# Validity of Compliance Calibration to Cracked Concrete Beams in Bending

The suitability of the compliance calibration technique to monitoring cracking in plain concrete beams was evaluated by using dye penetrant to determine average crack length. This was found to be less than that estimated by the compliance-calibration method

## by S.E. Swartz and C.G. Go

ABSTRACT—The method of compliance calibration for estimating crack growth in notched beams of metallic materials and nonmetallic materials such as rock has been used extensively with success. This method has also been used with concrete, but recently its suitability for this material has been questioned. The validity of this method has been evaluated using concrete beams in three-point bending in which the crack surface is revealed by a dye-penetrant technique.

The results of this study, which utilized twelve specimens precracked to varying depths and thirteen companion specimens using 0.076-mm thick Teflon notches of various depths, are presented. It was found that the compliance estimates of crack length agreed exactly with the actual length for the beams with Teflon notches. For the precracked beams the compliance estimates for crack length were in good agreement with the actual length observed at the beam surface (thus confirming previously reported results) but were greater than the average crack length revealed by dye.

## **List of Symbols**

- $a = \operatorname{crack} \operatorname{length}$
- CMOD = crack-mouth-opening displacement
  - $K_{I}$  = opening-mode stress-intensity factor
  - P = load on beam
  - P' = load on inverted beam used to open the crack for dye insertion
  - W = depth of beam

## Introduction

The method of compliance calibration for estimating crack growth in notched-beam and tension specimens is well known.<sup>1</sup> It has been successfully applied to metallic materials and nonmetallic materials such as rock, e.g., limestone.<sup>2</sup> This technique has also been used extensively to monitor crack growth in beams and other specimens of mortar, plain and fiber-reinforced concrete.<sup>3-8</sup> Recently, the validity of this method for concrete materials has been questioned, primarily because of the diversity of results obtained in trying to estimate the crack-opening-mode stress-intensity factor,  $K_1$ . Phenomena that influence these results include: material heterogeneity, crack clo

sure,<sup>2,3</sup> the process of microcracking through and/or around aggregate particles, slow vs. fast—or unstable crack growth. An attempt was made previously to demonstrate the feasibility of the compliance-calibration method to estimating crack growth in plain-concrete beams subjected to three-point or four-point bending.<sup>3-3</sup>

Nevertheless, it has been argued<sup>6-9</sup> that the effect of aggregate interlock at the crack interfaces will produce a higher stiffness (lower compliance) than would be associated with a beam with a clean notch of the same depth as the average crack depth. Or, to put it another way, since the compliance-calibration curve for notched specimens is always higher than that for cracked specimens for a given a/W (according to the argument) then use of a



Fig. 1—Compliance variation for notched beams and presumed compliance variation for cracked beams

S.E. Swartz (SESA Member) is Professor, Department of Civil Engineering, Kansas State University, Manhattan, KS 66506. C.G. Go (SESA Member) is Visiting Associate Professor, National Chung-Hsing University, Taichung, Taiwan, Republic of China.

Paper was presented at 1983 SESA Spring Meeting held in Cleveland, OH on May 15-19, 1983.

Original manuscript submitted: August 29, 1983. Final manuscript received: February 6, 1984.



Fig. 2—Cracked test specimen, loading arrangement and dye-application arrangement, (a) test specimen and crack, (b) inverted cracked specimen for dye application



calibration curve obtained from notched specimens with compliance values obtained from cracked specimens will always lead to an *underestimation* of the actual crack length if the argument is correct. This situation is illustrated in Fig. 1 where it is seen that, for instance, at  $\frac{a}{w} = 0.6$ , the compliance for the notched beam is presumed to be higher than for the precracked beam. It is also shown how use of the compliance curve based on the

notched specimens would appear to yield estimates of  $\frac{a}{w}$ 

that are too low for cracked specimens.

The methods used previously to correlate crack-length measurements with those obtained by compliance-calibration included<sup>3,4</sup> photoelastic coatings, wire-crack-propagation gages, and visual inspection. All of these techniques measure the crack length based on surface observations. The visual-inspection technique using a low-power magnifying glass gave consistent results which also agree well with the compliance method.<sup>4</sup>

Additional work is presented here which compares crack-length estimates for plain-concrete beams in threepoint bending obtained by compliance techniques with actual crack lengths determined by dye-penetrant methods and surface inspection.

## **Test Specimens**

All test specimens had the geometry shown in Fig. 2 and were tested in three-point bending. The maximumsize aggregate used in the concrete mix was 12 mm and the cylinder compressive strength was 36 MPa. Twelve specimens were precracked. Thirteen specimens were notched with Teflon strips to simulate very closely the widths of real cracks but not the aggregate-interlock effect. Teflon strips 0.076-mm thick of various lengths were used following the procedure given in Ref. 5.

The compliance-calibration specimens were made by notching a beam at midspan to various depths using a concrete saw. Using the procedure described in Refs. 3 and 4, a load vs. crack-mouth-opening displacement (CMOD) curve was obtained for a given notch. The inverse of the initial slope of this curve gives the compliance. Compliance values obtained in this way are plotted in Fig. 3.

## **Testing Procedure**

#### Precracked Beams

The precracked beams all had small starter notches at midspan and then were precracked to desired crack lengths using control of the CMOD. The procedure followed is described fully in Refs. 4 and 5. The crack length was obtained by drawing a line on the plotter with a slope corresponding to the desired compliance. The beam was then load-cycled until a similar slope was obtained on the load vs. CMOD plot.

After a beam was precracked it was removed from the machine, turned upside down and replaced in the machine [see Fig. 2(b)]. A small load (P') which was large enough to overcome crack closure<sup>3</sup> was applied and maintained while dye (Vanish blue dye) was inserted. As the dye was applied the load was varied from zero to P' to "work" the crack surfaces to enhance dye penetration.

Immediately after the dye application, which typically took 0.6-0.9 ks (10-15 min), the beam was removed, then

reinserted into the original testing arrangement [Fig. 2(a)] and loaded to failure using load control. A load-to-failure plot is shown in Fig. 4.

After failure, the dyed surface was outlined and sketched. A photograph of a precracked and failed beam is shown in Fig. 5 and sketches of all the precracked beams are given in Fig. 6.

Further details of the dye-application technique are given in Ref. 10. Failed specimens were examined to determine if the dye penetrated uncracked areasespecially normal to the crack front-but no evidence of this was found.



Fig. 4—Typical-load vs. CMOD curves for precracked and Teflon beams



Fig. 5-Failure appearance of specimen P6

## Teflon Beams

The Teflon beams were loaded to failure using load control. Following failure the actual depth of the Teflon insert was measured. A typical plot of load vs. CMOD for these beams is given in Fig. 4.

### Results

The test results are summarized in Table 1 for the precracked beams and in Table 2 for the Teflon beams. Results of average crack depth revealed by dye and those obtained by compliance calibration are presented in Fig. 6. The following may be seen. (1) In general, the crack depth estimated by compliance calibration is greater than the average value revealed by the dye, i.e., the compliance method *overestimates* the actual crack length. This contradicts the hypotheses described previously and in Refs. 6-9. The two sets of results can be correlated by

$$\left(\frac{a}{w}\right)_{dye} = 1.00 \left(\frac{a}{w}\right)_{compliance} - 0.14 \tag{1}$$

This correlation is compared to the data in Fig. 7.

(2) The *surface cracks* revealed by the dye correlate well with the crack depth predicted by compliance cali-

#### TABLE 1-PRECRACKED BEAMS

Specimen	(1) <i>a/W</i> Compliance	(2) Average <i>a/W</i> Dye-interior	(3) Average a/W Dye-surface	(4) Maximum Load kN	K, using results in (2) and (4) kNm <sup>-3/2</sup>
P1	0.30	0.20	0.26	5.52	887
P2	0.30	0.19	0.18	5.56	894
P3	0.46	0.26	0.40	3.42	654
P4	0.49	0.31	0.53	3.56	814
<b>P</b> 5	0.65	0.48	0.75	2.18	755
P6	0.66	0.64	0.88	*	*
P7	0.30	0.18	0.26	4.81	772
P8	0.28	0.18	0.20	4.81	772
P9	0.46	0.27	0.42	3.60	713
P10	0.49	0.31	0.53	3.52	738
P11	0.65	0.55	0.81	1.60	693
P12	0.62	0.47	0.78	•	*

\*Data not taken



Fig. 6-Crack surface revealed by dye penetration

bration. This also agrees with results presented in Refs. 3 and 4.

(3) For  $\frac{a}{w}$  values greater than about 0.26 the difference

between the average crack depth in the beam interior and the average crack depth on the surface is fairly constant and is about 25 mm. It is conjectured that this may in fact represent the depth of the zone of microcracking associated with crack growth.

(4) The compliance estimates for the Teflon-inserted beams are virtually identical to the actual Teflon depths as shown in Table 2.

Opening-mode stress-intensity values  $(K_i)$  were also determined. They were calculated using the value of  $\frac{a}{w}$  associated with the initial crack and the failure load

with the aid of the unit-load  $K_i$  curve<sup>10</sup> given in Fig. 8. This is the procedure typically used at present.<sup>5,8–10</sup>

The values of  $K_1$  obtained in this way for the precracked

TABLE 2-BEAMS WITH TEFLON INSERTS

Specimen	(1) <i>a/W</i> Compliance	(2) a/W Teflon	(3) Maximum Load kN	K <sub>1</sub> using results in (2) and (3) kNm <sup>-3/2</sup>
T1	0.35	0.31	2.00	446
T2	0.31	0.31	1.96	436
Т3	0.51	0.51	1.1 <b>1</b>	412
T4	0.49	0.51	1.07	409
T5	0.65	0.69	0.42	310
T6	0.68	0.69	0.42	327
Τ7	0.32	0.30	2.14	443
T8*	0.41	0.33	2.00	446
Т9	0.52	0.50	0.80	289
T10**	0.59	0.50	1.34	<b>48</b> 2
T11	0.54	0.50	1.34	482
T12	0.72	0.73	0.32	285
T13	0.70	0.71	0.42	366

\*Teflon strip was tilted

\*\*Teflon strip did not penetrate entirely through beam

beams using  $\frac{a}{w}$  as revealed by the dye (average, interior) are presented in Table 1 and Fig. 9. It is seen these values are fairly consistent and approximately independent of  $\frac{a}{w}$ . Also plotted in Fig. 9 are  $K_i$  values obtained when  $\frac{a}{w}$  is computed from eq (1).

The values of  $K_i$  for the beams with Teflon inserts are presented in Table 2 and averaged values are shown in Fig. 9 where they are compared to those obtained from



(<sup>a</sup>) Compliance

Fig. 7—Comparison of correlation equation with data for precracked beams







the precracked beams. It is seen that the values from the precracked beams are considerably higher than those from the Teflon beams. The discrepancy between the two results increases with  $\frac{a}{w}$ . This phenomenon was reported earlier.<sup>5</sup>

## Conclusions

Based upon the work presented here the following conclusions are made.

(1) The compliance-calibration method overestimates the actual crack length. However, a reasonable correlation between the estimated crack length and that actually present is given by eq (1).

(2) When using actual crack lengths and those from eq (1), good agreement among  $K_t$  values is obtained. Furthermore, these values are reasonably consistent and

independent of  $\frac{a}{w}$  values for the range tested.

3) Thus, it is concluded that the compliance-calibration method is suitable for concrete provided a correlation similar to eq (1) is made for a given beam geometry (and possibly mix design).

#### Acknowledgments

The work presented herein was supported by the En-

gineering Experiment Station, Kansas State University. This support is gratefully acknowledged.

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