Evaluation of Elastic Moduli of Bilaminate Filament-wound Composites

Crazed and uncrazed coupons from a rocket case and laboratory panel are subjected to static and dynamic loading

by Ronald G. Hill

ABSTRACT—This paper investigates the elastic behavior of bilaminate-composite coupons segmented from a filament-wound rocket case and of a laboratory prepared flat panel for direct tension–compression and flexural loading under static and dynamic conditions. New methods of testing have been developed which are primarily applicable to composite constructions. The dynamic test consists of exciting the primary free-free resonant mode of a specimen. Flexure tests utilizing a unique pure-bend system are employed for the static evaluations. The composite moduli determined from the static and dynamic test are compared with analytic values. The analytic values for the composite are derived from tension–compression and flexure analytical models using the material properties of the constituents. The measured elastic moduli compared favorably with analytical prediction and are indicative of the history of loading effects as well as the crazed condition of the composite constructions. The moduli determined by the dynamic test showed the closest agreement with analytic values, with a difference of 0 to 16 percent.

List of Symbols

a = distance between x_1 and x_2
A_i = area of lamina section
b = width of specimen
d = depth of neutral axis
dA = increment of area
E_a = assumed modulus of metallic member
E_C = compression modulus
E_F = flexural modulus
E_L = modulus of lamina one, longitudinal
E_T = tension modulus
E_{TC} = average tension–compression modulus
E_{TR} = modulus of lamina two, transverse
e = ratio of the moduli of a bilaminate
f = frequency
GL = gage length, distance between grips
g = acceleration of gravity
h = beam depth
I = moment of inertia
l = length of specimen
M = bending moment
m = variable in eqs (8) and (9)
n = mode constant
P_L = percent of section, lamina one
P_T = percent of section, lamina two
t = thickness of composite
y = distance from neutral axis to dA
y_i = centroidal distance
x_1, x_2 = output displacement transducers
σ = stress
θ = angle of bend

Introduction

The orthotropic nature of filament-wound composites has caused considerable difficulty in obtaining the experimental design properties for these materials. Standard ASTM laboratory tests are, in many cases, not suitable for the evaluation of the elastic properties of filament-wound composites, in particular specimens sectioned from a fabricated structure. This is due to the highly directional characteristics of their mechanical properties and the curvatures associated with specimen configurations.

The experimental techniques for determining the elastic moduli are described and test values obtained for coupons from a full-scale rocket case are presented. Data for rocket-case elastic properties are required for performance analysis and to evaluate the effects of ground handling and flight-load conditions.

As in the case of isotropic materials, the elastic parameters of filament-wound materials may be determined by either or both static and dynamic test. The utilization of both test methods provides a means of evaluating the sensitivity of the composite moduli to strain rate. The test methods that were used to determine the elastic moduli of the bilaminate composite specimens resulted in strain rates ranging from 10^{-3} to 10^{3} ips. A ramp function was used for loading in the static test; while the response in the dynamic test was sinusoidal at resonance, Fig. 1. In both dynamic and static tests, the composite specimens were subjected to direct tension–compression and flexural loading. Pure bending, in tension, compression, and flexural modes, was employed for the static test. The dynamic tests consisting of tension–compression and flexural loading were conducted in the free-free vibration mode.

General Considerations

Analytic Model

The orthorhombic symmetry of the bilaminate construction is used to develop a mathematical

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Ronald G. Hill is Senior Dynamics Engineer, Liquid Rocket Operations, Aerojet-General Corporation, Sacramento, Calif.

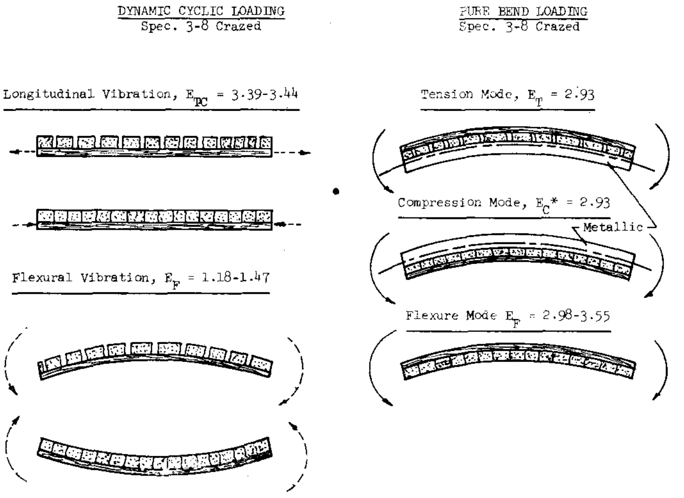
Paper was presented at 1967 SESA Spring Meeting held in Ottawa, Ont., Can., on May 16–19.

TABLE 1—SUMMARY OF MODULI TESTS—E × 10^{-6} PSI

			LAMINA ONE		LAMINA TWO		DYNAMIC - CYCLIC LOADING Experimental (Analytical Model)		STATIC - PURE BEND LOADING Experimental (Analytical Model)		
Specimen Number	Glass Type	Width inches	Angle of Wrap (Deg)	Thickness inches	Angle of Wrap (Deg)	Thickness inches	E_{T-C}	E_F	E_T	E_C	E_F
1	E	1.75	10	0.12	90	0.10	5.16 (5.37)	4.40 (4.40)	---	---	3.57 (4.40)
2	E	↕	10	↕	↕	0.10	5.13 (5.37)	4.40 (4.40)	---	---	3.57 (4.40)
3	S	↕	5.5	↕	↕	0.24	3.44 (4.03)	1.40 (1.02)	---	---	3.55 (3.71)
4	↕	↕	↕	↕	↕	↕	3.43 (4.03)	1.47 (1.02)	---	---	2.98 (3.71)
5	↕	↕	↕	↕	↕	↕	3.42 (4.03)	1.31 (1.02)	2.93 (3.08)	---	---
6	↕	↕	↕	↕	↕	↕	3.34 (4.03)	1.18 (1.02)	2.93 (3.08)	---	---
7	↕	↕	↕	↕	↕	↕	3.44 (4.03)	1.33 (1.02)	---	2.93*(4.98)	---
8	S	1.75	5.5	0.12	90	0.24	3.39 (4.03)	1.25 (1.02)	----	2.93*(4.98)	---

Notes

1. Lamina One-Analytic Modulus - Reference (1)
   Spec. 1-2, E_L = 7.45 Uncrazed
   Spec. 3-8, E_L = 8.40 Uncrazed
   Spec. 3-8, E_L = 8.40 Crazed
2. Lamina Two-Analytic Modulus - Reference (1)
   Spec. 1-2, E_{TR} = 2.85 Uncrazed
   Spec. 3-8, E_{TR} = 3.20 Uncrazed
   Spec. 3-8, E_{TR} = 0.32 Crazed
3. Density, Bilaminate-Measured
   Spec. 1-2, = 0.0745 lb/in.^{3}
   Spec. 3-8, = 0.0740 lb/in.^{3}
4. Metallic Backup - Pure Bend Test
   Mild Steel, E = 28.0, 0.5 in. x 1.75 in.
5. Gage Length - 8 in. - Pure Bend Test
6. *E_C is low - the applied strain may not have closed the cracks in lamina two



model for the filament-wound composite materials. Thus, the analytical composite moduli of the bilamina for direct tension–compression and flexure is defined by:

$$E_{T-C} = P_L E_L + P_T E_T \tag{1}$$

$$E_F = \frac{Ml}{2\theta e \Sigma y_i^2 A_i} \tag{2}$$

The analytical moduli of each of the lamina is computed from the material properties of the constituents, glass filaments and resin. The modulus in the direction parallel to the filaments is defined by an expression with similar terms to those used in eq (1) for the composite modulus of the bilaminate composite. Derivation of the modulus transverse to the filament is considerably more difficult; the interaction between the glass filaments and resin matrix is involved. The method is outlined in Ref. 1 and the properties for the individual lamina are shown in Table 1. The following assumptions were made for computation of the elastic moduli of individual lamina:

1. Square packing of round filaments
2. Infinitesimal deformations
3. Linear elastic behavior of the composite
4. Continuity between the constituents

The material properties of the constituents, as given by Ref. 1, were determined primarily from static test at low strain levels. Therefore, the composite analytical models built from the constituent material properties are designed for low static strain-rate effects.

Vibratory Loading

The mechanics that are utilized for defining the elastic behavior of homogeneous isotropic materials will be applied first to the case of a bilaminate beam. The frequency-modulus equations for the longitudinal and flexural vibration of a free-free prismatic beam are:

$$E_{T-C} = \frac{nf^2l^2}{g} \tag{3}$$

$$E_F = \frac{nl^4f^2}{gI} \tag{4}$$

Note that the modulus obtained from longitudinal vibration is independent of the cross section of the specimen; while the modulus derived from flexural vibration includes the effect of shear. This effect, however, has been shown to be negligible in most practical cases due to the high length-to-thickness ratio of the specimen.

Pure-bend Loading

Pure-bend loading* is used to determine the

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* *A special test fixture was used to apply a constant bending moment along the entire length of the specimen between grips.*

elastic modulus of a simple beam in flexure; it is then extended to the more complicated case of a multilaminate beam for determination of a tension or compression modulus. In each case, the equations of equilibrium for pure bending are applied. The two equations of bend equilibrium are:

$$\int \sigma dA = 0 \qquad (5)$$

$$\int \sigma y dA = M \qquad (6)$$

The flexural modulus of a prismatic beam is given by

$$E_F = \frac{Ml}{2\theta I} \qquad (7)$$

The tension or compression modulus of the bilaminate composite is obtained by combining it with a metallic member and loading the metallic-composite beam configuration in pure bending. The flexural mechanics are such that the composite material is loaded in uniaxial tension or compression depending on the placement of the metallic member. In this special case of an orthotropic beam, the elastic modulus of the metallic member is a known quantity and the modulus of the composite material is the unknown. The tension or compression modulus is determined by simultaneous solution of the following two equations which are derived from the equilibrium equations of bending:

$$d = \frac{(h-t)^2 + mt(2h-t)}{2(h-t+mt)} \qquad (8)$$

$$m = \frac{d^3 + [h-(d+t)]^3 - (3/bE_a)(M/\theta)}{[h-(d+t)]^3 - (h-d)^3} \qquad (9)$$

These equations are equally applicable to the determination of the constituent modulus of a bilaminate composite when one of the constituent moduli is the unknown.

Test Specimen

Eight bilaminate-composite samples were tested; two were segmented from a laboratory-prepared flat panel and six from a standard production rocket case. The laboratory-prepared panel was fabricated using U. S. Polymeric 787 E-HTS preimpregnated 20-end roving. Reference 2 describes the panel-fabrication procedures.

The standard full-scale rocket case was fabricated using 20-end S-994 HTS, Type I glass preimpregnated with a Shell 5868 resin system. Rocket-case curing was initiated through a programmed sequence of temperatures for a period of about 35 to 37 hrs. The highest curing temperature reached was 300° F. The specimens were sectioned from the barrel portion of the case with the principal axis of the specimen parallel to the longitudinal axis of the rocket case. The meridional and hoop wraps of the rocket case were wound in two separate operations; thus, a specimen cut through the total thickness of the barrel portion of the rocket case is of bilaminate construction. The rocket case, from which the bilaminate samples were obtained, had been previously subjected to a hydrostatic proof test and static firing.

The bilaminate laboratory-prepared sample was representative of a composite material which was fabricated under ideal condition and had no prior loading. The rocket-case samples demonstrate the condition of a composite material which had been fabricated utilizing standard production procedures and had been subjected to typical performance loads. The performance loads, which were biaxial in nature, had damaged the bilaminate constructions according to a specific history of loading. Crazing and/or microfractures of the resin were visually observed between the filaments in lamina two, Table 1, on all of the six specimens segmented from the rocket case.

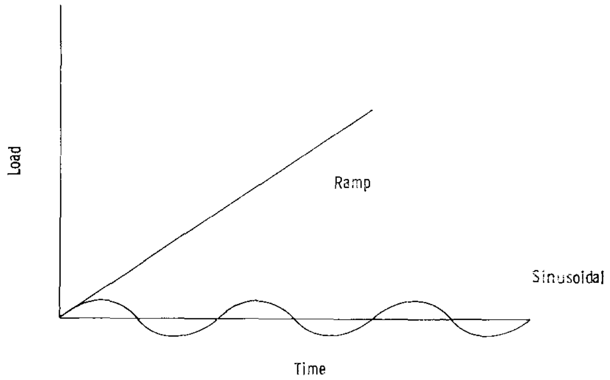

Fig. 1—Static and dynamic loading

Experimental Techniques

One of the major problem areas in the development of test methods for filament-wound materials has been the nonisotropic nature of their mechanical properties. The orthotropy and specimen configurations which characterize this anisotropic material are responsible for much of the difficulty encountered in applying standard metallic-test techniques to composites.

Some factors which were considered in the design of the test procedures used for the bilaminate specimens are:

1. Grips were designed to prevent slippage.
2. Simple uniaxial loading was applied.
3. Flexural loads were separated from tension and compression loads.
4. A condition of constant strain was imposed such that the elastic constants could be determined for simple flexure, tension and compression stress states.

In each of the experimental techniques applied, the test procedures were verified with tests of homogeneous isotropic materials with known elastic properties.† The static and dynamic tests were

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† *The results and approach used for the evaluation of an aluminum-steel beam are presented in the "Static-test Procedures" section.*

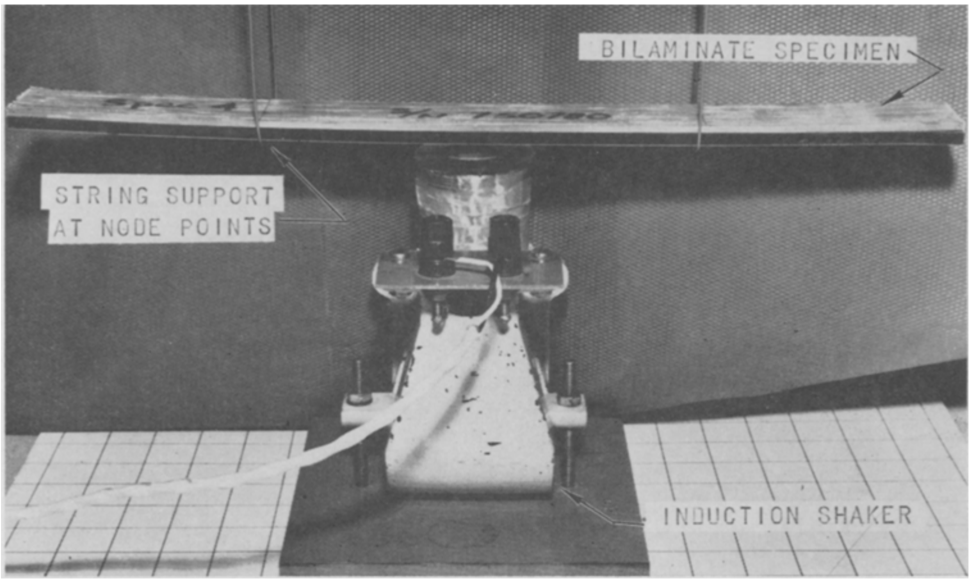

Fig. 2—Flexural-vibration test setup

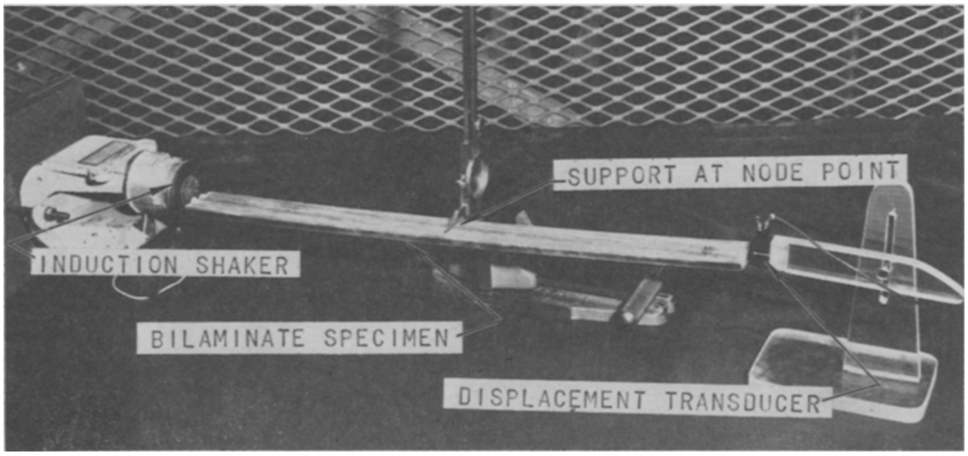

Fig. 3—Longitudinal-vibration test setup

set up to measure the primary properties of the beam being tested, frequency or stiffness. In the isotropic equations that were used to model the static and dynamic tests, the only unknowns were the moduli, which were computed as described. The other quantities in these equations consist of the geometry of the beam and its mass distribution.

Dynamic-test Procedures

Dynamic-test procedures were established to permit determination of free-free vibrational characteristics of the bilaminate-composite specimens. Accordingly, specimen excitation was achieved by induction shakers‡ which did not require contact with the specimen and could be placed in any position with respect to the specimen. The specimen's resonance response was detected by visual means in the case of the flexural-vibration test and by a displacement transducer for the longitudinal (tension–compression) vibration test. A standard phonograph magnetic cartridge and needle were used for the displacement transducer. The specimens were supported by means of strings or knife edges at the node points, depending on the method of loading, flexural or longitudinal. Figures 2 and 3 illustrate the flexural- and longitudinal-vibration test.

Static-test Procedures

Flexure tests utilizing a pure-bend system were employed for the static evaluations. The pure-bend fixture employed at Aerojet-General differs from the usual four-point bending in that (1) the specimen may be subjected to a controlled angle of bend through an included angle of 180 deg, and (2) the existence of pure moment is not a function of the specimen or its physical properties. This fixture design allows the bend specimen to deform freely in accordance with its elastic-plastic properties without frictional constrictions at supports or load points. A description of the pure-bend fixture follows.

The basic concept of the pure-bend-test fixture is illustrated in Fig. 4. It is seen that a pair of co-

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‡ *An electromagnetic induced force was imparted to a small steel disk bonded to the composite specimen. The mass of steel disk attached to the specimen is considered negligible.*

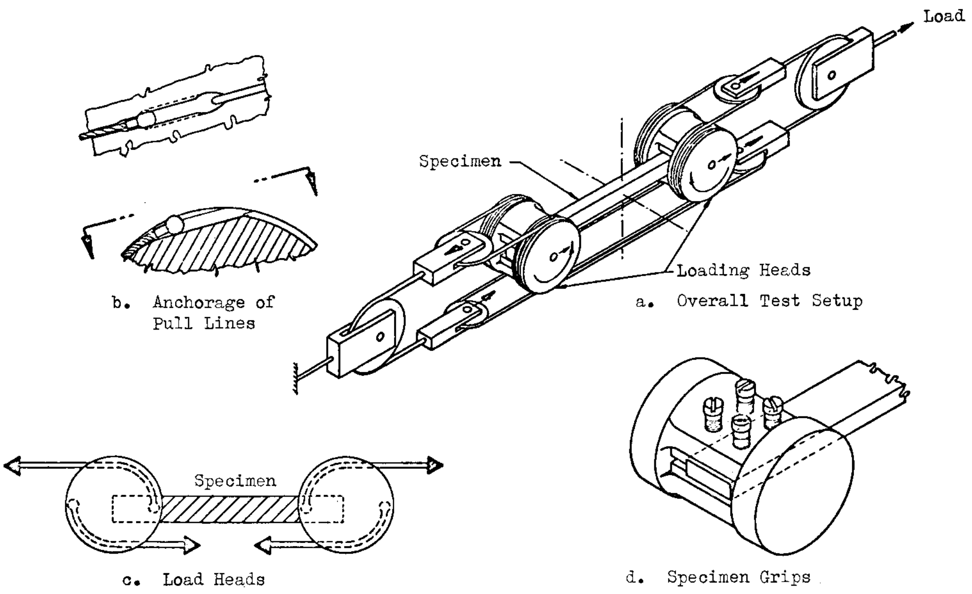

Fig. 4—Pure-bending test fixture

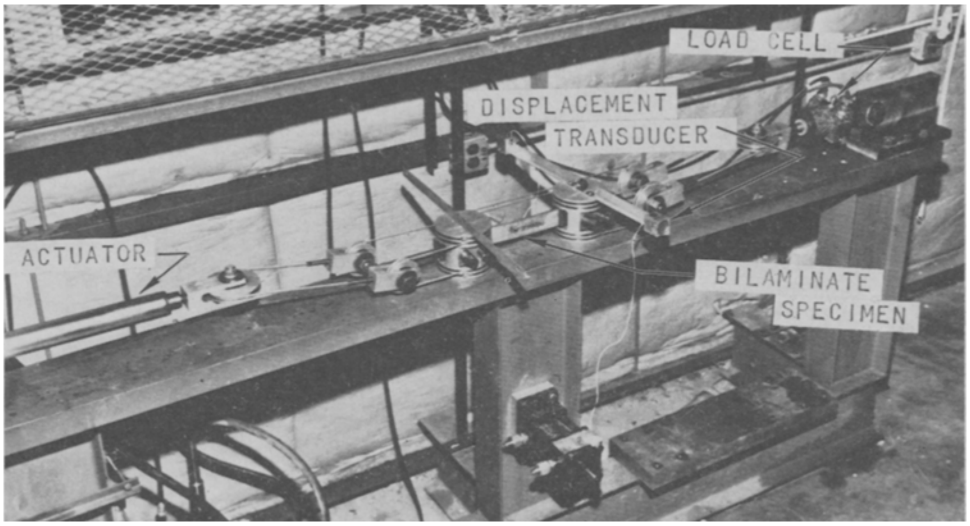

Fig. 5—Pure-bend test setup

planar load heads of pulley configuration grip the ends of the specimen. If the loads applied to the pulley load lines are equal, then zero shear and a constant moment will have been produced. The pull lines require equal loads for a pure moment to exist in the load heads. The balancing pulleys at right angles to the load heads maintain equal loads in all four pull lines. They permit the pull lines to stretch unequal amounts and a variable coefficient of friction in the bearings of the pulleys to exist without affecting the equality of the applied moment in the load heads. In Fig. 4, vectors indicate the relative movement of the parts responsive to the exerting of a pull on the loading lines. From these vectors, it can be seen that the load heads undergo equal and opposite rotation while the right head moves in the lineal direction of pull faster than the head on the left. As a consequence, the specimen is subjected to equal and opposite force couples resulting in pure bending.

The pure moment was imparted to the bilaminate specimen by applying a uniaxial tension load to the fixture. The moment is calculated from the uniaxial tension load and the mechanics of the fixture, Figs. 4 and 5. The angle of bend was measured as shown in Figs. 5 and 6, and is determined by the following equation:

$$\theta = \frac{x_1 + x_2}{2a} \qquad (10)$$

A uniaxial tension or compression loading on the bilaminate specimen was achieved by combining a metallic member with the bilaminate composite to form a multilaminate beam, Fig. 7. The metallic-composite specimen behaved as one beam in bending, because of the clamp-clamp condition* in the jaws. No bonding agent was used between the metallic and the bilaminate-composite material.

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* *Clamp-clamp, large bolts rigidly clamp the specimen between very stiff jaws to impart near theoretical "fixed-end" conditions.*

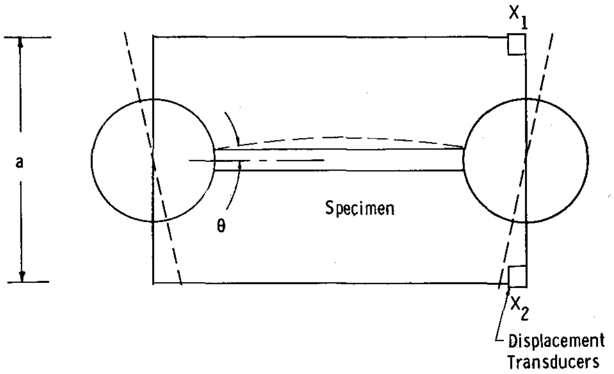

Fig. 6—Technique for angle-of-bend measurement

Two basic assumptions were made in deriving moduli data from the pure-bend test: first is the condition of a uniform strain along the gage length of the specimen, zero shear; and, secondly, that plane sections remain plane for a multilaminate beam as well as a single lamina. Graphical and grid-line checks indicated that these assumptions were valid for first-order approximation. Each of the eight specimens tested was subjected to only one static test, flexure, tension or compression. A clamped-clamped mode was used for all of the static test; the ends of the specimen located within the grips were not allowed to deform. A stress level of between 18,000 to 20,000 psi was imposed on each of the flexure specimens for determination of the slope of the moment vs. angle of bend curve, the modulus. The bilaminate composite specimen was oriented in the flexure test such that the outermost filaments of lamina two were in compression, causing the microcracks in the resin to close. In the tension or compression test, the bilaminate was oriented such that the filaments of lamina one experienced the maximum strain.

Figure 8 illustrates the approach that was used to determine the direct tension and compression moduli of the bilaminate composite. A composite beam consisting of aluminum in tension (simulating a low-modulus material) and steel in compression with the neutral axis of the composite beam in the area of the steel was tested in flexure. Using eqs (8) and (9), a modulus of 10 × 10^6 psi for aluminum and 28 × 10^6 psi for steel was computed. Therefore in subsequent test, Table 1, specimens numbered 5 to 8, a modulus of 28 × 10^6 psi was assumed for steel, E_a.

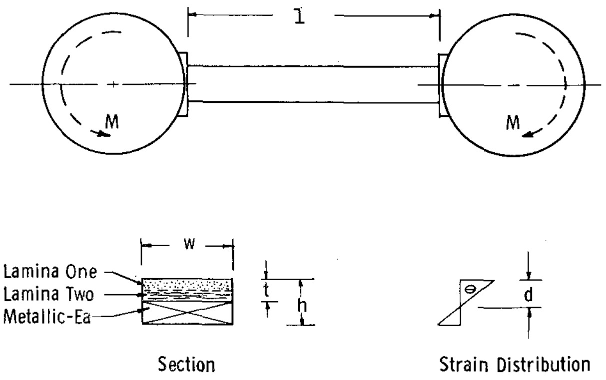

Fig. 7—Pure bending of a multi-laminate beam

Results and Discussion

The results of the static and dynamic tests are tabulated in Table 1. Analytic moduli in the crazed and uncrazed condition of the bilaminate and of the individual lamina, determined from the constituent properties[1] are also presented in Table 1. The analytic crazed modulus, 10 percent of the uncrazed value for E_{TR},[1] was computed from data obtained from hydroburst test of full-scale rocket cases.

A direct comparison of the static, dynamic and analytic moduli values is difficult because of the many factors that affected the loading and response of the bilaminate construction. The nonlinear effects of shear, when one of the lamina of a bilaminate has a modulus appreciably below that of the total section, are noted in Refs. 3 and 4. The effect of the different methods of mechanically loading a material are evaluated in Refs. 5 and 6.

The dynamic tests are considered to give true values, in that the driving and pickup systems employed are not directly coupled with the specimen (free-free vibration). In the case of the phonograph transducer, used in the longitudinal-vibration test, the specimen was coupled to the pickup system by a force of $^1/_2$ to 1 g, which can be considered as negligible. The dynamic tests are also considered to be nondestructive since the uncoupled input force of the induction shaker was between $^1/_4$ to 1 lb. In tests of over 300 filament-wound coupons that the author has conducted, the dynamic moduli have consistently shown very close agreement with analytic results.[7] The elastic moduli values listed in Table 1 for the uncrazed E-glass bilaminates specimens 1 and 2, show close agreement with the analytic bilaminate models, particularly the flexural values. However, for the crazed specimens, the dynamic-moduli values are high in comparison to the analytic flexural values, and low for tension–compression values. This is due to a combination of the crazed condition of the material and the cyclic loading imposed on the six S-glass specimens. The crazed resin or microcracks transverse to the direction of the filaments on lamina two, were subjected to a cyclic tension–compression loading both in the longitudinal and flexural modes, causing the microcracks to open on the tension portion of the load cycle and close on the compression cycle. Therefore, the dynamic modulus should be considered as an effective or average modulus that includes the nonlinear effects of the damaged material, microfracture of the resin, on the S-glass bilaminate constructions.

The static-moduli values were computed using deflection data; strain gages were not applied. In general, it has been noted that deflection results reveal modulus values that are 3 to 5 percent lower than the values determined by strain gages.[1, 6] This may account for some of the discrepancy between the analytic and experimental values. The influence of interlamina shear within the grip area may produce nonlinear effects that would reveal an apparent low modulus.

Both composite materials tested, E-glass and S-glass, demonstrated a linear elastic response. The dynamic response of the bilaminate specimens was displayed by curves which were sharp and discrete, revealing low damping. The low-damping characteristic of the composite was also confirmed by the moment-vs.-angle of bend hysteresis curves.

It is generally accepted that, for an isotropic material, the tension–compression and flexural moduli are equal. However for a bilaminate beam, Table 1 shows that the experimental and analytic tension–compression and flexural moduli are of different values. This difference between flexural and tension–compression moduli data for laminated beams can be related to the "transformed section theory" eq (2), that was used to develop the flexural modulus. Equating the tension–compression and flexural moduli relations, it can be shown[1] that:

$$\frac{E_{T-C}}{E_F} = \frac{A_{\text{transformed}}/A_{\text{section}}}{I_{\text{transformed}}/I_{\text{section}}} \tag{11}$$

The analytic and experimental modulus ratios for the uncrazed E-glass specimens are:

Specimen No.	Analytic E_{T-C}/E_F	Experimental E_{T-C}/E_F
1	1.22	1.18
2	1.22	1.17

There does not appear to be a large difference in the elastic moduli of the bilaminate composites for low and high strain rates. The dynamic flexural and tension–compression moduli of the uncrazed E-glass are in agreement with the respective analytic values which were modeled for low-strain-rate effects. There may have been some deterioration due to aging in all of the composite specimens. The specimens were fabricated and cured well over one year before testing. The dynamic flexural modulus was obtained at a cyclic strain rate of 100 ips and the dynamic tension–compression modulus at a rate of 10^3 ips.

Conclusions

1. The utility of the mechanical tests described in this paper, for the determination of the elastic moduli of filament-wound composite specimens cut from actual structures, has been demonstrated.

2. The dynamic tests provide a simple, direct and nondestructive means to measure the primary elastic properties of a composite construction to assess environmental degradation as well as for design and other evaluation purposes.

3. Flexure in the form of a pure-bend test provides an expedient means for measurement of the static moduli of a bilaminate construction. A direct tension or compression composite modulus can be derived from a pure-bend test which utilizes a metallic-composite beam. This method is particularly applicable to the determination of the tension or compression modulus of a curved composite beam.

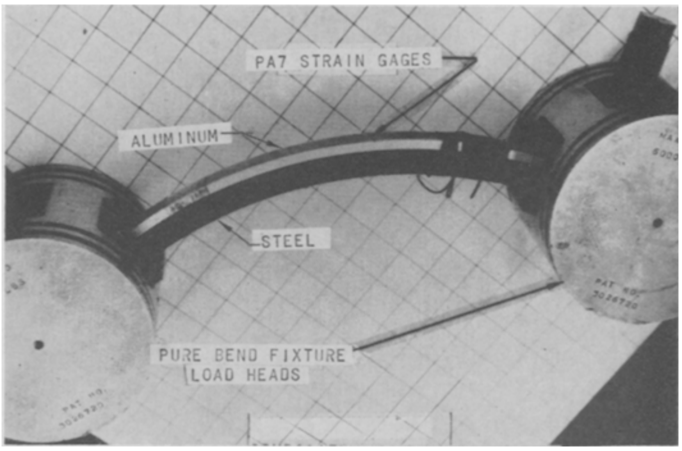

Fig. 8—Aluminum–steel composite specimen

4. The measured elastic moduli are indicative of "history of loading," the crazed condition and degradation of the reinforcement or matrix of composite constructions.

5. These tests do not permit broad generalization as to differences between static and dynamic moduli for bilaminate composites, but suggest the difference is small for types of composites tested.

6. Near-future improvements in bend-test techniques are expected to permit refined generalizations with respect to low and high-strain-rate moduli in composites.

Acknowledgments

The author wishes to acknowledge the suggestions and contributions to the analysis and techniques presented, given by W. T. Walker and F. E. Peterson.

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