Constitutive Modeling of Plastic-bonded Explosives

Two analytical models are developed for high-explosive materials and the predictions of these models are compared with experimental results

by R.L. Peeters and R.M. Hackett

ABSTRACT—Typically, elastic and elastic-plastic theory are used in structural-analysis computer programs to model the mechanical behavior of high explosives; these models, however, do not fit the observed behavior of plastic-bonded explosives. This paper discusses the development of an equation-of-state creep model and a linear viscoelastic model for the analysis of these material systems and shows comparisons between experimental results and analytical-model predictions.

Symbols

a_0 through a_6 = material parameters
c = subscript referring to uniaxial creep
F, R, G = derived constants
g_0, g_1, g_2 = functions of stress at constant temperature
J = creep compliance, 1/MPa
t = time, h
a_σ = shift factor
s, τ = generic times
α, β = rate-dependent coefficients
δ_1 through δ_5 = prony-series time constants
ϵ = strain
ξ_σ, ξ_σ' = reduced time, h
σ = stress, MPa
ϱ, ϱ_1 through ϱ_6 = prony-series coefficients
ϕ = creep function, MPa

Introduction

In applications involving the use of high-explosive (HE) materials, recent design emphasis has resulted in plastic-bonded explosives serving as structural components. The use of HE as load-bearing members requires the assessment of the state of stress and strain in these components. The objective of this research is to improve the constitutive models for high-explosive materials.

The first step in this effort has been to develop a capability to properly test HE materials. Accurate determination of the mechanical properties of plastic-bonded explosives is difficult because of their physical characteristics. For example, the high-solids loading with a plastic binder results in a stiff quasi-brittle polymeric material. The mechanical behavior is characterized by small strain capability combined with a creep-like response.

This paper addresses the room-temperature stress-strain characterization of an inert plastic-bonded material, with particular attention given to its creep and recovery behavior. Three accepted methods of modeling this type of time-dependent behavior are: (1) the phenomenological or equation-of-state creep theory, (2) the memory or hereditary theory, and (3) the nonlinear viscoelastic approach. The constitutive models developed in this paper are based upon methods 1 and 3.

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Test Specimens

The material reported on in this study is an inert plastic-bonded material known as 900-10. It is frequently used as a structural substitute for explosives in environmental tests because it displays similar mechanical behavior. This material contains 46-percent barium nitrate, 48-percent pentaerythritol, 2.8-percent cellulose nitrate, 3.2-percent tris beta chloroethyl phosphate, and .05-percent red dye by weight. It is produced by first being formed into a molding powder in a Baker-Perkins sigma-blade mixing machine. Next, the powder is preheated to 90°C and pressed to a cylindrical shape in a steel die at 140 MPa. The final specimens are machined from these cylinders to the desired dimensions. For tensile tests, the active length is a right circular cylinder 12.2 mm in diameter and 39.4 mm long. The compression and creep specimens are also right circular cylinders with diameters of 41.4 mm and 15.9 mm, respectively, and heights of 46.0 mm and 44.5 mm, respectively. The creep specimens are carefully machined and checked to assure that their ends are parallel.

Experimental Procedures

Creep data were obtained using a Luster-Jordan cold-flow testing device (designed in accordance with ASTM D-621), a directly linked data-acquisition system, MTS extensometers, and an Endevco strain-gage conditioning unit. The Hewlett-Packard (H-P) 3052A automatic data-acquisition system, with peripheral disk units, printer and plotter, proved to be a crucial component of the apparatus. Not only did this system provide great flexibility in the postprocessing of experimental data, but it also eliminated the use of amplifiers and their inherent drift in the load and extensometer output signals. Two extensometers, spaced 180 deg apart, are attached to the specimen's axial direction and provide a check on the uniformity of the axial-strain field. The effect of any bending strain present is minimized by using the average of the two extensometer outputs. To reduce off-axis loading components, extreme care is taken to assure parallel platens

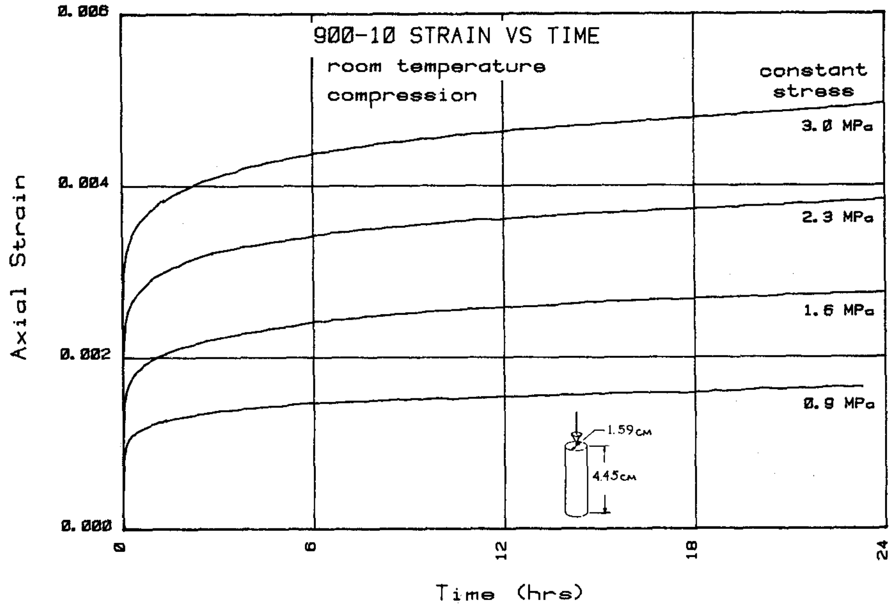

Fig. 1—Uniaxial creep vs. time for 900-10

in the cold-flow testing device, thus matching as closely as possible the parallel-ended creep specimens.

Stress–strain behavior of the material under study was determined using a 44,000-N load frame in conjunction with the same instrumentation discussed above. For tensile tests, off-axis loading components are reduced through the use of a gimbaled ball fixture at each end of the specimen. For compression tests the load-frame platens are shimmed to compensate for nonparallel specimen ends.

Experimental Data

A set of creep tests was conducted for the purpose of developing simple creep and viscoelastic-characterization models. Figure 1 shows the results of room-temperature creep tests for 900-10 as a function of various constant stress levels. These creep data are normalized to a unit stress and replotted vs. log time in Fig. 2, resulting in the creep compliance curves as shown by the solid lines. The average creep compliance, shown as a dashed line in the same figure, is the compliance used in the mathematical models. Room-temperature stress–strain results, in both tension and compression, as a function of strain rate are shown in Fig. 3. A pronounced stiffening with rate is observed in both loading quadrants.

In order to gain further insight into the creep behavior and linearity of this material, a cyclic-loading test was conducted. This test consisted of a triangular-shaped loading at a constant strain rate, followed by a 1-h rest phase. This cycle was repeated for the original stress value, and then at multiples of 2 and 4 times the initial stress value. The final cycle was loaded to failure at the same constant strain rate. The results of this room-temperature test are compared in Fig. 4 with the constant rate-to-failure result from Fig. 3. Stress-relaxation tests were also conducted at room temperature and are reported in Fig. 5. In this test, the specimen was loaded at a constant strain rate to an initial stress value and held at nearly constant strain for 2 h. Next, the load was incremented to achieve twice the original stress level and held for 20 h followed by 4 times the initial stress level held for 2 h. Finally, the specimen was loaded to failure at the same constant strain rate. The amount of stress relaxation is a strong function of the level of stress present in the specimen. This effect is clearly seen by comparing the drop in stress during the last 2-h relaxation period with the somewhat smaller stress change during the previous 20-h relaxation period. Also shown in Fig. 5, for comparative purposes, is the constant rate-to-failure result from Fig. 3.

The mechanical behavior of 900-10 is very similar to a plastic-bonded explosive known as PBX 9501. A previous paper[1] describes its stress–strain behavior.

Creep-model Formulation

Initial studies centered on the equation-of-state creep theory. Creep models of this type are widely used in nuclear-reactor technology[2] to predict the deformation, under sustained loading of metal structures subjected to high temperatures. Such models exist in numerous nonlinear finite-element-computer codes, an example being the ADINA (Automatic Dynamic Incremental Nonlinear Analysis) code.[3] Two different one-dimensional creep models are contained in ADINA and have the following mathematical forms:

$$\epsilon_c = a_0 \sigma^{a_1} t^{a_2} \tag{1}$$

where ϵ_c is the uniaxial creep strain, σ is the uniaxial stress at a sustained load, t is the time since the application of the sustained load, and a_0 through a_2 are material parameters obtained from a series of uniaxial creep tests conducted at different stress levels; and

$$\epsilon_c = F[1 - \exp(-RT)] + Gt \tag{2}$$

where

$$F = a_0 \exp(a_1, \sigma) \tag{3a}$$

$$R = a_2(\sigma/a_3)^{a_4} \tag{3b}$$

$$G = a_5 \exp(a_6 \sigma) \tag{3c}$$

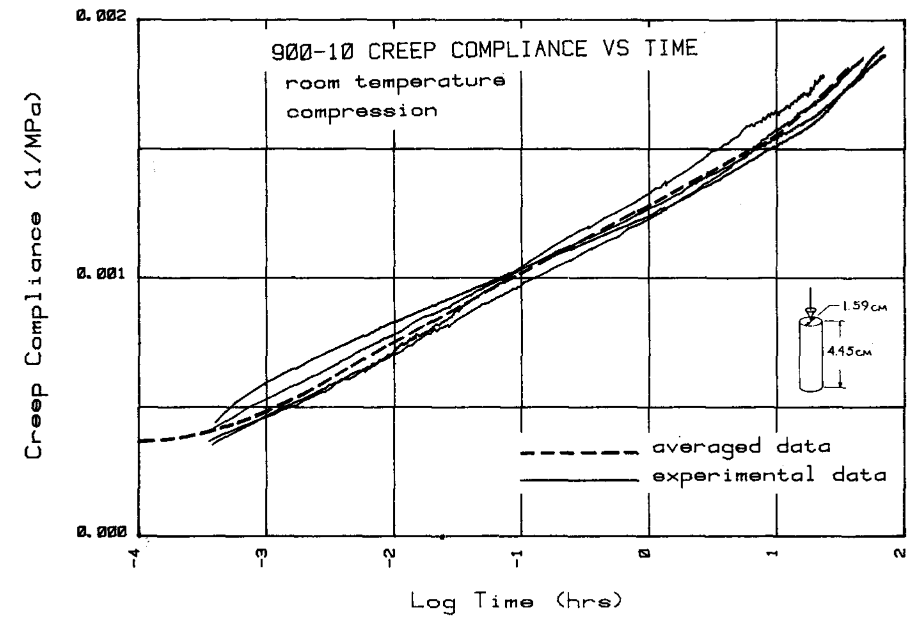

Fig. 2—Uniaxial creep compliance vs. log time for 900-10

where a_0 through a_6 are material parameters. The ADINA code also provides for the inclusion of a user-developed creep model. Using the creep data shown in Fig. 1, an attempt was made to model the response of the 900-10 inert material using eqs (1) and (2). In order to efficiently perform the necessary computations, a finite-element computer-analysis package for the H-P 9825A calculator, employing the flexible disk drive, printer and plotter, was developed. The program uses the 4-node, isoparametric, axisymmetric, quadrilateral element and the same elastic-plastic thermal-creep analysis found in the ADINA code. For simplicity, the initial analyses were limited to elastic, rather than elastic–plastic, material behavior.

A preliminary investigation indicated that eqs (1) and (2) could not be successfully employed in the ADINA program to model this material. The parameter a_2 from eq (1), as calculated from the creep data, had a value less than unity, which is not permissible in the creep-strain-

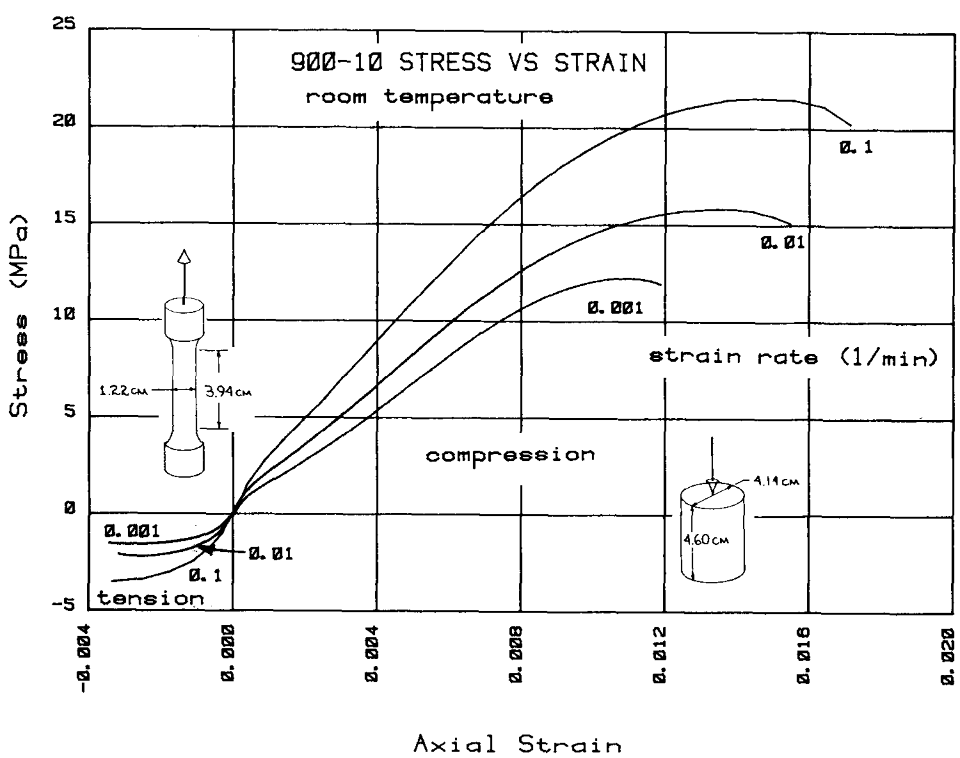

Fig. 3—900-10 uniaxial tension and compression as a function of strain rate

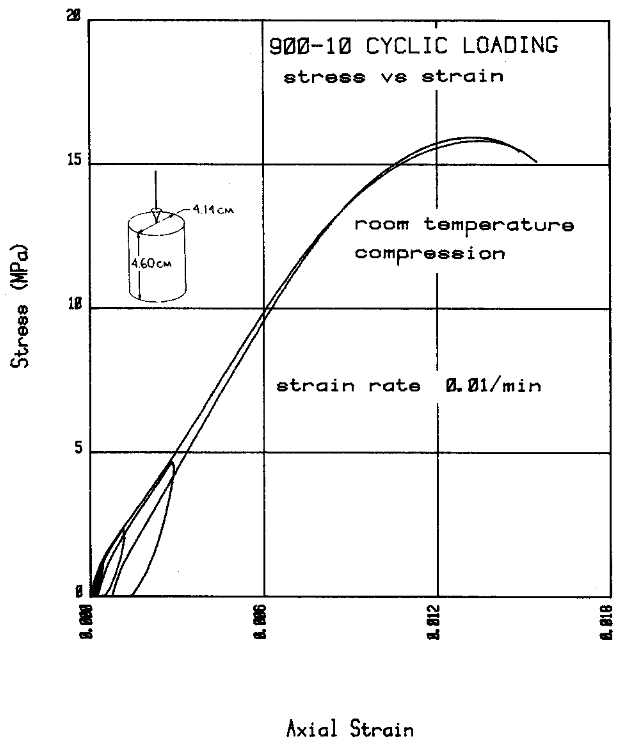

Fig. 4—900-10 cyclic-loading test

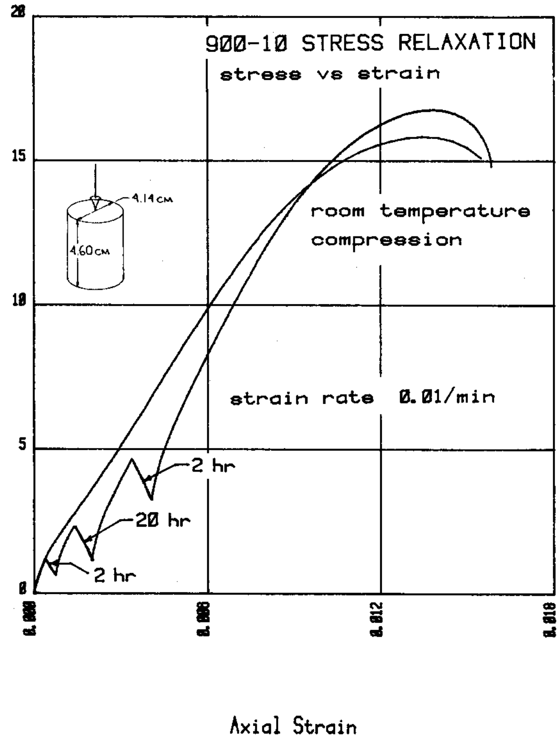

Fig. 5—900-10 stress-relaxation test

rate calculation step in ADINA. It was also found that eq (2) could not be provided with values of the constants a_0 through a_6 that would result in an adequately close fit of the creep data.

An attempt to develop a new creep model that could be inserted into the ADINA code was then undertaken. This model was based upon the creep results shown in Fig. 1. In order to test the validity of the model, it was employed in the H-P finite-element program, and an attempt was made to reproduce the compression results for the three curves of constant strain-rate-to-failure shown in Fig. 3. The developed model is expressed in the form:

Fig. 6—Creep-model prediction for 900-10

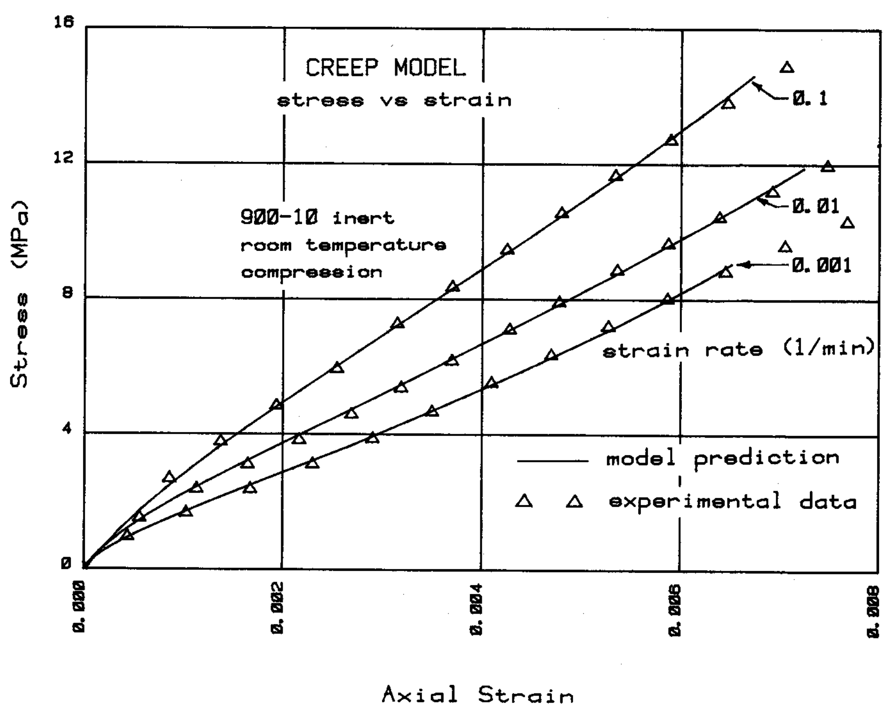

TABLE 1—RATE-DEPENDENT PARAMETRIC VALUES

	Strain Rate		
	0.001/min	0.01/min	0.1/min
α	14	80	400
β	1.16	4.64	19.14

$$\epsilon_c = a_0\sigma^{a_1}\,[1 - \exp(-a_2 t)] + a_3 t \qquad (4)$$

where a_0 through a_3 are material parameters. In applying this model, it was found that values of 1.843×10^{-3}, 0.7 and 7.5×10^{-5} for a_0, a_1 and a_3, respectively, were satisfactory but that the parameter a_2 was rate sensitive. This sensitivity is expressed in the following form:

$$a_2 = \alpha - \beta\sigma \qquad (5)$$

The rate dependence of the coefficients α and β is shown in Table 1.

Figure 6 compares the predicted stress-strain behavior using this creep model with the experimental data and excellent agreement is observed. In the H-P program, the values of the initial modulus of elasticity and Poisson's ratio were determined from prior testing to be 4137 MPa and 0.36, respectively. The analysis was also based upon the assumption of incompressible creep strains, consistent with the formulation in the ADINA code.

Viscoelastic-model Formulation

Additional effort was expended on the development of viscoelastic models. This effort was partly based upon the equation-of-state creep models' inability to predict recovery behavior. Because the degree of linearity of the material studied was unknown, a generalized nonlinear approach was considered. Nonlinear viscoelastic theory is highly formulated,[4] but applied with difficulty. It was, therefore, felt that the initial attempt should be to formulate a linear viscoelastic model that would provide the basis for a future nonlinear model.

The nonlinear viscoelastic constitutive theory developed by Schapery[5] was derived by the use of principles of the thermodynamics of irreversible processes. In this formulation, the nonlinearity is contained in a reduced time, which is an implicit function of stress. It is based, in part, on the observation that the nonlinear stress relaxation of various materials can be described in terms of the same time-dependent properties found in the linear range. The formulated expression is:

$$\epsilon(t) = g_0 J_0 \sigma(t) + g_1 \int_0^t \phi(\xi_\sigma - \xi_\sigma')\,\frac{dg_2\sigma(\tau)}{d\tau}\,d\tau\,. \qquad (6)$$

This equation is employed by Schapery under conditions of constant temperature and uniaxial stress σ, where

$$\xi_\sigma = \xi_\sigma(t) = \int_0^t \frac{ds}{a_\sigma[\sigma(s)]} \qquad (7a)$$

and

$$\xi_\sigma' = \xi_\sigma(\tau) = \int_0^\tau \frac{ds}{a_\sigma[\sigma(s)]} \qquad (7b)$$

are reduced times and a_σ is a shift factor. The terms g_0, g_1, g_2, and a_σ are functions of stress at constant temperature. The terms J_0 and $\phi(t)$ are the time-independent creep compliance and the time-dependent creep function, respectively, and s and τ are generic times. None of the parameters in eq (6) are fundamental physical or thermodynamic constants and, therefore, all must be determined from creep, relaxation or other mechanical tests of the

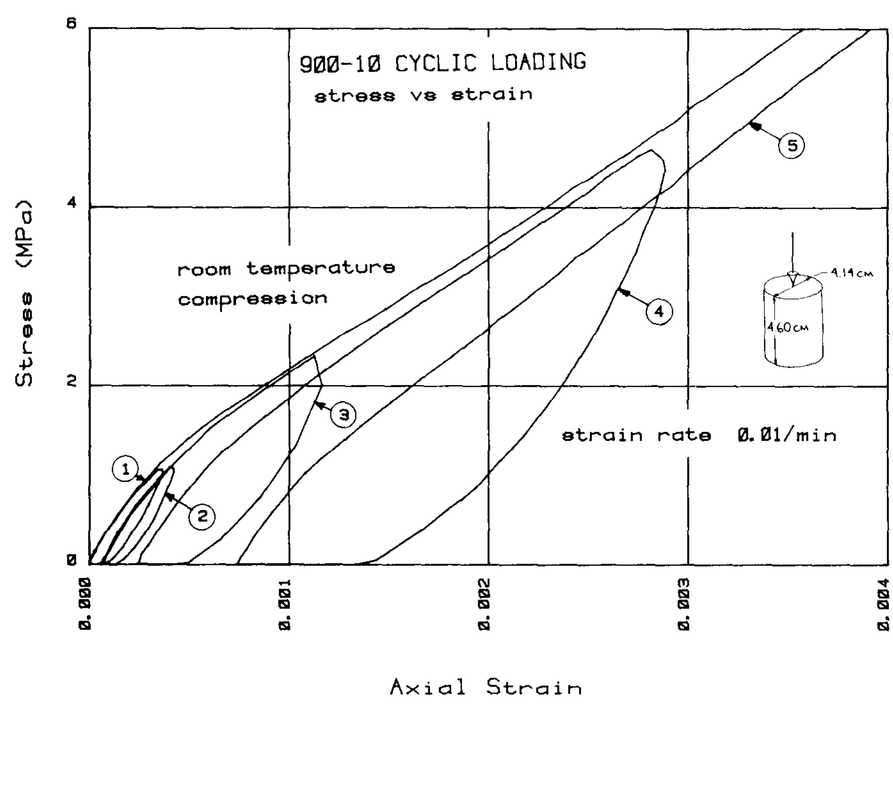

Fig. 7—900-10 cyclic-loading test enlargement

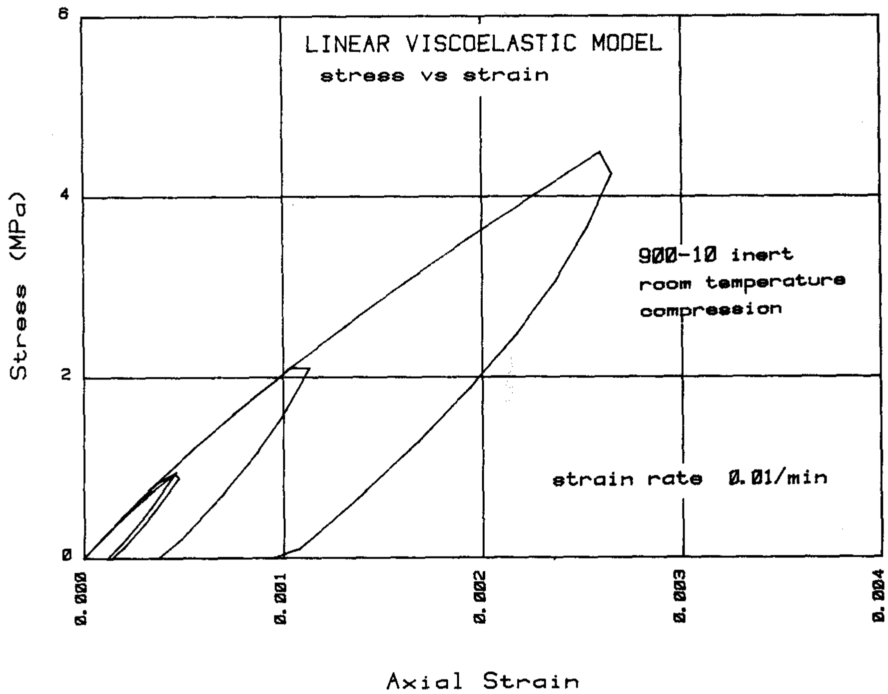

Fig. 8—Linear viscoelastic-model prediction for 900-10

material being considered. For linear viscoelastic behavior, eq (6) reduces to the expression;

$$\epsilon(t) = \int_0^t J(t-\tau)\frac{\partial\sigma(\tau)}{\partial\tau}d\tau \qquad (8)$$

In developing an expression for $J(t)$, the test data shown in Fig. 2 were fit using a prony-series resulting in

$$J(t) = \varrho + \varrho_1\exp(-\delta_1 t) + \varrho_2\exp(-\delta_2 t) + \varrho_3\exp(-\delta_3 t) + \varrho_4\exp(-\delta_4 t) + \varrho_5\exp(-\delta_5 t) + \varrho_6 t \qquad (9)$$

where, $\varrho = 3.074 \times 10^{-3}$, $\varrho_1 = -1.622 \times 10^{-3}$, $\varrho_2 = -6.567 \times 10^{-5}$, $\varrho_3 = -3.406 \times 10^{-4}$, $\varrho_4 = -1.983 \times 10^{-4}$, $\varrho_5 = -5.126 \times 10^{-4}$, $\varrho_6 = -3.568 \times 10^{-6}$, with units of 1/MPa, and the values of δ_1 through δ_5 preset to 0.01, 0.1, 1.0, 10.0 and 100.0.

The linear viscoelastic model using this expression for $J(t)$ was then programmed for the H-P calculator. The results of the cyclic-loading test shown in Fig. 4 were then predicted using this model. Figure 7 shows an enlarged plot of the experimental results for the cyclic-loading test, and Fig. 8 shows the linear viscoelastic model's prediction for this test. The analytical prediction is quite similar to the data, the major exception being the model's inability to predict the strain offset occurring after each cycle.

Summary and Conclusions

Two exploratory time-dependent constitutive models have been formulated. Complete details of these material models, as applied to the 900-10 inert material, are found in Ref. 6. The excellent agreement between the experiment and the equation-of-state creep model shown in Fig. 6 supports the formulation of eq (4) as being representative of the material behavior over a restricted range. The linear viscoelastic model of eq (8) provides an interesting comparison with experimental data. Notably, the shapes of the two curves as shown in Figs. 7 and 8 are approximately the same throughout the loading cycles. However, the viscoelastic model predicts complete strain recovery between cycles, whereas the experimental data show that this is not the case. Comparison of the results obtained with the linear viscoelastic model with experimental data indicates that a nonlinear viscoelastic theory incorporating a damage criterion might be appropriate. These analytical models provide valuable insight into the constitutive behavior of this class of materials and provide a solid foundation for future development.

Acknowledgment

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