σ_f'	133 ksi (917 MPa)	170 ksi (1170 MPa)	155 ksi (1070 MPa)
b	−.095	−.12	−.11
ϵ_f'	.26	.9	1.0
c	−.47	−.6	−.61
RQC-100			
Reference	1	2	10
σ'_f	168 ksi (1160 MPa)	180 ksi (1240 MPa)	200 ksi (1380 MPa)
b	−.075	−.07	−.094
ϵ_f'	1.06	.66	1.00
c	−.75	−.69	−.75

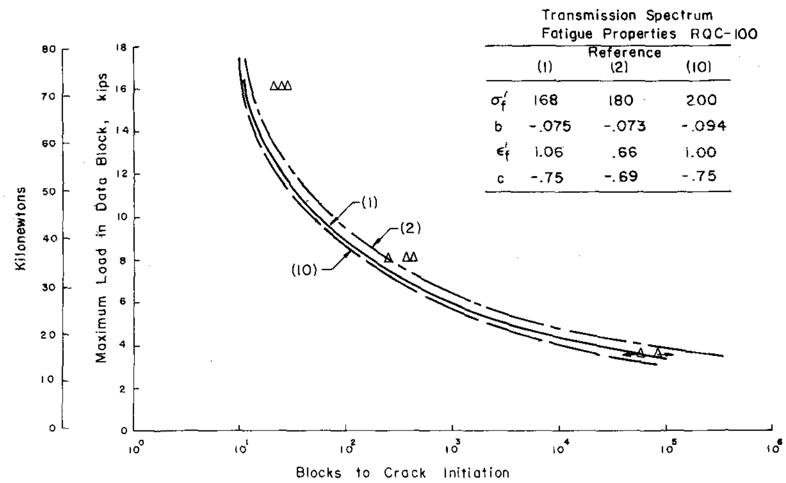


Fig. 9—Effect of material properties on life prediction

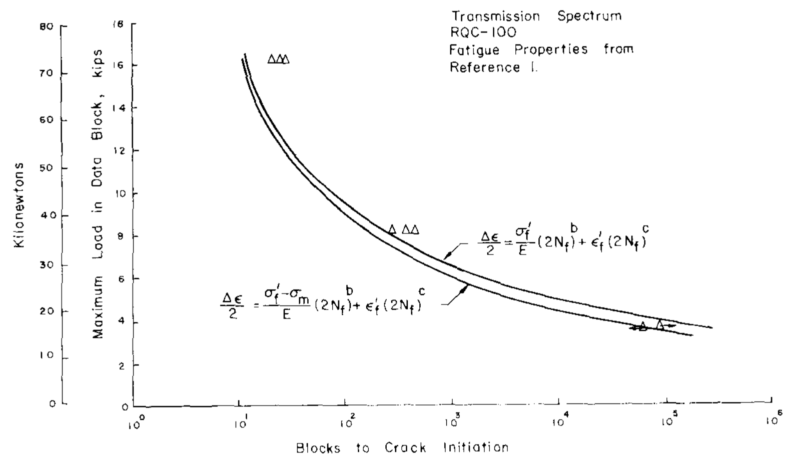


Fig. 10—Effect of mean stress on life prediction

It is well known that surface finish has an effect on fatigue life. Cracks initiated from a machined surface in the test specimen and no surface correction is required. However, in many practical applications the effect of surface finish cannot be neglected. Lipson has suggested that surface finish could be accounted for by introducing a fatigue-notch factor that varies from 1.0 (for a ground and polished surface) to 2.0 (for a forged material). This notch factor is also dependent on hardness, environment and other related variables. The results for a notch factor of 1.1 are shown in Fig. 11. A factor of two-to-three between lives, with and without the notch factor or surface-finish correction, is noted at long lives. The selection of a fatigue-notch factor is an important part of the analysis.

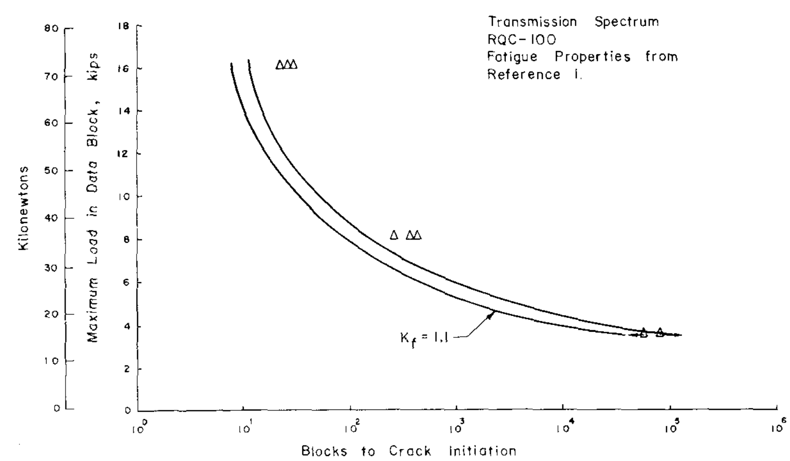


Fig. 11—Effect of notch factor on life prediction

Summary and Conclusions

1. A simple computer algorithm for cumulative-fatigue-damage analysis has been presented. The basic algorithm, listed in the Appendix, can be expanded for multiple data input and histograming to suit the individuals needs.
2. Fairly accurate fatigue-life predictions can be obtained for variable-amplitude loading spectra, provided considerable care is taken in the selection of material properties.
3. Accurate determination of the local stresses and strains is the most important part of the analysis as demonstrated by the large effect a small notch factor has on fatigue life.

Acknowledgments

The author is grateful to M. R. Mitchell and J. Morrow, University of Illinois, for their review of the manuscript.

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APPENDIX
Fortran Listing of Cumulative-damage Program

```
      DIMENSION G(6ØØØ)
      REAL*8 DMG(2),DM
1     FORMAT(F1Ø.Ø)
2     FORMAT(F8.Ø)
3     FORMAT(F5.Ø)
4     FORMAT(2I5)
5     FORMAT(2F1Ø.Ø)
6     FORMAT(I5)
7     FORMAT(14F5.Ø)
8     FORMAT(' BLOCKS TO FAILURE,NOMINAL',1PE1Ø.3,/
     1 ' BLOCKS TO FAILURE,NOTCH',1PE1Ø.3)
C *********
C ********* READ IN MATERIAL PROPERTIES
C *********
      READ(5,1)ELAS
      READ(5,2)SF
      READ(5,3)B
      READ(5,3)EF
      READ(5,3)C
      READ(5,3)AKF
      AN=B/C
      SCC=SF/EF**AN
C **********
C ********** READ AND FORM LOAD STRAIN ELEMENTS
C *********
      READ(5,4) LI,LSI
      NI=(LI-1)*LSI+1
      DO 1ØØ I=1,5
      E(1,I)=Ø.
1ØØ    CONTINUE
      DO 1Ø1 I=1,NI,LSI
      READ(5,5) AL,ET
      DAL=(AL-E(I,1))/FLOAT(LSI)
      DET=(ET-E(I,2))/FLOAT(LSI)
      DO 1Ø1 J=1,LSI
      E(I+J,1)=E(I+J-1,1)+DAL
      E(I+J,2)=E(I+J-1,2)+DET
1Ø1   CONTINUE
C *********
C ********* FORM STRESS STRAIN ELEMENTS
C *********
      NI=NI+LSI
      DO 11Ø I=2,NI
      SIG=E(I-1,3)
111   Y1=E(I,2)-SIG/ELAS-(SIG/SCC)**(1./AN)
      Y2=-1./ELAS-(1./AN)*(SIG/SCC)**(1./AN-1.)/SCC
      SIG=SIG-Y1/Y2
      IF(ABS(Y1).GT.Ø.Ø1*E(I,2)) GO TO 111
      E(I,3)=SIG
11Ø   CONTINUE


C *********
C ********* FORM NEUBER ELEMENTS
C *********
      DO 12Ø I=2,NI
      ANEUB=E(I,2)*E(I,3)*AKF**2
      SIG=E(I,3)
121   Y1=ANEUB-SIG**2/ELAS-SIG**(1./AN+1.)/SCC**(1./AN)
      Y2=-2.*SIG/ELAS-(1./AN+1.)*SIG**(1./AN)/SCC**(1./AN)
      SIG=SIG-Y1/Y2
      IF(ABS(Y1).GT.Ø.Ø1*ANEUB) GO TO 121
      E(I,5)=SIG
      E(I,4)=SIG/ELAS+(SIG/SCC)**(1./AN)
12Ø   CONTINUE
C *********
C ********* READ AND SCALE DATA LARGEST VALUE LAST
C *********
      READ(5,5) SCALE
      READ(5,6) NPEAKS
      READ(5,7)(A(I),I=1,NPEAKS)
      DO 2ØØ I=1,NPEAKS
      A(I)=A(I)*SCALE/999.
2ØØ   CONTINUE
C *********
C ********* INITALIZE AVAILABILITY AND STRESS
C *********
      SGN=A(NPEAKS)/ABS(A(NPEAKS))
      DO 21Ø I=1,NI
      IF(ABS(A(NPEAKS)).LE.E(I,1)) GO TO 211
21Ø   CONTINUE
211   S(1)=E(I,3)*SGN
      S(2)=E(I,5)*SGN
      ALD=E(I,1)*SGN
      DO 212 I=1,NI
      AB(I)=SGN
212   CONTINUE
      DMG(1)=Ø.
      DMG(2)=Ø.
C *********
C ********* CALCULATION LOOP
C *********
      DO 999 J=1,NPEAKS
      SGN=(A(J)-ALD)/ABS(A(J)-ALD)
      I=1
3ØØ   I=I+1
      ALD=ALD+(E(I,1)-E(I-1,1))*(SGN-AB(I))
      S(1)=S(1)+(E(I,3)-E(I-1,3))*(SGN-AB(I))
      S(2)=S(2)+(E(I,5)-E(I-1,5))*(SGN-AB(I))
      AB(I)=SGN
      IF(AB(I+1).EQ.SGN) GO TO 4ØØ
      GO TO 399


C *********
C ********* DAMAGE CALCULATION
C *********
4ØØ   DO 41Ø K=1,2
      SM=S(K)-E(I,K*2+1)*SGN
      EST=(E(I,K*2+1)/(SF-SM))**(1./B)
4Ø1   Y1=E(I,K*2)-EF*EST**C-(SF-SM)/ELAS*EST**B
      Y2=-C*EF*EST**(C-1.)-B*(SF-SM)/ELAS*EST**(B-1.)
      EST=EST-Y1/Y2
      IF(EST.LT.1.) EST=1.
      IF(ABS(Y1).GT.Ø.Ø1*E(I,K*2)) GO TO 4Ø1
      DM=DBLE(2./EST)
      DMG(K)=DMG(K)+DM
41Ø   CONTINUE
411   I=I+1
      IF(SGN.EQ.AB(I)) GO TO 411
      I=I-1
399   IF((A(J)-ALD)*SGN.GT.Ø.Ø) GO TO 3ØØ
999    CONTINUE
C **********
C ********** OUTPUT BLOCKS TO FAILURE
C **********
      DMG(1)=1./DMG(1)
      DMG(2)=1./DMG(2)
      WRITE(6,8) DMG(1),DMG(2)
      STOP
      END
```