The Grid Method

Review of the grid method as used by several investigators indicates that a large variety of techniques is available to determine strain in nonhomogeneous fields

by Vincent J. Parks

ABSTRACT—Determining strain with grids is one of the oldest and simplest methods of experimental stress analysis. Here, the method is reviewed. Various techniques that have been developed to print, record and analyze grids are discussed, and the types of problems to which the grid method has been applied are presented.

Introduction

Hooke determined strain in tensile specimens by noting the change in length between two scribed lines. This is such a simple and direct approach, it hardly warrants designation as a method. However, as the number and direction of lines is increased, and as the geometry becomes more complex, there are certain techniques that aid in the process of measuring the distance between two lines.

The grid method will be defined as a method of applying a grid to the surface of a specimen (or sometimes on an interior plane), measuring the distance between discrete points on the grid both before and after loading, and an analysis of these measurements. The analysis of the data consists in (1) obtaining the difference in distances before and after loading to determine displacements, and (2) dividing these displacements by the distance to determine strain.

The grid itself will be defined as an array of lines or dots which indicate discrete points in the specimen. The array usually has a repeated motif, with lines or dots occurring at some regular frequency. Often the array is rectangular, with dots or lines repeated in two perpendicular directions. Polar grids of radial and circumferential lines are also common, and have special usefulness in axisymmetric problems. Occasionally irregular scratches are used; however, the more common grid has a regular frequency of lines which is specified to some extent by the problem.

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The current interest in the moiré method somewhat overshadows interest in the grid method. The two methods have much in common. However, because of the wealth of current literature on moiré, this paper will restrict itself only to the grid method.

Hooke used grids to determine strains in a tensile specimen, a homogeneous field of strain. As such he could make his specimen very long to obtain an accurate value of strain. Grids are often used in this way to obtain the stress–strain curve of the material and the material properties. Because the strain field is homogeneous, many of the difficulties usually associated with the grid method are not encountered. In this paper the emphasis will be put on the grid method applied to the nonhomogeneous field, with the understanding that some of the techniques discussed do have application to the problem of material characterization.

Choosing the Grid Pitch

It is necessary that the grid-line spacing (pitch) be such that the desired displacements and strains can be obtained. This has to do with the specimen geometry. The smaller the specimen, and the smaller the discontinuities (such as fillets, slots, holes, etc.) on the specimen, the smaller the pitch needs to be. If strain is defined as

$$\epsilon = (\ell_f - \ell_i)/\ell_i \qquad (1)$$

this strain is the average over the base length ℓ (the subscripts f and i signify final and initial), and from the mean value theorem, it is the value of the strain at least at one point in the pitch interval. It is necessary that this interval be such that the strain does not vary appreciably over the interval. In the case of obtaining strain in a region of large strain gradient, as at the edge of a circular hole, this would mean that the pitch should be only a small fraction of the hole radius, probably ¼ radius or less.

This limitation on pitch because of strain gradient conflicts with the linear increase in error associated with shortening the base length. If a model of the specimen can be used, increasing the scale and selecting a more deformable material can make up for the limitation of a short base length.

Pitches as fine as 0.006 in.[1] have been analyzed and a pitch of 0.01 in. (100 lines per inch) is not uncommon. But grid spacings up to ½ in. to 1 in. can be used on large specimens or regions of low strain gradient.

Pitch Precision

In choosing a grid, it has to be decided if it will be measured before loading the specimen. If it is not measured before loading, the pitch is assumed to have a certain value. The precision of this value specifies a limit of accuracy, and the measurement of the loaded specimen need not be any more precise. If measurements are made both before and after loading, then the line spacing is not critical, and allows even random scratches to be used as grid lines.

Grid-line Visibility

Usually the limitation in obtaining an accurate measurement from the grid is not in the measuring instrument (normally some optic-mechanical system) but rather in the investigator's ability to see a precise point. (In grid analysis, it is necessary to talk about the distance between two points rather than between two lines. If a system of lines is analyzed, it is the line intersections which serve as points.) In general, a dot or intersection is larger than the measuring system units, and covers a number of measurable divisions. Thus, the width of the dot becomes a limiting factor on the accuracy. It is necessary to estimate to the center of the point or to one edge of the dot, or to one corner of the intersection to improve the accuracy. If measurements are made before and after loading, it is also important that the chosen point of measurement be recognizable under both the loaded and unloaded conditions. This can be a problem if the dot or intersection is warped or greatly distorted in loading.

The grid should be chosen such that the operator can return the measuring system to the desired point within the desired degree of accuracy. This can be first tried by repeated reading of the grid of the unloaded specimen, and then by readings on an unloaded and loaded specimen with a known displacement. Grid-line visibility is also aided if records of the grid before and after loading are superimposed and viewed simultaneously.

It was noted above that the pitch is chosen with regard to specimen geometry. Here it is seen that the grid line should be chosen for accuracy. The time spent in selecting the proper grid may be more than made up in the subsequent analysis. Grids of various pitches and overall size, as well as various degrees of precision, are available from a number of commercial sources, both from the field of graphic arts and from those companies specializing in stress-analysis equipment.

Applying the Grid

Printing

Photographic printing is probably the most common method of putting a grid on a specimen. The grid method was introduced to the SESA by Brewster[2] as a means to measure plastic strains in formed metals. He referred to the method as "photogrid,"* because of the photographic method he had earlier developed to print a 100 line per inch (lpi) grid on metal specimens. Other investigators have suggested a number of variations. MacLaren[3] itemized several solutions used to print photogrids, including a non-soluble solution to use under water. A diazo compound is recommended by Hart.[4] Knauss[1] describes an inking technique. Photoengraver techniques have developed to the point where a good photoengraver can make a contact copy of a 100-lpi grid or cross grid on a flat hard surface with little difficulty.

Etching

Etching techniques were described recently by Holister and Luxmoore[5] in a paper on moiré. Many commercial products are now available for etching. As indicated by Holister, today etching is in many cases a photographic process. Many of the products used to etch can be used also to print the grid, without etching.

Scribing

Scribing is a common approach to apply grids to metal and plastic models. Bell developed a technique to scribe grids with densities of up to 30,000 lpi. Because his technique uses diffraction of light, it is not considered here. Box[6] presented a technique to machine scribe grid lines. Any competent machine shop can suggest numerous hand and machine-scribing methods.

Applying random scratches with emery can be considered a type of scribing. Usually an area is rubbed with emery in two directions to provide intersection points. Random scratches are often used in conjunction with the replica technique described below, although neither technique is necessary to the other.

Embossing

Closely akin to scribing is a technique proposed

* *The term "photogrid" was retained in the index of the SESA Proceedings to refer to any paper on grids.*

by Taggert, Polonis and James[7] who stamped groups of three closely set points in right triangular arrays into a metal specimen. Essentially they treated each set of points as a point analysis and stamped the array at all required positions.

Embedding

For three-dimensional analysis, grids have been embedded in transparent castings.[8, 9] To obtain enough response, the casting is usually a soft material (either rubber or frozen epoxy), and rubber threads are used to avoid reinforcing the model. (The author has found it is also possible to cast a fine wire in rubber and remove it after casting to leave a sharp visible line.) Threads are usually strung on a frame like a tennis racket except with a finer pitch, and a split mold used, with the split along the plane which is to be analyzed.

A two-dimensional application of embedding is described in Ref. 10. The method has the advantage of placing the grid in the central plane of the two-dimensional model. The distinctness of the edge and intersections of the rubber thread is also an advantage. The author, in one problem, cemented rubber threads on the surface of a two-dimensional rubber model to take advantage of the sharp line images of the rubber-thread.

Embedding pellets has also been suggested as a technique by Bynum, Rastrelli and deHart.[11]

Reflection

Polishing a specimen surface and viewing the reflection of a grid was suggested by Duncan[12] as a method of analysis for determining plate curvature under load.

Cementing

In some cases, it may be convenient to cement a grid film on the specimen. The film material should be such as not to reinforce the specimen material. Commercial stripping film minimizes this reinforcing effect.

A novel approach was suggested by Durelli[13] who cemented a commercially available rubber stamp with a Cartesian grid directly on the flat face of a solid-propellant rocket material. By inking the stamp in the usual way, a copy was made on paper, before and after loading.

Recording the Grid Images

Recording the grid is not always necessary since measurements may be made directly on the specimen. However, it is a common and convenient approach to record or copy the undeformed and deformed grid images.

Photographic recording is common, and any of a number of high-contrast films and glass plates are now commercially available (e.g., Contrast Process Ortho, Kodalith, High Resolution Plate, etc.). Beside obtaining a grid image of high contrast, it is necessary to insure that no distortion is produced in photographing. This leads to the use of glass-plate negatives. Further, the lens must not distort, and the image and object planes should be parallel. It is worthwhile to have known lengths in the plane of the specimen, and at several places and directions, to guarantee faithful copying. A known scale in the photograph allows the control of the desired magnification of the negative.

The other method of recording the grid is the replica technique developed by Hickson.[14] The grid is indented (by scribing) so that a material can be cast over it and removed as a replica of the grid. Replicas can be obtained before and after loading.

Hickson combined several features to make the replica technique a valuable contribution to the grid method; emery is used to scratch lines which give fine, distinct intersections; a special gun is designed to use with low-melting alloys for ease and accuracy in replicating the initial and deformed image; and the replicas can be viewed side by side to increase accuracy.

Measuring and Analyzing the Grid

A great variety of instruments is available to record the sighting in the photographic film or on the specimen. These include: microscopes with graduated reticles, microscopes with calibrated movable crosshairs, microscopes mounted on traveling heads, comparators with scales on a screen, or comparators with movable calibrated tables, and the ordinary caliper and scale.

Three approaches to reading and analyzing grids are presented here.

(1) Point-per-point

Probably the simplest conceptual method to determine strain is to treat each interval individually and make measurements before and after loading and apply eq (1). Note that each strain obtained this way requires four data, two values of position before loading and two values of position after loading. For the complete analysis at a point in a two-dimensional problem, intervals in 3 directions are needed. The rosette equations can then be applied in the usual way to obtain the principal strains and directions. In a rectangular grid, measurement of the two perpendicular intervals, plus the 45-deg diagonal, is the most common arrangement. If the point is on a line of symmetry, then only two intervals, parallel and perpendicular to the line, are needed.

(2) Whole Line (Line of Symmetry)

If the strain along a whole line of symmetry is

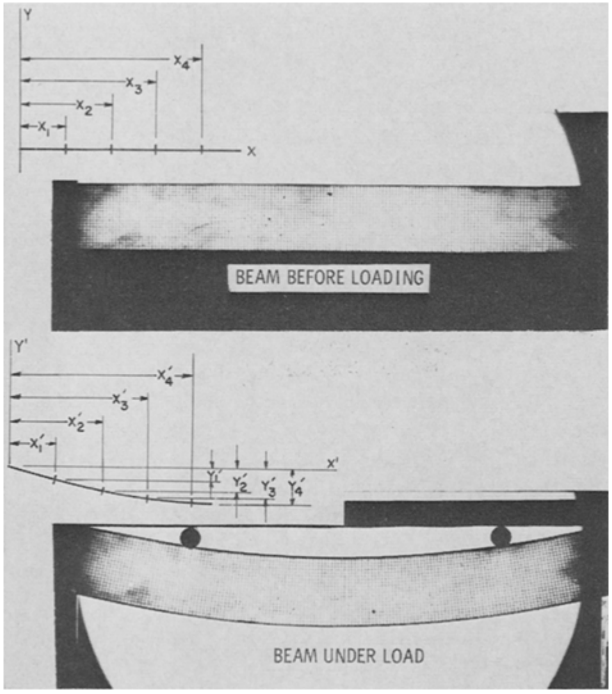

Fig. 1—Grid patterns in a beam before loading and when subjected to a deformation producing uniform curvature

required, it is practical to read the position of points in a continuous manner, that is, from a continuous scale.

If the line running along the line of symmetry is considered as the x-direction, two sets of x-coordinates can be obtained, one before and one after loading at all discrete points on the line. The differences in these two sets gives the displacement along the line.

The analysis can revert at this stage to point-per-point by subtracting the displacement values at each end of the interval to obtain the change in displacement, and subtracting the coordinate values at each end of the interval to obtain the interval base length. The ratio of the change in displacement to the base length give the average strain in the interval. This approach has an advantage over the individual point-per-point approach in that any error in measurement which gives a high strain is compensated for in the adjacent interval by a low strain.

When the displacements have been computed, an alternate to this modified point-per-point approach is Fischer's method.[15] It consists of plotting the displacement values with respect to position. A smooth curve is drawn through the points and by constructing tangents to the curve, the strain at *any* point can be determined (as opposed to the average strain in an interval). This is one technique to determine a peak strain from data taken at intervals.

The strain in the direction perpendicular to the line of symmetry must, of course, still be analyzed in the point-per-point fashion.

(*Nonsymmetric Line*)

For lines which are not lines of symmetry, two approaches are suggested.

GEOMETRIC APPROACH. The strain in terms of the change in length along the original direction and the angle of rotation can be obtained from geometry as

$$\epsilon_x = \frac{\Delta\ell_x}{\ell_i \cos\theta_x} + \frac{1 - \cos\theta_x}{\cos\theta_x} \qquad (2)$$

where

ϵ_x = as defined in eq (1)
ℓ_i = the initial length for the interval, which lay in the x-direction
$\Delta\ell_x$ = the change in length *in the original x-direction* obtained by subtraction of the four data points as described above obtained from a continuous line reading
θ_x = the rotation of the line

for small angles of θ_x

$$\epsilon_x = \frac{\Delta\ell_x}{\ell_i} + (1 - \cos\theta_x) \qquad (3)$$

The analyst can see, for any small θ_x, what degree of error is sustained by dropping θ altogether.

$$\epsilon_x = \frac{\Delta\ell_x}{\ell_i} \qquad (4)$$

Where the measurement of θ_x is required, it would require some auxiliary, but simple, measuring device.

STRAIN-DISPLACEMENT APPROACH. The same results obtained from eq (2) are available from the classic nonlinear theory of elasticity.

In some fixed reference system, the coordinates of the points on a line are measured both before and after loading the specimen. Then, by subtraction, components of the displacements at each point can be obtained. If these values are plotted along a coordinate system associated with the grid network, two curves are obtained which can be differentiated graphically or using the finite-interval method (as explained above) to give the partial derivatives of displacement components necessary to compute the strain in the direction of the grid. (The fixed-coordinate system used for measuring displacements need not be the same, or even the same type, as the grid system; however, in practice, both are often aligned Cartesian systems.) As illustration, Fig. 1 shows a Cartesian grid system on a beam subjected to bending. The fixed measuring system is also Cartesian and is illustrated by the coordinates x-y and x'-y'. The coordinates for a particular grid line on the undeformed beam are shown as $(x_1,0)$, $(x_2,0)$, $(x_3,0)$ and $(x_4,0)$. The corresponding coordinates on the same grid line of the deformed beam are shown as (x_1',y_1'), (x_2',y_2'), (x_3',y_3') and (x_4',y_4'). Subtracting these two sets of pairs gives the displacement components for the four points. The displacement components in the x and y directions are called u and v. The displacement of the four points can be written $(u,v) = (x_1' - x_1, y_1')$, $(x_2' - x_2, y_2')$, $(x_3' - x_3, y_3')$, and $(x_4' - x_4, y_4')$. Plotting the two sets of displacement components individually, with respect to the position (x_1, x_2, x_3 and x_4) which was already obtained,* gives two curves. The slope of these curves are $\partial u/\partial x$ and $\partial v/\partial x$. And the expression for strain is

$$\epsilon_x = \sqrt{1 + 2\frac{\partial u}{\partial x} + \left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial x}\right)^2} - 1 \qquad (5)$$

which corresponds to eq (2).

Again, similar simplifications are possible for small strain

$$\epsilon_x = \frac{\partial u}{\partial x} + 1/2\left[\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial x}\right)^2\right] \qquad (6)$$

or in some cases

$$\epsilon_x = \frac{\partial u}{\partial x} + 1/2\left(\frac{\partial v}{\partial x}\right)^2 \qquad (7)$$

and for small angles

$$\epsilon_x = \frac{\partial u}{\partial x} \qquad (8)$$

Again, as with the line of symmetry, the v components are obtained in a discontinuous way. Here, this has more significance since it enters into the computation of ϵ_x.

(3) Whole Field

The whole-field approach extends the method of the nonsymmetric whole-line approach (classic nonlinear theory) to the whole field. All the grid lines in both directions are analyzed. The analysis of both sets of lines allows all displacement components to be plotted as continuous curves. Each point to be analyzed is on four curves which give the four partial derivatives of displacement. Equation (5), (6), (7) or (8) can be applied in both directions as appropriate.

A set of diagonal lines would give the strains at an angle and provide a rosette analysis for the principal strains and directions. Or, the four Cartesian partial derivatives can be used to compute the Cartesian shear strain.

$$\gamma_{xy} = \text{arc sin}\,\frac{\dfrac{\partial v}{\partial x} + \dfrac{\partial u}{\partial y} + \left(\dfrac{\partial u}{\partial x}\right)\left(\dfrac{\partial u}{\partial y}\right) + \left(\dfrac{\partial v}{\partial x}\right)\left(\dfrac{\partial v}{\partial y}\right)}{(1 + \epsilon_x)\,(1 + \epsilon_y)}$$

Whole-field graphical methods are presented elsewhere.[16] It is shown that, by mapping the intersections of undeformed and deformed lines over the field for various amounts of shift, the two fields of displacement components are obtained. It is further shown that mapping intersections of the deformed lines with adjacent lines in a duplicate image of the deformed grid for various amounts of shift leads to development of the four fields of the partial derivatives. Figure 2 shows an example of one value of a partial derivative obtained by shifting, and the field of two of the partial derivatives.

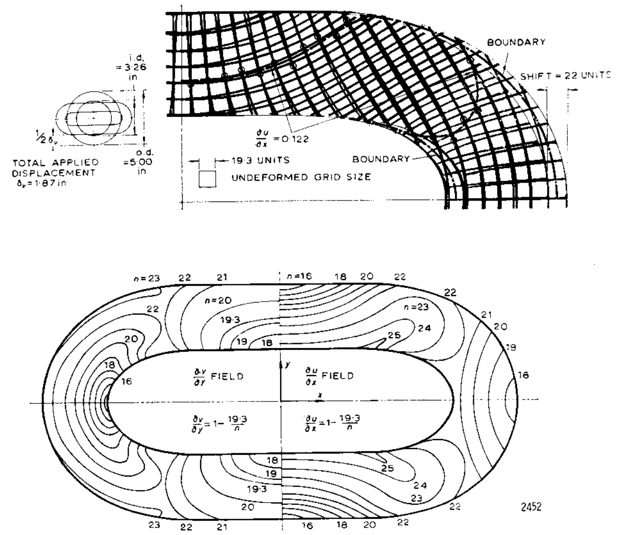

Fig. 2—Eulerian description of the field of partial derivatives of displacement components obtained by shifting the distorted grid of a ring on an image of itself

Lagrangian and Eulerian Descriptions

Since the grid is applied to the undeformed specimen, the grid method lends itself to Lagrangian analysis. By Lagrangian description is meant that the strain is based on the initial length and that, when associated with a direction, the direction refers to the initial direction of the fiber. In cases where the deformed specimen can be returned to its deformed shape and it is possible to put the grid on the deformed specimen, then the Eulerian description will be the easiest to compute.[16]

Solution of Problems

There are numerous papers in which grids are used to obtain mechanical properties of materials at various load levels, loading rates, temperatures and other special environmental conditions. As noted above, this type of problem is not considered here. Aside from this type of problem, the main value of the grid method seems to be for problems of large displacement and large strain. Most of the published solutions of nonuniform strain fields using grids are in three classes: rubber elasticity, plasticity or structures.

Rubber Elasticity

Grids are used regularly to study deformation in rubber and rubber-like materials. Rubber is an engineering material and often requires analysis.

* *If the grid were another type of coordinate system, say radial, then independent position measurements would have to be made on the undeformed specimen.*

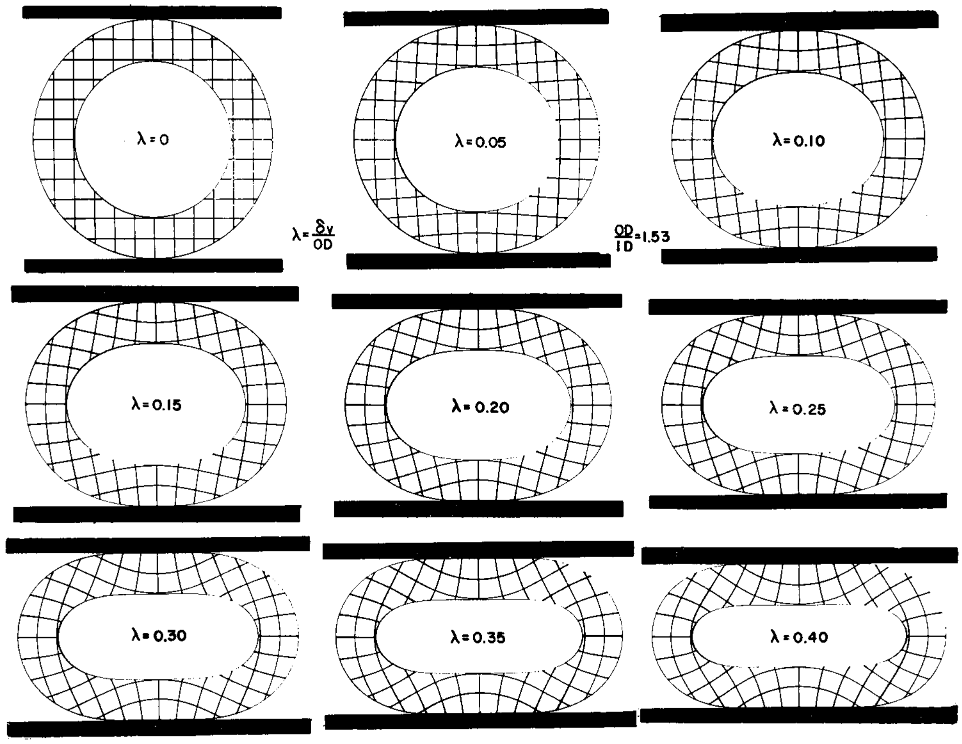

Fig. 3—Cartesian grid on a circular ring subjected to several levels of diametral compression

Durelli and Mulzet[17] reported the distribution of large strains in a rubber disk under compression. Knauss[1] analyzed a plate subjected to very large strains with an elliptical hole. He also suggested means to obtain stresses from the strains. Durelli, Parks and Feng[18] used grids to determine large strains in a ring. Figure 3 shows the grid patterns used in that analysis.

Rubber is also a convenient model material. Grids may be used to determine strain in rubber models of specimens of more rigid materials. This approach generally treats the linear small-strain problem in the specimen, and the rubber model must be such that both material and geometric linearity is preserved in loading the rubber model. For static and quasi-static problems, the material linearity in the rubber is easy to preserve. The preservation of geometric linearity reduces to the restriction of requiring that there be no gross changes in the overall geometry of the model, that is, a circular hole must still "look" circular after loading.

Figure 4 is an example from Ref. 8 of an embedded grid in a rubber model used to solve the small-strain problem of the sphere under diametral compression. The grid is in the meridian plane of the sphere. It is viewed by placing the sphere in a transparent tank of oil with the same index of refraction as the model material.

Figure 5 is an example from Ref. 19 showing a two-dimensional grid analysis of a dynamic load on a rubber specimen.

Epoxy heated to its critical temperature, or semi-cured epoxy, may also be used as a model material for three-dimensional grid analysis. The soft epoxy provides sufficient grid response. The deformations can be frozen as in three-dimensional photoelasticity and slices can be cut from the model for measurement. Such an analysis is presented in Ref. 9.

Rubber models are also used with grids to demonstrate principles of mechanics. Every student of mechanics should remember Timoshenko's[20] reference to Bach's twisted rectangular bar. The hole in a rubber plate subjected to uniaxial load is presented by Durelli, Tsao and Jacobson.[21]

Plasticity

A number of papers on analysis of metal forming illustrate the grid method applied to plasticity.[22–25] A three-dimensional analysis of a metal-forming problem is described by Thomsen and Lapsley.[26] The grid in this case was scribed on a meridian plane of an axisymmetric specimen which had been cut in half. The specimen was rejoined and subjected to an extrusion process.

Structures

Grids, or single lines, have been used for a long time to study displacements in structures. A paper by Rocha and Borges[27] describes such an approach. Small targets were placed on frame structures, and photographic records before and after loading were used to determine displacements and influence lines.

Summary

A review has been made of the grid method as used by several investigators, with emphasis on work

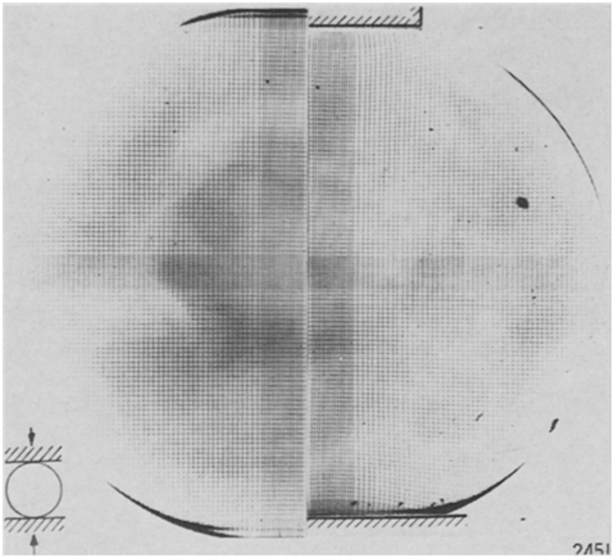

Fig. 4—Grid embedded in the meridian plane of a 6.9-in. rubber sphere. Left and right images are for 10 and 200-lb loads

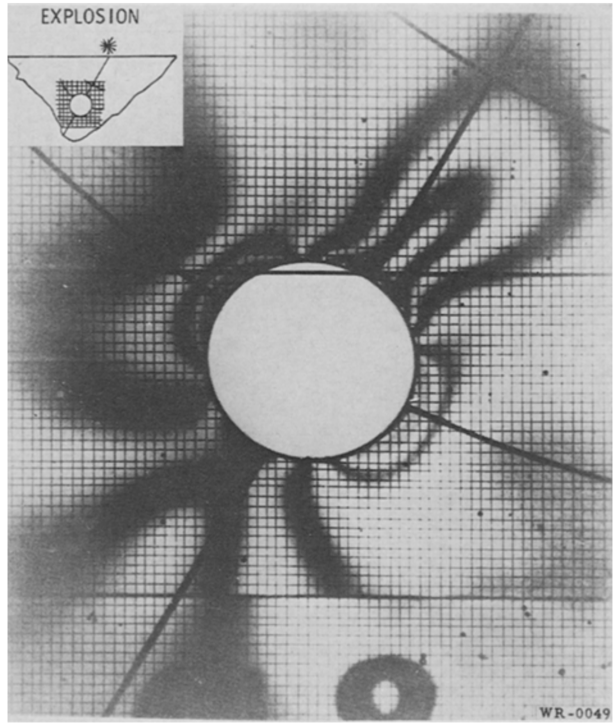

Fig. 5—Embedded grid around hole in a plate 1200 μsec after the explosive charge was detonated

done by members of the Society of Experimental Stress Analysis. A large variety of techniques was found available to determine strain in nonhomogeneous fields. The chief limitation of the method seems to be the lower level of measurable strain. This limit depends in part on the technique used. Here, engineering judgment is necessary in choosing a technique for a given problem.

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